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International correspondence school
Electric engineering.

v. 1

ELECTRICITY AND MAGNETISM
ELECTRICAL MEASUREMENTS
APPLIED ELECTRICITY
BATTERIES

SCRANTON:
INTERNATIONAL TEXTBOOK COMPANY

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PREFACE.

The International Library of Technology is the outgrowth of a large and increasing demand that has arisen for the Reference Libraries of the International Correspondence Schools on the part of those who are not students of the Schools. As the volumes composing this Library are all printed from the same plates used in printing the Reference Libraries above mentioned, a few words are necessary regarding the scope and purpose of the instruction imparted to the students of—and the class of students taught by—these Schools, in order to afford a clear understanding of their salient and unique features.

The only requirement for admission to any of the courses offered by the International Correspondence Schools, is that the applicant shall be able to read the English language and to write it sufficiently well to make his written answers to the questions asked him intelligible. Each course is complete in itself, and no textbooks are required other than those prepared by the Schools for the particular course selected. The students themselves are from every class, trade, and profession and from every country; they are, almost without exception, busily engaged in some vocation, and can spare but little time for study, and that usually outside of their regular working hours. The information desired is such as can be immediately applied in practice, so that the student may be enabled to exchange his present vocation for a more congenial one, or to rise to a higher level in the one he now pursues. Furthermore, he wishes to obtain a good working knowledge of the subjects treated in the shortest time and in the most direct manner possible.

In meeting these requirements, we have produced a set of books that in many respects, and particularly in the general plan followed, are absolutely unique. In the majority of subjects treated the knowledge of mathematics required is limited to the simplest principles of arithmetic and mensuration, and in no case is any greater knowledge of mathematics needed than the simplest elementary principles of algebra, geometry, and trigonometry, with a thorough, practical acquaintance with the use of the logarithmic table. To effect this result, derivations of rules and formulas are omitted, but thorough and complete instructions are given regarding how, when, and under what circumstances any particular rule, formula, or process should be applied; and whenever possible one or more examples, such as would be likely to arise in actual practice—together with their solutions—are given to illustrate and explain its application.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and to try and anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear, is to give a picture or a diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives, have been used, according to which will best produce the desired results. Half-tones have been used rather sparingly, except in those cases where the general effect is desired rather than the actual details.

It is obvious that books prepared along the lines mentioned must not only be clear and concise beyond anything heretofore attempted, but they must also possess unequaled value for reference purposes. They not only give the maximum of information in a minimum space, but this information is so ingeniously arranged and correlated, and the indexes are so full and complete, that it can at once be made available to the reader. The numerous examples and explanatory remarks, together with the absence of long demonstrations and abstruse mathematical calculations, are of great assistance in helping one to select the proper formula, method, or process and in teaching him how and when it should be used.

This volume treats of the elements of electricity and magnetism, including a detailed description of primary and secondary batteries, and a full and complete discussion of the physical theory of the dynamo. As the subject matter here presented forms the groundwork of electrical engineering, every effort has been made to bring out those points that are essential and impress them on the mind by means of numerous examples and illustrations. The various electrical measurements have been given in an unusually clear manner, so that they can readily be understood and applied to every-day work, even by those who are not accustomed to making such measurements. Special attention has been paid to storage batteries, owing to their large and increasing use in connection with central stations, and primary batteries have been described much more fully than in ordinary textbooks. Besides being of great value to those making a specialty of electrical work, this volume will be found an excellent textbook by persons connected with electrical enterprises who wish to gain a general knowledge of electricity and magnetism.

As mentioned above, this volume is printed from the plates used in printing the Reference Libraries of the International Correspondence Schools. On account of the omission of certain papers, the material contained in which is given in better form elsewhere, there are several breaks in the continuity of the page numbers, formula numbers, article numbers, etc. This, however, does not impair the value of the volume, as the index has been reprinted and made to conform to the present arrangement.

International Textbook Company.

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PRINCIPLES OF ELECTRICITY AND MAGNETISM.

INTRODUCTORY.

2201. Electricity is the name given to that which directly causes all electrical phenomena. The word is derived from the Greek word *elektron*, meaning amber.

Although electrical science has made great advances in the last few years, the exact nature of electricity is unknown. Recent researches tend to demonstrate *that all electrical phenomena are due to a peculiar state or stress of a medium, called ether* (see Art. **1126**); that, when in this condition, the *ether* possesses *potential energy* or *capacity for doing work*, as is manifested by attractions and repulsions, by chemical decomposition, and by luminous, heating, and various other effects.

2202. All researches tend to prove that electricity is not a form of *matter*, for the only physical properties it possesses in common with material substances are *indestructibility* and *elasticity;* it does not possess *weight*, *extension*, nor any of the other physical properties of matter.

2203. Electrical science is founded upon the effects produced by *the action of certain forces upon matter*, and all knowledge of the science is deduced from these effects. The study of the fundamental principles of the science is an analysis of a series of experiments and the classification of the results, under laws and rules. It is not necessary to keep in mind any hypothesis as to the exact nature of electricity; its effects and the laws which govern them are quite similar to those of well-known mechanical and natural phenomena, and will be best understood by comparison.

2204. Electricity may appear either to reside upon the surfaces of bodies as a *charge*, under high pressure, or flow through their substance as a *current*, under comparatively low pressure.

That branch of the science which treats of charges upon the surfaces of bodies is termed **electrostatics,** and the charges are said to be **static charges.**

Electrodynamics is that branch which treats of the action of *electric currents.*

ELECTROSTATICS.

PRODUCTION OF STATIC ELECTRICITY.

2205. When a glass rod or a piece of amber is rubbed with a piece of silk or fur, the parts rubbed will be found to have the property of attracting light bodies, such as pieces of silk, wool, feathers, gold-leaf, pith, etc., which, after momentary contact, are again repelled. These attractions and repulsions are caused by a *static charge of electricity* residing upon the surfaces of those bodies. A body in this condition is said to be **electrified.**

A better experiment for demonstrating this action is to suspend a small pith-ball by a silk thread from a support or bracket, as shown in Fig. 901. Such an apparatus is spoken of as an **electric pendulum.** If a static charge of electricity be developed on a glass rod by rubbing it with silk, and the rod be brought near the pendulum, the ball will be attracted to the rod, but after momentary contact will be repelled. By this contact the ball becomes electrified, and so long as the two bodies retain their

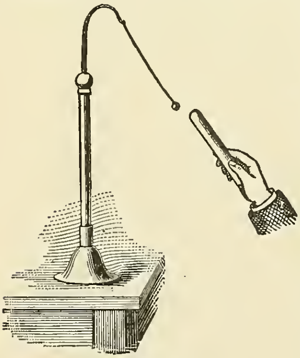


FIG. 901.

charges mutual *repulsion* will take place whenever they are brought near each other. If a stick of *sealing-wax*, electrified by being rubbed with fur, is approached to another pendulum, the same results will be produced—the ball will fly towards the wax, and after contact will again be repelled. But the charges respectively developed in these two cases are not in the same condition. For if after the pith-ball has been touched with the glass rod and repelled, the electrified sealing-wax be brought in the vicinity, *attraction* takes place between the ball and sealing-wax. Similarly, if the pendulum be charged with the electrified sealing-wax, the ball will be repelled by the wax and attracted by the glass rod.

We have, therefore, to distinguish between two kinds of electrification—that produced by rubbing glass with silk and that produced by rubbing sealing-wax with fur.

To make this distinction clear, the following designations have been adopted:

An electric charge excited upon glass by rubbing it with silk has been termed a **positive charge** (+), and that developed on resinous bodies by friction with flannel or fur a **negative charge** (−).

2206. Neither charge is produced alone, for there is always an equal quantity of both charges produced, one charge appearing on the body rubbed, and an equal amount of the opposite charge upon the rubber.

2207. The *intensity of the charge* developed by rubbing the two substances together is evidently independent of the *actual amount of friction* which takes place between the bodies. For, in order to obtain the highest possible degree of electrification from two dissimilar substances, it is only necessary to bring every portion of one surface into intimate contact with every particle, or every portion of the other surface; when this is done, no extra amount of rubbing can develop any greater charge upon either substance.

2208. From these experiments are derived the following laws:

When two dissimilar substances are placed in contact, one of them always assumes the positive and the other the negative condition, although the amount may sometimes be so small as to render its detection very difficult.

Electrified bodies with similar charges are mutually repellent, while electrified bodies with dissimilar charges are mutually attractive.

2209. Table 71 gives a list called the **electric series,** where the substances are arranged in such order that each receives a *positive* charge when rubbed with any of the bodies following, and a *negative* charge when rubbed with any of those which precede it:

TABLE 71.

THE ELECTRIC SERIES.

1. Fur.
2. Flannel.
3. Ivory.
4. Crystals.
5. Glass.
6. Cotton.
7. Silk.
8. The body.
9. Wood.
10. Metals.
11. Sealing-wax.
12. Resin.
13. Sulphur.
14. Gutta-percha.
15. Gun-cotton.

For example, *glass* when rubbed with *fur* receives a *negative* charge; but when rubbed with *silk*, it receives a *positive* charge.

ELECTROSTATIC INSTRUMENTS.

2210. The **electroscope** is an instrument for detecting static charges of electricity and for determining their condition, whether positive or negative; but not for measuring the intensity of the charges.

The pith-ball suspended by a silk thread acts as a simple electroscope. A more sensitive electroscope is shown in Fig. 902, and consists of two gold leaves suspended within a glass jar *J*, which serves to protect them from drafts of air and to support them from contact with the earth. The gold leaves *a* are supported side by side in the jar by a brass rod or wire *b* which passes through a cork in the mouth of the jar. The upper end of the brass rod is furnished with a flat metallic plate or ball *c*. An electrified body, such as the rod *d*, brought into the vicinity of the electroscope, will cause the leaves to repel one another, due to the fact that they are both similarly electrified.

To determine the condition of a charge by the electroscope: First, charge the gold leaves with a known charge, such as that developed upon glass when rubbed with silk. The leaves will spread apart, being electrified with a *positive* charge. When they are thus charged, the approach of a body which is *positively* charged will cause them to open still more widely; while on the approach of one negatively charged, they will close together.

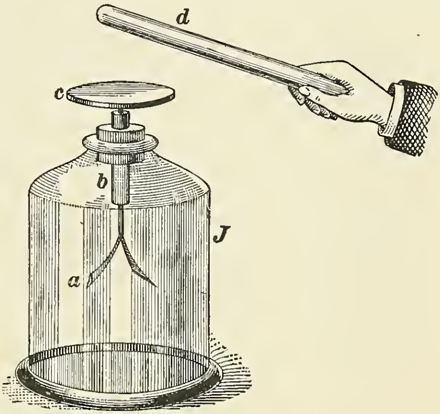


FIG. 902.

2211. The **torsion balance** is an instrument used to measure the *force* exerted between two electrified bodies.

It consists of an arm or lever of some light insulating material, such as a straw or piece of wood, provided at one end with a gilt pith-ball *n*, Fig. 903, and suspended in a glass jar by a fine silver wire. The wire passes up through a glass tube and is fastened to a brass stopper *b*, called the **torsion head.** The torsion head is graduated in degrees, and is capable of being revolved around upon the glass tube. Another gilt pith-ball *m* is fastened to the end of the vertical glass rod *a*, which is inserted through an opening in the top of the jar. A narrow strip of paper, also divided into degrees, encircles the glass jar at the level of the two pith-balls.

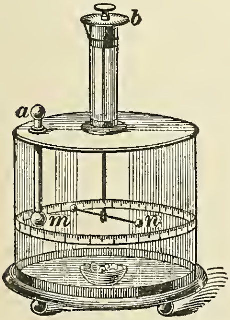


FIG. 903.

2212. *To use the torsion balance:* Turn the torsion head around until the two pith-balls *m* and *n* just touch each other. Remove the glass rod *a*, and communicate the charge to be measured to the gilt ball *m*. Replace the glass rod in the jar. The two gilt balls will touch each other momentarily, and half of the charge will pass from *m* to *n*. As both balls possess similar charges, they will immediately repel each other; the ball *n*, being driven around, twists up the wire to a certain extent. The force of torsion in the wire will eventually balance the force of repulsion, and the ball *n* will come to rest when the balls are separated by a certain distance. *In any wire, the force of torsion is proportional to the amount of twist, or, in this case, to the angle of torsion;* hence, the force exerted between the two balls can be measured by the angle described by the ball *n*.

2213. By means of the torsion balance, it is proven *that the force exerted between two bodies statically charged with electricity varies inversely as the square of the distance between them.*

Thus, suppose two electrified bodies one-fourth inch apart repel each other with a certain force; at a distance of one inch the force would only be *one-sixteenth* as great. This law is equally true for the force of attraction between two bodies with dissimilar charges.

2214. In either case, whether of attraction or repulsion, the force at any given distance is equal to the product of the two quantities of electricity on the bodies. But a unit quantity of electricity is that charge which, when placed in air at a distance of *one centimeter* from another equal and similar charge, will be repelled with a force of one *dyne*. (For values of the centimeter and dyne, see Arts. **2255** and **2262**.)

Therefore, if a certain body were charged with 4 *unit quantities* of electricity and another with 3 *unit quantities*, then the force exerted between them would be 12 times greater than if each had contained a charge of one *unit*.

CONDUCTORS AND INSULATORS.

2215. Only that part of a dry glass rod which has been rubbed will be electrified; the other parts will produce neither attraction nor repulsion when brought near an electroscope. The same is true of a piece of sealing-wax or resin. These bodies do not readily *conduct* electricity; that is, they *oppose* or *resist* the passage of electricity through them. Therefore, electricity can reside only as a *charge* upon that part of their surfaces where it is developed. Experiments show that when a *metal* receives a charge at any point, the electricity immediately passes or flows through its substance to all parts. Metals, therefore, are said to be *good conductors* of electricity. Bodies have accordingly been divided into two classes; namely, *non-conductors* or *insulators*, those bodies which offer an infinitely high *resistance* to the passage of electricity; and *conductors*, or those which offer a comparatively low resistance to its passage. This distinction is not absolute, for all bodies conduct electricity to some extent, while there is no known substance that does not offer some resistance to its flow.

2216. Electrical resistance may be defined as a general property of matter, varying with different substances, by virtue of which matter *opposes* or *resists* the passage of electricity.

2217. Conductivity is the facility with which a body transmits electricity, and is the reciprocal, or opposite, of resistance. For instance, copper is of low resistance and high conductivity; wood is of high resistance and low conductivity.

Table 72 gives a list of conducting and non-conducting substances.

2218. In dividing the different substances into two classes, it should be understood that it is done only as a guide for the student. Between these classes are many substances which might be included in either, and no hard or fast line can be drawn. The list is arranged in order of the *conductivity* of the different substances, beginning with silver, which is the best conductor known.

TABLE 72.

CONDUCTORS AND INSULATORS IN ORDER OF THEIR VALUE.

Conductors.	Insulators (Non-Conductors).	
Silver.	Dry Air.	Glass.
Copper.	Shellac.	Mica.
Other Metals.	Paraffin.	Ebonite.
Charcoal.	Amber.	India-rubber.
Plumbago.	Resin.	Silk.
Moist Earth.	Sulphur.	Paper.
Water.	Wax.	Oils.

A general idea of these values may be obtained from the fact that *water* has 6,754 million times greater resistance than copper.

ELECTROSTATIC INDUCTION.

2219. An electric charge will be *induced* in a *conductor* when that conductor is brought into the vicinity of an electrified body. This effect is termed **electrostatic induction,** and the range of space in which it can take place is an **electrostatic field.**

2220. If the conductor *A B*, Fig. 904, is supported from contact with the earth by insulators, and is then brought into the electrostatic field of the conductor *C*, but not touching *C*, which is electrified with a *positive* charge, then:

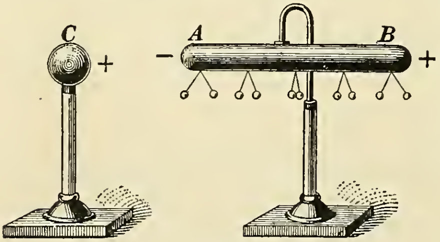


FIG. 904.

1. A charge will be produced on *A B*, as is shown by the pith-balls spreading apart.

2. This charge will be *negative* at the end A nearest C and *positive* at the end B farthest from C, as can be shown by an electroscope.

3. The charges at A and B are equal to each other ; for if the conductor $A\,B$ be removed from the vicinity of the conductor C without having touched C, the opposite charges immediately neutralize each other; that is, no electrification will be indicated by the pith-balls.

4. Again, as C is brought nearer and nearer A, the charges of opposite signs on the approaching surfaces attract each other more and more strongly until C is approached very near, and then a spark darts across the intervening space. Two charges rushing together neutralize one another, leaving the induced *positive* charge, which was formerly repelled to the end B of the conductor, as a permanent charge over all the surface of $A\,B$.

5. Or, if the conductor $A\,B$ be touched by a conductor connected to the earth when it is under the influence of C, the *positive* charge will neutralize with the earth and the *negative* charge will remain when $A\,B$ is removed from the field of C. The charge which passes to the earth from $A\,B$ is called a **free charge,** while that charge which is held by the inductive influence of C is a **bound charge.** Both free and bound charges can be *negative* or *positive*, depending upon the sign of the charge on C.

2221. When two conducting bodies, both electrified with equal dissimilar charges, are touched together momentarily, the two charges will neutralize each other, no trace of either remaining ; but if they are unequal, the smaller charge will neutralize an equal amount from the larger and leave a charge which is equal to the difference between the two original charges, the sign of the remaining charge being the same as that of the larger one. Before the bodies can be separated, the remaining charge will divide equally between the two bodies. For example, two gilt balls A and B are charged respectively with $+20$ and -4 units of electricity. When the balls are placed in contact, the -4

charge on B will neutralize a $+4$ charge on A and leave a $+16$ charge, which immediately divides equally between the two balls; that is, a charge of $+8$ units remains on each ball when they are separated.

It is found that the effect of this electrostatic induction is greatly increased by placing some other substance, such as glass or paper instead of air, between the two bodies.

2222. The facility with which a body allows electrostatic induction to act across it is called its **inductive capacity.** The inductive capacity varies with different substances, but almost all non-conductors are better than air.

2223. Any substance which allows electrostatic induction to act across it is termed a **dielectric.** All dielectrics are non-conductors. Table 73 gives a list of several non-conductors in the order of their inductive capacity values, from which it will be seen that, with two exceptions, air has the lowest inductive capacity.

TABLE 73.

INSULATORS IN ORDER OF THEIR INDUCTIVE CAPACITY VALUES.

Glass.
Shellac.
Sulphur.
Ebonite.
India-rubber.
Petroleum.
Paraffin (solid).
Carbonic Acid.
Air.
Hydrogen.
Vacuum.

2224. The **electrophorus,** Fig. 905, is an instrument devised for the purpose of obtaining an almost unlimited number of static charges of electricity from one single charge, and is based upon the principle of electrostatic induction.

It consists of two main parts: a thin cake of resinous material cast in a round metal dish or pan B, about one foot in diameter; and a round disk A, of slightly smaller diameter, made of metal and provided with a glass handle. In using the electrophorus, the resinous cake must first be beaten or rubbed with a warm piece of woolen cloth or fur. The disk or cover is then placed upon the cake, touched momentarily with the finger to liberate the *free* charge, then removed by taking it up by the handle. It is now found to be powerfully electrified with a *positive* charge; so much so, indeed, as to yield a considerable spark when the hand is brought near it. The cover may be replaced, touched, and again removed, and will thus yield any number of sparks ; the original charge on the resinous plate meanwhile remaining practically as strong as ever.

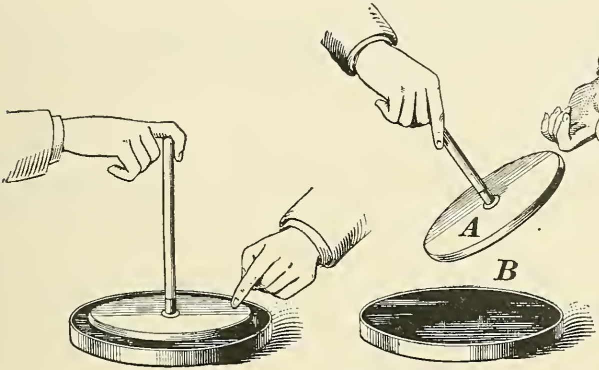


FIG. 905.

2225. A static charge of electricity is not usually distributed uniformly over the surface of conducting bodies. Experiments show that there is more electricity on the edges and corners than upon their flatter parts.

The term **electric density** is used to signify the amount or quantity of electricity residing on a small area of any part of a body, the distribution being supposed to be uniform over that small part of the surface.

The electric density is the quotient arising from dividing the total charge of electricity in units of quantity residing upon the surface of a body, by the area of the surface in square inches. For example, a charge of 240 units of electricity is imparted to a sphere, the surface area of which is 40 square inches ; then, the electric density over the surface of the sphere is $\frac{240}{40} = 6$ units of electricity per square inch.

ELECTROSTATIC MACHINES.

2226. Electrostatic machines have been devised for the purpose of obtaining larger static charges than can be developed by rubbing a glass rod or by the electrophorus. They consist, mainly, of two parts, one for producing and the other for collecting the charges.

There are three important kinds of electrostatic machines—the *cylinder*, the *plate*, and the *induction* machines.

2227. The **cylinder machine,** as usually constructed, consists of three principal parts: (1) a cylinder of glass revolving upon a horizontal axis; (2) a rubber or cushion of horsehair, to which is attached a long silk flap, and (3) an insulated metallic cylinder called a **prime conductor.** In Fig. 906 the cushion of horsehair *a*, covered with a coating of amalgam of zinc, presses against the glass cylinder *b* from behind, allowing the silk flap *s* to rest upon the upper half of the glass. The prime conductor *C* is provided at one end with a row of fine metallic spikes, and is placed in front of the machine with the row of spikes projecting towards the glass cylinder. When the glass cylinder is revolved, a *positive* charge is produced upon the glass and a *negative* charge upon the rubber. The *positive* charge is carried around upon the glass cylinder, and just before reaching a position opposite the row of spikes it acts inductively upon the prime conductor, attracting a *negative* charge to the near end and repelling a *positive* charge to the far end. When the *positive* charge arrives in front of the row of

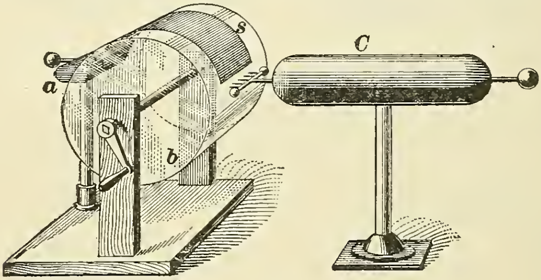


FIG. 906.

spikes, it will be neutralized by the attracting *negative* charge from the conductor, leaving the glass in a neutral condition ready to be excited again. A *positive* charge now remains upon the prime conductor, and can be utilized for other experiments.

2228. The **plate machine** is similar in all respects to the cylinder machine, with the exception that a glass or ebonite plate is used instead of the glass cylinder, and there are usually two sets of rubbers or cushions instead of one. Each set of cushions is double; that is, it is made in two parts, with the plate revolving between them. One set of cushions is placed at the top of the machine, and the other at the bottom, with silk flaps extending from each over a quadrant of the plate. The charge is collected on two prime conductors connected by a metal rod, and each is provided with a row of fine spikes at one end. They are placed in such a position that the two rows of fine spikes project towards the glass plate at opposite sides of its horizontal diameter. The electrostatic action of the machine is in all respects the same as that of the cylinder machine.

2229. The **induction machine** differs widely in its action from the two machines previously described. It requires an initial charge from some exterior source to start its action. The initial charge acts inductively across a revolving glass plate and produces other charges; these charges in turn are conveyed by the moving parts to some other point, where they increase the initial charge, or furnish a supply of electricity to a prime conductor.

The two principal machines of this class are the **Holtz** and the **Wimshurst.**

THE CONDENSER.

2230. It has been shown that opposite charges attract and hold one another; that electricity can not flow through glass, and yet can act across it by induction. If a piece of tin-foil is stuck upon the middle of each face of a thin plate of glass, and one of the pieces is electrified with a *positive* charge and the other with a *negative* charge, the two charges will attract one another, or, in other words, they are held or *bound* by each other. It will be found that these two pieces of tin-foil may be charged a great deal stronger in this manner than either of them could possibly be if they were stuck to the glass alone and then electrified. This property of retaining and accumulating a large quantity of static charges which two conductors possess when placed side by side and separated from each other by a non-conductor, is called their **capacity.**

2231. A **condenser** is an apparatus for *condensing* or accumulating a large quantity of static charges of electricity on a comparatively small surface, and consists of two conductors separated by a thin layer of some non-conducting material. One of the plates is entirely insulated from the earth, and the other is connected to it by a conductor.

The capacity of a condenser depends upon (1) the size and form of the condensing plates, (2) the thinness of the insulating material between them, and (3) the *inductive capacity* of the insulating material.

2232. A convenient form of condenser is called the **Leyden jar,** Fig. 907. It consists of a glass jar *J* coated up to a certain height on the inside and outside with tin-foil. A brass knob *a* is fixed on the end of a stout brass wire, which passes downwards through a lid or stopper of dry, well-varnished wood, and connected by a loose bit of brass chain with the inner coating of the jar.

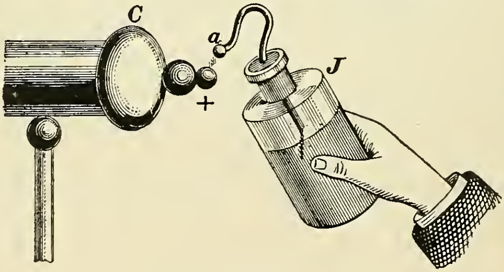


FIG. 907.

To charge the jar, the knob is held to the prime conductor *C* of an electrical machine, the jar being either held in the hand by the outer tin-foil coating or connected to the earth by a wire or chain. When a *positive* charge is thus imparted to the inner coating, it acts inductively on the outer coating, attracting a *negative* charge in the face of the outer coating nearest the glass, and repelling a *positive* charge to the outside of the outer coating. This outer charge then passes through the hand or any conductor to the earth.

2233. An **electrostatic battery** consists of a number of Leyden jars whose inside coatings are all connected together and whose outside coatings are all connected to the earth.

ELECTRODYNAMICS.

POTENTIAL AND CURRENT.

2234. In dealing with electric currents, the word *potential* will be substituted for the general and vague phrase *electrical condition.*

The term potential, as used in electrical science, is analogous with *pressure* in gases, *head* in liquids, and *temperature* in heat.

When an electrified body, *positively* charged, is connected to the earth by a conductor, electricity is said to *flow from* the body *to* the earth; and, conversely, when an electrified body *negatively* charged is connected to the earth, electricity is said to *flow from* the earth *to* that body. That which determines the *direction of flow* is the relative electrical *potential* or *pressure* of the two charges in regard to the earth.

2235. It is impossible to say with certainty in which direction electricity really flows, or, in other words, to declare which of two points has the higher and which the lower electrical potential or pressure. All that can be said with certainty is, that when there is a *difference* of *electrical* *potential*, or *pressure*, an electric current tends to flow *from* the point of higher *to* that of lower potential or pressure.

For convenience, it has been arbitrarily assumed and universally adopted that that electrical condition called *positive* is at a *higher potential* or *pressure* than that called negative, and that an electric current flows *from* a positively *to* a negatively electrified body.

2236. The zero or normal level of water is taken as that of the surface of the sea, and the normal pressure of air as that of the atmosphere at the sea-level; similarly, there is a *zero pressure* or *potential* of electricity in the earth itself. It may be regarded as a reservoir of electricity of infinite quantity, and its pressure or potential taken as zero. For this reason all electric currents have the tendency to reach this zero level, exactly as the water on the mountain top tends to flow down to the sea-level. For this reason it becomes necessary to insulate most electrical apparatus, otherwise the electric current it generates or carries will leak away to the earth. In Art. **2234** the condition which is called *positive* is assumed to be at a higher potential than the earth, and that called *negative* is assumed to be at a lower potential than the earth.

It must be understood that electricity is a condition of matter and not matter itself, for it possesses neither *weight* nor *extension*. Consequently, the statement that electricity is flowing through a conductor must not be taken too literally; it must not be supposed that any material substance, such as a liquid, is actually passing through the conductor in the same sense as water flows through a pipe. The statement that electricity is flowing through a conductor is only another way of expressing the fact that the conductor and the space surrounding it are in different conditions than usual, and that they possess unusual properties. The action of electricity, however, is quite similar in many respects to the flow of liquids, and the study of electric currents is much simplified by the analogy.

2237. *In order to produce what is called an electric current, it is first necessary to cause a difference of electrical potential or pressure between two bodies or between two parts of the same body.*

In Art. **2208** it was stated that when two dissimilar substances are simply placed in contact, one always assumes the *positive* and the other the *negative* condition; in other words, a *difference of electrical potential* is developed between the two bodies.

Placing a piece of copper and zinc in contact will develop a difference of electrical potential which can easily be detected. The same results will follow if the plates are slightly separated from each other and placed in a vessel containing saline or acidulated water, leaving a small portion of one end of each plate exposed. The exposed ends of the zinc and copper are now electrified to different degrees, or, in other words, there is a difference of electrical *potential* between the plates, one plate being at a higher potential than the other.

When the exposed ends are connected together by any conducting material, the potential between the plates tends to equalize, and a momentary rush or discharge of electricity passes between the exposed ends through the conducting material and between the submerged ends through the liquid. During its passage through the liquid, the electricity causes certain chemical changes to take place; these chemical reactions cause in their turn a fresh difference of potential between the plates, which is followed immediately by another equalizing discharge, and that by a further difference, and so on. These changes follow one another with great rapidity—so rapidly, in fact, that it is impossible to distinguish them apart, and they appear absolutely *continuous*. The equalizing flow which is constantly taking place from one plate to the other is known as a *continuous current* of electricity. Consequently, *an electric current becomes continuous when the difference of potential is constantly maintained.*

By the use of a very delicate electroscope, the exposed end of the copper will be found to be electrified with a *positive* charge and the submerged end with a *negative* charge; in the case of the zinc, the opposite conditions exist, namely, the exposed end is electrified with a *negative* charge and the submerged end with a *positive* charge. The current, therefore, will flow *from* the exposed end of the copper through the conductor *to* the exposed end of the zinc, and *from* the submerged end of the zinc through the liquid *to* the submerged end of the copper.

VOLTAIC ELECTRICITY.

2238. The two Italian physicists, *Volta* and *Galvani*, first constructed the so-called **simple voltaic** or **galvanic cell,** as shown in Fig. 908. It is an apparatus for developing a continuous current of electricity, and consists, essentially, of a vessel *A*, containing saline or acidulated water, into which are submerged two plates of dissimilar metals, *C* and *Z*, or one metal and a metalloid.

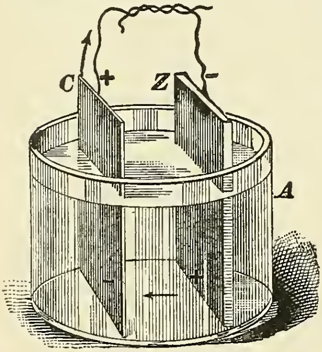


Fig. 908.

Electrolyte is the name given to the liquid which, as it transmits the current, is decomposed by it.

The two dissimilar metals, when spoken of separately, are called **voltaic elements;** when taken collectively, they are known as a **voltaic couple.**

2239. A **voltaic battery** is a number of simple voltaic cells properly joined together.

Electrodes or **poles** of a cell or battery are metallic *terminals* attached to the plates, and are used to connect the cell or battery to any exterior conductor or to another cell or battery.

It should be remembered that the polarity of that end of the plate or voltaic element which is acted upon by the electrolyte is always of opposite sign to its *electrode*. For instance, in the case of the zinc and copper, the electrode fastened to the zinc would be spoken of as the *negative* electrode of the cell, while the zinc itself would be the *positive* element of the cell, its submerged end being *positive*.

CHEMICAL ACTION IN A SIMPLE CELL.

2240. When a piece of ordinary zinc is placed alone in *sulphuric acid* diluted with water, the zinc is attacked by the acid, and a part of it is dissolved into a salt of that metal, called *sulphate of zinc*. At the same time the liquid is decomposed and *hydrogen gas* is liberated from it, coming up from around the zinc in small bubbles, and the whole mass of the liquid becomes heated. If the zinc is absolutely pure, the chemical actions take place more slowly; the bubbles of *hydrogen* do not immediately rise to the surface, but form around the zinc, protecting it from further action of the acid. By placing another metal in the water, say a piece of copper, and connecting its exposed end with that of the zinc by a conductor, the chemical actions become exceedingly vigorous again. Large quantities of *hydrogen gas* are again liberated, but instead of the bubbles appearing around the zinc, they form around the copper and come to the surface at that place; the energy which in the former case was expended in heating the liquid now appears in the form of electric energy. Whenever the connection between the exposed ends is broken, all chemical actions cease and remain inactive until the two metals are again connected.

2241. In any voltaic cell the element which is acted upon by the electrolyte will always be the positive element, and its electrode the negative electrode of the cell.

The differences of **electric potential,** however, between the different pairs of metals are not all equal. In Table 74 various materials are arranged in a series, such that each substance enumerated becomes positively electrified when placed in contact with any one below it in the series.

TABLE 74.

THE ELECTROMOTIVE SERIES.

1. + Sodium.
2. Magnesium.
3. Zinc.
4. Lead.
5. Tin.
6. Iron.
7. Copper.
8. Silver.
9. Gold.
10. Platinum.
11. – Graphite (carbon).

2242. *The term* **electromotive force,** *usually written* ***E. M. F.,*** *is employed to denote that which moves or tends to move electricity from one place to another.*

In the case of two substances placed in contact, either directly or by a liquid, the resulting electromotive force is due to the *difference of potential.* Just as in water-pipes a *difference of level* produces a *pressure*, and the pressure produces a *flow*, as soon as the water is turned on, so *difference of potential* produces *electromotive force*, and electromotive force sets up a *current*, as soon as the circuit is completed through which the electricity may flow.

2243. Any two of the substances of Table 74 form a *voltaic couple*, and produce a difference of potential when submerged in saline or acidulated water; the one standing first on the list being the *positive* element or plate and the other the *negative.* For example, if *iron* and *graphite* are used, the *iron* will be acted upon by the liquid, and will form the *positive* element; but if *iron* and *zinc* are used, the *zinc* will be acted upon by the liquid, and will form the *positive* element.

The difference of potential will be greater in proportion to distance between the positions of the two substances in the list. For example, the difference of potential developed between *zinc* and *graphite* is much greater than that developed between *zinc* and *iron;* in fact, the difference of potential developed between *zinc* and *graphite* is equal to the difference of potential developed between *zinc* and *iron* plus that developed between *iron* and *graphite.*

2244. Electricity flowing as a current differs from static charges in three important degrees, namely, its *potential* is much lower, its *actual quantity* is larger, and it is *continuous*.

A strong voltaic battery of several cells produces only a slight effect upon a gold-leaf electroscope, and, apparently, none of its parts possesses the property of attracting light substances. The *potential* of a current of electricity is comparatively so small that a voltaic battery composed of a large number of cells is not sufficient to produce a spark of more than one or two hundredths of an inch in air, whereas a small electrostatic machine will produce sparks several inches in length. If, however, the *actual quantity* of electricity is measured by its effects in decomposing water, then the quantity produced by a simple voltaic cell as small as a thimble would give greater results than that from an electrostatic machine with plates two or three feet in diameter.

An electric current can not be developed upon the surfaces of *non-conducting* substances by current electricity, as in the case of static charges, and it will never flow unless the conducting path is made entirely of *conducting* material.

2245. A number of contacts of dissimilar metals can be so arranged as to add their electrical effects together; the difference of potential then developed will be greater in proportion to the number of contacts. Such an arrangement is called a **voltaic pile.** (See Fig. 909.) It is made by placing a pair of disks of zinc (chemical symbol, *Zn*) and copper (chemical symbol, *Cu*) in contact with one another, and then laying a piece of flannel or blotting-paper, moistened with brine, upon the copper disk. The pair of disks now form a voltaic couple. Several voltaic couples are placed together, and each pair separated by a moistened piece of

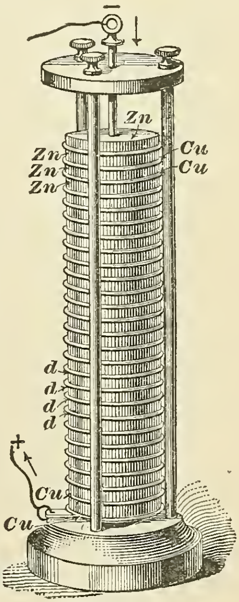


FIG. 909.

flannel or blotting-paper. One end of such a pile would then be terminated by a disk of copper and the other by a disk of zinc. The copper forms the *positive electrode* and the zinc the *negative electrode.* By joining these two electrodes together with a conductor, a current will flow *from* the positive *to* the negative *through* the conductor, and *from* the negative *to* the positive through the contacts.

THERMOELECTRIC CURRENTS.

2246. The difference of potential developed by the mere contact of two dissimilar metals varies, not only with the kind of metals and the physical condition of each, but also with their *temperature.*

The greater difference of potential developed by heat can be shown by soldering one end of a bar of copper to one end of a bar of zinc, and applying heat to the juncture so as to raise its temperature above that of the other parts of the bars. By joining the free ends together with a conductor, a current of electricity will be found to flow *from* the zinc through the contact *to* the copper; then *from* the free end of the copper *to* the free end of the zinc through the conductor. If the junction be cooled below the other parts of the bars, a current is produced in the opposite direction, that is, *from* the copper through the contact *to* the zinc, etc. Even the same metal in different physical conditions will develop a difference of potential if heated in a certain place. For instance, take a copper wire, part of which is straight and the remainder bent into a spiral, and heat the place where the spiral begins. Under these conditions, a difference of electrical potential will be developed between the two free ends.

In general, the difference of potential is larger in proportion as the difference of temperature increases. With extreme temperatures, however, this condition changes, and at a certain temperature of the junction no difference of potential whatever is noticed. This temperature is called the **neutral temperature.** When the junction is heated beyond the neutral temperature, inversion takes place, that is, the direction of the current changes.

2247. Electric currents produced by a change of temperature are called **thermoelectric currents.**

On account of the small difference of potential of thermoelectric currents, they have not been found of great practical value; in fact, they often become a source of great annoyance and error in accurate measurements with delicate instruments.

CIRCUITS.

2248. A **circuit** is a path composed of a conductor, or of several conductors joined together, through which an electric current flows from a given point around the conducting path back again to its starting-point

A circuit is **broken** or **opened** when its conducting elements are disconnected in such manner as to prevent the current from flowing.

A circuit is **closed** or **completed** when its conducting elements are so connected as to allow the current to pass.

A circuit in which the conductors have come into contact with the ground, or with some electric conductor leading to the ground, is said to be a **grounded circuit,** or is called **an earth.**

The **external circuit** is that part of a circuit which is outside or external to the electric source.

The **internal circuit** is that part of a circuit which is included within the electric source.

In the case of the simple cell, the internal circuit consists of the two metallic plates, or elements, and the liquid, or electrolyte; an external circuit would be a wire or any conductor connecting the free ends of the electrodes together.

2249. A circuit divided into two or more branches, each branch transmitting part of the current, is a **divided circuit;** the conductors forming these branches are said to be connected in **parallel** or **multiple arc.** Each branch taken separately is called **a shunt.**

Conductors are said to be connected in **series** when they are so joined as to allow the current to pass through each successively.

2250. A battery of voltaic cells is said to be connected in **multiple arc** or **parallel** when the positive electrodes of all the cells are connected to one main positive conductor and all the negative electrodes are connected to one main negative conductor, as shown in Fig. 910.

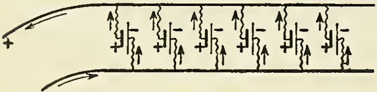


FIG. 910.

A battery of voltaic cells is said to be connected in **series** when the cells are arranged in one circuit by joining the positive electrode of one cell to the negative electrode of the adjacent one, so that the entire current passes successively through each, as shown in Fig. 911.

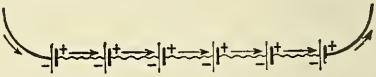


FIG. 911.

When the series and multiple connections are combined, the battery is said to be connected in **multiple-series** or **parallel-series.** This is accomplished by joining several groups in **multiple** or **parallel,** the cells in each group being connected in **series,** as shown in Fig. 912.

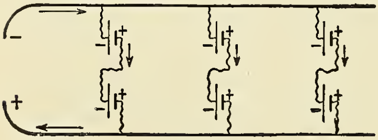


FIG. 912.

ELECTRICAL UNITS.

2251. To properly measure the various factors of an electric circuit, certain definite standards or units must be adopted, to which these factors can be compared.

In every electrical circuit there are particularly three factors, the true relation of which must be clearly understood before they can be measured.

These three factors are:

1. The force tending to move the electricity.
2. The rate of flow of the electricity.

3. The resistance which the force must overcome to produce the flow of electricity.

These factors are respectively termed:

1. The **electromotive force** (written **E. M. F.** or **E.**).
2. The **current** (written **C.**).
3. The **resistance** (written **R.**).

2252. The relation of the three principal factors will be better understood by comparison with the flow of water through a pipe. The force which causes the water to flow through the pipe is due to the *head* or *pressure;* that which *resists* the flow is the friction of the water against the inside of the pipe, and varies with circumstances. The *rate of flow*, or the *current*, may be expressed in *gallons per minute*, and is a ratio between the *head* or *pressure* and the *resistance* caused by the friction of the water against the inside of the pipe. For, as the pressure or head increases, the rate of flow or current increases in proportion; as the resistance increases, the flow or current diminishes.

In the case of electricity flowing through a conductor, the electromotive force corresponds to the pressure or head of water, and the resistance which a conductor offers to the current to the friction of the water in the pipe. The *strength of an electric current* or the *rate of flow of electricity* is also a ratio—a ratio between the electromotive force and the resistance of the conductor through which the current is flowing. This ratio, as applied to electricity, was first discovered by Dr. G. S. Ohm, and has since been called **Ohm's law.**

2253. Ohm's Law.—*The strength of an electric current in any circuit is directly proportional to the electromotive force developed in that circuit and inversely proportional to the resistance of the circuit; i. e., is equal to the quotient arising from dividing the electromotive force by the resistance.*

Ohm's law is usually expressed algebraically, thus:

$$\text{Strength of current} = \frac{\text{electromotive force}}{\text{resistance}},$$

and may be written, by utilizing the symbols given in Art. **2251,**

$$C = \frac{E}{R}.$$

When the values of any two such quantities are known, the third can be readily found ; for, by transposing,

$$E = CR \text{ and } R = \frac{E}{C}.$$

Before giving examples of the application of Ohm's law, the value and significance of the various units will be treated upon. There are two principal systems of units employed in electrical science. They are, respectively, the **fundamental units** and the **practical units.**

FUNDAMENTAL UNITS.

2254. The fundamental electrical units from which the practical units are derived, as shown later, are based on the three factors *mass*, *length*, and *time*. They are, therefore, absolutely independent of all other considerations, and the system which they form is hence termed the **system of absolute units.**

These fundamental units are, respectively,

1. The **centimeter** as the unit of **length.**
2. The **gram** as the unit of **mass.**
3. The **second** as the unit of **time.**

This system is hence often termed the **centimeter-gram-second system,** and is written **C. G. S. system.**

2255. The **centimeter** represents $\frac{1}{1{,}000{,}000{,}000}$ of the distance from the pole to the equator on the surface of the earth, and is equal to .3937 inch. Hence, 1 inch equals 2.54 *centimeters*, nearly.

2256. The **unit of mass or quantity of matter** is the **gram,** and represents the quantity of matter contained in a cubic centimeter of pure water at the temperature of its maximum density, which is 4° C., or 39.2° F., and is equal in weight to 15.432 grains.

2257. The **unit of time** is the **second,** and represents $\frac{1}{86,400}$ part of a mean solar day.

The **secondary units** derived from these fundamental units are defined as follows :

2258. The **unit of area** is the **square centimeter,** and is the area contained in a square, each of whose sides is one centimeter in length.

1 square centimeter equals .155 square inch.
1 square inch equals 6.45 square centimeters.

2259. The **unit of volume** is the **cubic centimeter,** and is the volume contained in a cube, each of whose edges is one centimeter in length.

1 cubic centimeter equals .06102 cubic inch.
1 cubic inch equals 16.387 cubic centimeters.

2260. The **unit of velocity,** or the rate at which a body moves from one position to another, is defined as the velocity of a body moving through unit distance (one centimeter) in unit time (one second). The unit of velocity is, therefore, **one centimeter per second.**

NOTE.—The word *per* in such expressions denotes that the quantity named before it is to be divided by the quantity named after it. Thus, to compute the velocity in *centimeters per second*, divide the number of centimeters by the number of seconds.

2261. The **unit of acceleration** is that acceleration which imparts unit velocity to a body in unit time, or an acceleration of **one centimeter-per-second per second.** The acceleration due to gravity imparts in one second a velocity considerably greater than this, for the velocity it imparts to falling bodies is about 981 centimeters per second (or about 32.2 feet per second). The value differs slightly in different latitudes. At New York City the acceleration of gravity is $g = 980.26$; at the Equator, $g = 978.1$; at the North Pole, $g = 983.1$.

2262. The **unit of force** is the **dyne,** and is that force which, acting on a mass of one gram for one second, gives to it a velocity of one centimeter per second. For an example of force and the application of the unit of force, see Art. **2214.**

2263. The **unit of work** is the **erg,** and is that amount of work performed when a force of one dyne is overcome through a distance of one centimeter; that is, the work done in pushing a body through a distance of one centimeter against a force of one dyne; the unit of work, the **erg,** therefore equals **one dyne centimeter.**

2264. The **unit of energy** is also the **erg;** for the energy of a body is measured by the work it can do. The unit of energy, the erg, is therefore also **one dyne centimeter.**

2265. The **unit of power** has no particular name in the C. G. S. system. It is defined as the **rate of doing work,** and is hence equal to **one erg-per-second.**

2266. The **unit of heat** (sometimes called a **calorie**) is the amount of heat required to warm one gram mass of water from 0° to 1° C.

2267. The **unit of electric-current strength** is a current of such a strength that when passing through a circuit one centimeter in length, arranged in an arc having a radius of one centimeter, it will exert a force of one dyne on a unit magnet pole placed at the center. (See Art. **2379.**)

2268. The **unit of quantity of an electric current** is that quantity which is conveyed by unit current in one second.

2269. The **unit of difference of potential** (or of **electromotive force**) is defined as the *work* done on a unit of electricity; hence unit difference of potential exists between two points when it requires the expenditure of one *erg* of work to bring a unit of + electricity from one point to the other against the electric force.

2270. The **unit of resistance** is that resistance which a conductor possesses when unit difference of potential between its two ends will allow a current of unit strength (that is, one unit of quantity per second) to flow through it.

PRACTICAL UNITS.

2271. Several of the above *absolute* units would be inconveniently large and others inconveniently small for practical use. The following *practical* units have therefore been adopted and named after distinguished men of science, such as **Ampere, Coulomb, Volta, Ohm, Joule,** and **Watt.**

THE AMPERE.

2272. The practical unit of electric current is the **ampere.** The ampere is smaller than the absolute unit of current. (Art. **2267.**)

1 absolute unit equals 10 amperes.
1 ampere equals $\frac{1}{10}$ absolute unit.

2273. The strength of an electric current can be described as a quantity of electricity flowing continuously every second, or, in other words, it is the *rate* of flow of electricity, just as the current expressed in *gallons per minute* is the rate of flow in liquids. When one *practical unit quantity of electricity* is flowing every second, continuously, then the rate of flow or the strength of the current is one *ampere;* if two unit quantities are flowing continuously every second, then the strength of the current is two amperes, and so on. It makes no difference in the number of amperes whether the current flows for a long period or for only a fraction of a second; if the quantity of electricity that would flow in one second is the same in both cases, then the strength of current in *amperes* is the same.

2274. Electricity possesses neither *weight* nor *extension*, and, therefore, an electric current can not be measured by the usual methods adopted for measuring liquids or gases. In liquids the strength of current is determined by measuring or weighing the actual quantity of the liquid which has passed between two points in a certain time and dividing the result by the time. The strength of an electric current, on the contrary, is determined *directly by the effect it produces*, and the actual quantity of electricity which has passed between two points in a certain time is afterwards calculated by multiplying the strength of the current by the time.

The principal effects produced by an electric current are magnetic attractions and repulsions, chemical decomposition, and heating and luminous effects; of these, the two most generally used for measuring are: (1) its action before a magnetic needle, and (2) its chemical actions. These methods will be treated upon in detail in the section on Electrical Measurements ; the following, however, will give an illustration of one of the methods used in measuring electric currents, and also one mode of determining the value of one *ampere:*

2275. A current of electricity, when passing through water, decomposes it into its two elements, *hydrogen* and *oxygen.* The quantity of water decomposed is proportional to the strength of the current flowing, and also to the time during which it flows. For example, if a current of two amperes flowing for one second decomposes a certain quantity of water, then a current of four amperes flowing for one second will decompose *twice* that quantity, and if it flows for two seconds it will decompose four times the original quantity. Consequently, a unit strength of current can be conventionally adopted by agreeing that it is that strength of current which will decompose a certain quantity of water in a certain time, and agreeing furthermore upon the quantity of water and the time.

2276. By universal agreement, one *ampere* is that strength of current which will decompose .00009324 gram or .0014388 grain of water in one second.

Rule.—*To find the strength of an electric current in amperes by the decomposition of water, divide the weight of the quantity of water decomposed by the time in seconds required to decompose it; if the mass of water is expressed in grams, divide the quotient by .00009324; but if expressed in grains, divide by .0014388.*

Let W = weight of water decomposed in grams;
w = weight of water decomposed in grains;
t = time in seconds required for decomposition;
C = current in amperes.

Then the strength of the current in amperes is given by the formulas:

$$C = \frac{W}{t \times .00009324}. \qquad (401.)$$

$$C = \frac{w}{t \times .0014388}. \qquad (402.)$$

2277. Rule.—*To find the quantity of water which an electric current of a given strength can decompose in a given time, multiply the strength of the current in amperes by the time in seconds during which the current flows; if the quantity of water is to be expressed in grams, multiply the product by .00009324; but if in grains, multiply by .0014388.*

Let q = quantity of water in grams;
q' = quantity of water in grains;
t = time in seconds of current flow;
C = current in amperes.

Then the quantity of water which can be decomposed by a current of C amperes in t seconds is given by the formulas:

$$q = .00009324\, C\, t. \qquad (403.)$$

$$q' = .0014388\, C\, t. \qquad (404.)$$

Example.—The current from a voltaic cell decomposes water at the rate of 1.29492 grains per hour; what is the strength of current in amperes?

Solution.— 1 hour = 3,600 seconds. By formula **402,** the strength of current

$$C = \frac{1.29492}{3{,}600 \times .0014388} = .25 \text{ ampere. Ans.}$$

Example.—Find the number of grains of water decomposed in 3 hours by a current of .6 ampere.

Solution.— 3 hours = 10,800 seconds. By formula **404,** the quantity of water decomposed

$$q' = .0014388 \times .6 \times 10{,}800 = 9.3234 \text{ grains. Ans.}$$

THE COULOMB.

2278. The practical unit of quantity of an electric current is the **coulomb.**

The coulomb is smaller than the absolute unit of quantity of current. (Art. **2268.**)

1 absolute unit equals 10 coulombs.
1 coulomb equals $\frac{1}{10}$ absolute unit.

2279. Relation of Ampere and Coulomb.—The relation of the ampere and the coulomb may be made clear by the water-flow analogy :

When a current of **water** *flows through a pipe, then the current must have a certain fixed strength, if a definite quantity of water is to be delivered at any point in a given time.*

When a current of **electricity** *flows through a conductor, then the current must have a certain* **fixed ampere strength,** *if a* **definite number of coulombs** *of current is to be delivered at any point in a given time.*

2280. *The coulomb may be further defined as being such a quantity of electricity as would pass in one second through a circuit in which the strength of the current is one ampere.*

One coulomb delivered per second therefore represents a current of one ampere.

One ampere flowing for one second will deliver one coulomb.

2281. If Q = quantity of electricity in *coulombs ;*
C = strength of current in amperes;
t = time in seconds,

then, $Q = Ct.$ **(405.)**

By transposition, $C = \frac{Q}{t}$ and $t = \frac{Q}{C}$.

EXAMPLE.—Find the quantity of electricity in coulombs that flows around a circuit in $1\frac{1}{2}$ hours, when the strength of current is 12 amperes.

SOLUTION.—By formula **405,** the quantity of electricity

$Q = Ct = 12 \times 1.5 \times 3{,}600 = 64{,}800$ coulombs. Ans.

EXAMPLES FOR PRACTICE.

1. Find the quantity of electricity in coulombs that passes in a circuit in which a current of 40 amperes flows for 55 seconds.
Ans. 2,200 coulombs.

2. Find the quantity of electricity in coulombs that passes in a circuit in which a current of 13 amperes flows for 15 minutes.
Ans. 11,700 coulombs.

3. 36,000 coulombs of electricity pass through a closed circuit in 1 hour. If the flow is uniform during that time, what is the strength of the current? Ans. 10 amperes.

4. How long will it take 72,000 coulombs of electricity to pass in a circuit in which the strength of current is 4 amperes? Ans. 5 hours.

THE OHM.

2282. The practical unit of resistance is the **ohm.**

The ohm is greater than the absolute unit of resistance. (Art. **2270.**)

1 absolute unit equals one-billionth $\left(\frac{1}{1{,}000{,}000{,}000}\right)$ of an ohm.

1 ohm equals 1 billion (1,000,000,000) absolute units.

2283. The *ohm* is the only unit in electrical measurements for which a material standard can be adopted. The basis of any system of physical measurements is generally some material standard conventionally adopted as the unit; physical measurements in each system are made by comparison with the unit of that system.

As a basis for the measurement of resistance, *Siemens* originally proposed a column of mercury having a height of 100 centimeters and a cross-section of one square millimeter, at the temperature of 0° C.; that is, at the temperature of freezing water. This column of mercury he claimed had a resistance of one ohm.

2284. The idea of utilizing a column of mercury of 1 square millimeter cross-section at 0° C. as the practical unit of resistance has been universally adopted, but the height of this column has never been exactly determined. There are, therefore, various values of the unit often found quoted. The following list gives these various values in tabular form with annotations denoting their use.

TABLE 75.

VARIOUS VALUES OF THE OHM.

Name.	Height of Mercury Column.	Cross-Section of Mercury Column.	Use.
Siemens' Unit..	100 cm.	1 sq. mm.	Out of use, because incorrect.
British Association Unit, written B. A. U...	104.8 cm.	1 sq mm.	Out of use, because incorrect.
Legal Ohm (commonly called Ohm)........	106.0 cm.	1 sq. mm.	In all technical measurements and calculations, as well as in this Course.
International Ohm.........	106.3 cm.	1 sq. mm.	Latest and most exact determination. Correct within $\frac{1}{5,000}$ part. Not yet in general use.

2285. The relative values of these units are given by the following list:

1 legal ohm = 1.0112 B. A. U.
1 legal ohm = 1.0600 Siemens' Unit.
1 B. A. U. = .9889 legal ohm.
1 B. A. U. = 1.0483 Siemens' Unit.
1 Siemens' Unit = .9540 B. A. U.
1 Siemens' Unit = .9434 legal ohm.

2286. As stated in Table 75, the legal ohm, commonly called the **ohm,** is used as yet in all technical measurements and throughout this Course, so that when the ohm is mentioned we understand thereby the resistance of a column of mercury 106 cm. (or 41.7323 inches) high, having a cross-section of 1 sq. mm. (or .00155 sq. in.) at 0° C. (or 32° F.).

2287. It very often occurs in practical work that exceedingly small resistances are to be measured, for which the ohm as a unit causes unnecessary labor, because so very large. The absolute unit of resistance, on the other hand, is too small to do very well. Therefore, to facilitate calculations and measurements, a unit is used for such work having the value of one-millionth $\left(\frac{1}{1,000,000}\right)$ of an ohm.

2288. This derived practical unit is called the **microhm.** Therefore, to express the resistance in *microhms*, multiply the resistance in ohms by 1,000,000; and, conversely, to express the resistance in ohms, divide the resistance in *microhms* by 1,000,000. For example, $.75$ ohm $= .75 \times 1,000,000 =$ 750,000 microhms, or 750,000 microhms $= \frac{750,000}{1,000,000} = .75$ ohm.

2289. Another similarly derived practical unit is the **megohm,** devised to facilitate calculations and measurements of exceedingly large resistances, and is equal to 1,000,000 ohms. Therefore, to express the resistance in *megohms*, divide the resistance in ohms by 1,000,000; and, conversely, to express the resistance in ohms, multiply the resistance in megohms by 1,000,000.

For example, 850,000 ohms $= \frac{850,000}{1,000,000} = .85$ megohm, or .85 megohm $= .85 \times 1,000,000 = 850,000$ ohms.

The megohm is used mainly in the determination of the resistance of non-conductors and insulators.

EXAMPLES FOR PRACTICE.

1. Give the equivalent resistance in microhms of .00425 ohm.
Ans. 4,250 microhms.
2. Give the equivalent resistance in ohms of 375 microhms.
Ans. .000375 ohm.
3. Give the equivalent resistance in megohms of 4,560,000 ohms.
Ans. 4.56 megohms.
4. Give the equivalent resistance in ohms of 62.5 megohms.
Ans. 62,500,000 ohms.

RESISTANCE.

2290. The resistance which all substances offer to the passage of an electric current is one of the most important quantities in electrical measurements. Resistance is that attribute of a conductor or of a circuit which determines the strength of the electric current that can be sent through the conductor or the circuit, around which a constant difference of potential is maintained, as shown by *Ohm's* law, Art. **2253.**

2291. If a given conductor offers a resistance of 2 ohms to a current of 1 ampere, it offers the same amount, no more nor no less, to a current of 10 amperes. Hence we have the

Rule.—*The resistance of a given conductor is always constant at the same temperature, irrespective of the strength of current flowing through it or the electromotive force of the current.*

2292. When it is required to find the resistance of a conductor of which the length is varied, though all other conditions remain unchanged, the following formula may be used :

$$r_1 : r_2 :: l_1 : l_2, \text{ or } r_2 = \frac{r_1 \times l_2}{l_1}. \qquad \textbf{(406.)}$$

In this formula,

r_1 = the original resistance;
r_2 = the required or changed resistance;
l_1 = the original length;
l_2 = the changed length.

2293. As in all examples of proportion, the two lengths must be reduced to the same unit. We then have the

Rule.—*The resistance of a given conductor increases as the length of the conductor increases; that is, the resistance of a conductor is directly proportional to its length.*

EXAMPLE.—Find the resistance of 1 mile of copper wire, if the resistance of 10 feet of the same wire is .013 ohm.

SOLUTION.— $r_1 = .013$ ohm; $l_1 = 10$ feet, and $l_2 = 1$ mile $= 5{,}280$ feet. Then, by formula **406,** the required resistance

$$r_2 = \frac{.013 \times 5{,}280}{10} = 6.864 \text{ ohms. Ans.}$$

EXAMPLE.—Find the resistance of 11 in. of a German silver wire, if the resistance of 100 feet of the same wire is 2.4 ohms.

SOLUTION.— $r_1 = 2.4$ ohms ; $l_1 = 100 \times 12 = 1{,}200$ in. ; $l_2 = 11$ in. By formula **406,** the required resistance

$$r_2 = \frac{2.4 \times 11}{1{,}200} = .022 \text{ ohm. Ans.}$$

EXAMPLES FOR PRACTICE.

2294. 1. Find the resistance per foot of a wire, if the resistance of 1 mile of the wire is 14.75 ohms. Ans. .002793 ohm.

2. If the resistance of 18 in. of a certain piece of wire is .027 ohm, what is the resistance of 1,020 feet of the same wire ? Ans. 18.36 ohms.

2295. If the sectional area of a conductor is increased, and other conditions remain unchanged, the resistance of the conductor will be decreased. For instance, if the sectional area be doubled the resistance is halved, and, conversely, if the sectional area is halved the resistance is doubled. The resistance of a conductor, therefore, grows with decreasing sectional area, and diminishes with increasing sectional area. This may be expressed by the general rule:

2296. Rule.—*The resistance of a conductor varies inversely as its sectional area.*

The value of the resistance of a conductor for any change in its sectional area may be obtained from the following formula :

$$r_1 : r_2 :: a_2 : a_1, \text{ or } r_2 = \frac{r_1 a_1}{a_2}. \qquad \textbf{(407.)}$$

In this formula,

r_1 = the original resistance;
r_2 = the required resistance;
a_1 = the original sectional area;
a_2 = the changed sectional area.

EXAMPLE.—The resistance of a conductor whose sectional area is .025 sq. in. is .32 ohm ; what would be the resistance of the conductor if its sectional area were increased to .125 sq. in., other conditions remaining unchanged ?

SOLUTION.— $r_1 = .32$ ohm ; $a_1 = .025$ sq. in., and $a_2 = .125$ sq. in. Then, by formula **407,** the required resistance

$$r_2 = \frac{.32 \times .025}{.125} = .064 \text{ ohm. Ans.}$$

EXAMPLE.—The sectional area of a conductor is .01 sq. in. and its resistance is 1 ohm ; if its sectional area is decreased to .001 sq. in., and other conditions remain unchanged, what will be its resistance ?

SOLUTION.— $r_1 = 1$ ohm ; $a_1 = .01$ sq. in., and $a_2 = .001$ sq. in. By formula **407,** the required resistance

$$r_2 = \frac{1 \times .01}{.001} = 10 \text{ ohms. Ans.}$$

2297. The resistance of a conductor is independent of the shape of its cross-section. For example, this cross-section may be of circular, square, rectangular, or irregular shape; if the sectional area is the same in all cases, the resistances will be the same, other conditions being similar. When comparing the resistances of copper wires of circular cross-section, it is usually simpler to express the copper wire by its diameter than by its area. The sectional area of any circular cross-section is, however, proportional to the square of the diameter; for the sectional area = diameter^2 × .7854. We therefore have the rule:

2298. *The resistance of a conductor of circular cross-section is inversely proportional to the square of its diameter.*

Formula **407** may, therefore, be rewritten as follows:

$$r_1 : r_2 :: d^2 : D^2, \text{ or } r_2 = \frac{r_1 D^2}{d^2}. \qquad \textbf{(408.)}$$

In this formula,

r_1 = the original resistance;
r_2 = the required resistance;
D = the original diameter;
d = the changed diameter.

EXAMPLE.—The resistance of a round copper wire .12 in. in diameter is .64 ohm; find the resistance of the conductor when its diameter is increased to .24 in., the other conditions remaining unchanged.

SOLUTION.— $r_1 = .64$ ohm; $D = .12$ in., and $d = .24$ in.
Then, by formula **408,** the required resistance

$$r_2 = \frac{.64 \times .12^2}{.24^2} = .16 \text{ ohm. Ans.}$$

EXAMPLE.—The diameter of a round wire is .1 in. and its resistance is 2 ohms; what would be its resistance if its diameter were decreased to .02 in., and the other conditions remained unchanged?

SOLUTION.— $r_1 = 2$ ohms; $D = .1$ in., and $d = .02$ in.
By formula **408,** the required resistance

$$r_2 = \frac{2 \times .1^2}{.02^2} = \frac{2 \times .01}{.0004} = 50 \text{ ohms. Ans.}$$

EXAMPLES FOR PRACTICE.

The resistance of a piece of round copper wire .001 in. in diameter and 1 foot long is 10.8 ohms; use the same quality of copper, and solve the following problems:

1. Find the resistance of 1,200 feet of round copper wire .102 in. in diameter. Ans. 1.2457 ohms.

2. Find the resistance of 1 mile of round copper $\frac{1}{8}$ in. in diameter. Ans. 3.6495 ohms.

3. Find the resistance of 1,500 feet of square copper wire .1 in. on a side. Ans. 1.2723 ohms.

4. Find the resistance of 100 yards of copper wire .12 in. wide by .09 in. thick. Ans. .23562 ohm.

Note.—The temperature of the copper in all the above problems is assumed to be equal.

2299. The Resistance of Metals.—It was stated in Art. **2216** that the resistance varies in different substances; that is, one substance offers a higher resistance to a current of electricity than another. In order to compare the resistances of different substances, however, the dimensions of the pieces to be measured must be equal. For, by changing its dimensions, a good conductor may be made to offer the same resistance as an inferior one. Under like conditions, annealed silver offers the least resistance of all known metals or conductors. Soft annealed copper comes next on the list, and then follow all other metals and conductors.

2300. The resistance of a given conductor, however, is not always constant; it changes with the temperature, and also with the physical condition of the conductor. In all metals the resistance *increases* as the temperature rises; in liquids and carbons the resistance *decreases* as the temperature rises; and in non-conductors the resistance decreases as the temperature rises. The amount of variation in the resistance caused by a change in temperature will be treated upon under Electrical Measurements; it is a small factor, and can be neglected for the present.

2301. A list of the common metals is given in Table 76 in the order of their relative resistances, beginning with silver as offering the least resistance. The first column of figures gives the actual resistance in *microhms* of 1 cubic inch of the corresponding metal at 32° Fahrenheit, or the freezing-point of water. By the resistance of 1 cubic inch is meant the resistance of a piece of the conductor, the length of which is 1 inch, and whose sectional area is 1 sq. in. Therefore, the resistance of any conductor of known dimensions which is made of one of the metals in the list can be determined by applying the formulas in Arts. **2296**

and **2298.** The second column of figures gives the relative resistances of the different metals compared with silver. For example, the resistance of mercury is 62.73 times the resistance of silver, or the resistance of iron is 6.46 times the resistance of silver.

TABLE 76.

Name of Metal.	Resistance in Microhms of 1 Cu. In. at 32° F.	Relative Resistance.
Silver, annealed	.5921	1.000
Copper, annealed	.6292	1.063
Silver, hard drawn	.6433	1.086
Copper, hard drawn	.6433	1.086
Gold, annealed	.8102	1.369
Gold, hard drawn	.8247	1.393
Aluminum, annealed	1.1470	1.935
Zinc, pressed	2.2150	3.741
Platinum, annealed	3.5650	6.022
Iron, annealed	3.8250	6.460
Nickel, annealed	4.9070	8.285
Tin, pressed	5.2020	8.784
Lead, pressed	7.7280	13.050
German Silver	8.2400	13.920
Antimony, pressed	13.9800	23.600
Mercury	37.1500	62.730
Bismuth, pressed	51.6500	87.230

EXAMPLES FOR PRACTICE.

1. Find the resistance in ohms of a round column of mercury 70 inches high and .05 inch in diameter. Ans. 1.3244 ohms.

2. Find the resistance in ohms of 1,000 feet of round German silver wire .2 inch in diameter. Ans. 3.1476 ohms.

3. Find the resistance in microhms of a cubic foot of bismuth, pressed. Ans. 4.3042 microhms.

4. Find the resistance in ohms of 1 mile of square iron wire (annealed) .1 inch on a side. Ans. 24.2352 ohms.

2302. In a simple voltaic cell, the *internal* resistance, that is, the resistance of the two plates and the electrolyte, is of great importance, for it determines the maximum strength of current that can possibly be obtained from the cell. In the common forms of cells, the internal resistance may be excessively large, owing to the resistance of the electrolyte, the relative resistance of ordinary liquids used as electrolytes being from 1 to 20 million times that of the common metals. In liquids, as in all conductors, the resistance increases as the length of the circuit increases, and diminishes as its sectional area increases. Consequently, the internal resistance of a simple voltaic cell is reduced by decreasing the distance between the two plates or elements and by increasing their active surfaces.

The internal resistance of the ordinary forms of cells varies from about .2 to 20 ohms.

THE VOLT.

2303. The practical unit of electromotive force, or difference of potential, is the **volt.**

The volt is greater than the absolute unit of electromotive force. (Art. **2269.**)

1 absolute unit equals one one-hundred-millionths $\left(\frac{1}{100,000,000}\right)$ of a volt.

1 volt equals one hundred million (100,000,000) absolute units.

2304. *The volt may be further defined as being that E. M. F. which will cause a current of one ampere to flow against the resistance of one ohm.*

2305. The volt is the measure of the electromotive force, which has been defined and explained in Arts. **2242** and **2252.**

The various terms **electromotive force, pressure, difference of potential,** and **voltage** are, in general, used to signify the same thing; namely, that force which tends to move a current of electricity against the resistance of a conductor.

2306. The maximum difference of potential developed by any voltaic couple (see Art. **2243**) placed in any electrolyte is about 2.25 volts ; in the common forms of cells, the difference of potential developed averages from .75 to 1.75 volts.

2307. The determination of the value of the E. M. F. in any circuit is made by applying Ohm's law (see Art. **2253**), which gives the E. M. F. accurately when the resistance and current are known. Measuring instruments, which will be described under Electrical Measurements, have been devised upon the principle of Ohm's law, to indicate the E. M. F. directly.

OHM'S LAW APPLIED TO CLOSED CIRCUITS.

2308. Ohm's law, as shown in Art. **2253,** expresses the relation between the three fundamental units of resistance, electrical pressure, and current. If any two of these values are known, the third is found by solving the simple equation of their relation. Before applying this law, however, the following four facts should be carefully noted :

2309. **I.**—*The strength of a current* (C) *is the same in all parts of a closed circuit, except in the case of divided circuits.*

II.—*In the case of a divided circuit, the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

III.—*The resistance* (R) *is the total resistance of the circuit, that is, the sum of the resistances of the internal circuit and of the external circuit, or its equivalent.*

IV.—*The electromotive force* (E) *in a closed circuit is the total generated difference of potential in that circuit.*

The law may now be stated by the following rules and formulas:

2310. **Rule I.**—*The strength in amperes of a current* (C) *flowing in a closed circuit, when the electromotive force* (E) *and the total resistance* (R) *are known, is found by dividing the electromotive force in volts by the total resistance in ohms; that is,*

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}, \text{ or } C = \frac{E}{R}. \qquad \textbf{(409.)}$$

Rule II.—*The total resistance* (R) *in ohms of a closed circuit, when the electromotive force* (E) *and the current* (C) *are known, is found by dividing the electromotive force in volts by the current in amperes; that is,*

$$\text{Resistance} = \frac{\text{electromotive force}}{\text{current}}, \text{ or } R = \frac{E}{C}. \qquad \textbf{(410.)}$$

Rule III.—*The total electromotive force* (E) *in volts developed in a closed circuit, when the current* (C) *and the total resistance* (R) *are known, is found by multiplying the current in amperes by the total resistance in ohms; that is,*

$$\text{Electromotive force} = \text{current} \times \text{resistance, or}$$

$$E = C R. \qquad \textbf{(411.)}$$

2311. The following examples show the application of Ohm's law as given by the formulas of the preceding article:

Example.—What current can be made to flow through a circuit having a resistance of 10 ohms, if an E. M. F. of 100 volts is applied?

Solution.— $E = 100$; $R = 10$; hence, by formula **409,** the required current

$$C = \frac{100}{10} = 10 \text{ amperes. Ans.}$$

Example.—What resistance can be overcome by a current of 50 amperes, if the electromotive force is 500 volts?

Solution.— $C = 50$; $E = 500$; hence, by formula **410,** the required resistance

$$R = \frac{500}{50} = 10 \text{ ohms. Ans.}$$

Example.—What voltage is required to send a current of 25 amperes through a resistance of 4 ohms?

Solution.— $C = 25$; $R = 4$; hence, by formula **411,** the required voltage

$$E = 25 \times 4 = 100 \text{ volts. Ans.}$$

Example.—The two electrodes of a simple voltaic cell are connected together by a copper wire, the resistance of which is 1 ohm. If the internal resistance of the cell is 4 ohms and the electromotive force developed is 2 volts, what is the strength of the current in the circuit?

Solution.—Let $r_i =$ the internal resistance and $r_e =$ the external resistance; that is, the resistance of the copper wire. Then,

$$R = r_i + r_e = 4 + 1 = 5.$$

By formula **409,** the current

$$C = \frac{E}{R} = \frac{2}{5} = .4 \text{ ampere flowing through the circuit. Ans.}$$

Example.—The total electromotive force developed in a closed circuit is 1.2 volts and the strength of the current flowing is .3 ampere; find the total resistance of the circuit.

Solution.—By formula **410,**

$$R = \frac{1.2}{.3} = 4 \text{ ohms. Ans.}$$

Example.—The internal resistance of a certain dynamo-electric machine is 10.9 ohms and the external resistance is 73 ohms; the voltage of the machine is 839 volts. Find the strength of the current flowing in the circuit.

Solution.— $r_i = 10.9$; $r_e = 73$; $R = 10.9 + 73 = 83.9$. By formula **409,**

$$C = \frac{839}{83.9} = 10 \text{ amperes. Ans.}$$

EXAMPLES FOR PRACTICE.

1. The current from a simple voltaic cell decomposes water at the rate of 2.58984 grains per hour, and the total resistance of the circuit through which the current flows is 2 ohms. Find (*a*) the strength of the current, and (*b*) the total electromotive force developed by the cell.

$$\text{Ans.} \begin{cases} (a)\ .5 \text{ ampere.} \\ (b)\ 1 \text{ volt.} \end{cases}$$

2. A battery of 10 cells connected in *series* generates a total electromotive force of 12 volts. If the resistance of each cell is 4 ohms and the resistance of an external circuit is 8 ohms, what is the strength of current flowing in the circuit? Ans. .25 ampere.

3. Given,

Internal resistance = 4 ohms.
Electromotive force = 1.5 volts.
Current = .2 ampere.

Find the external resistance. Ans. 3.5 ohms.

4. Given.

Electromotive force = 24 volts.
Current = .6 ampere.

If the external resistance is 3 times the internal, what is the resistance of each?

Ans. { External, 30 ohms.
Internal 10 ohms.

DROP OF POTENTIAL.

2312. Referring again to the flow of water in pipes, we may tabulate the analogies as given in Table 77, a careful study of which will do much to assist the understanding of what is to follow.

2313. The fourth analogy of the table states that the loss of pressure or E. M. F., due to the resistance of conductor, is termed **drop of potential.** This drop may be made clearer by the following:

Let Fig. 913 represent a tank *T* of water with a horizontal discharge-pipe *E N*, which is provided with open vertical tubes at *a*, *b*, *c*, etc. If the outlet at *N* is closed the water in the vertical tubes will rise to the height of the water in the tank; but if the water is allowed to flow freely from the outlet at *N*, then the height of the water in the tubes will be represented by the inclined line at *a'*, *b'*, *c'*, etc. The *pressure* or *head* of the water, which is measured by the height of the water in the tubes, decreases in the direction in which the water is flowing, so that the water which leaves the discharge outlet at *N* has considerably less pressure than the water entering at *E*.

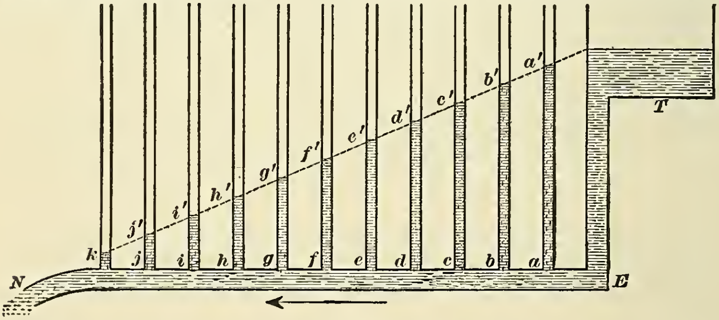


FIG. 913.

TABLE 77.

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY.

	Water in Pipes.	Electricity in Conductors.
I.	Difference of level tends to make water fall from the upper level to the lower level.	Difference of potential tends to make electric current fall from point of high potential to point of low potential.
II.	Difference of level hence acts as a pressure tending to cause a flow.	Difference of potential or E. M. F., hence acts as a pressure tending to cause a flow.
III.	If not entirely obstructed, this pressure actually produces a flow of water.	If not entirely obstructed, this pressure or E. M. F. actually produces a flow of current.
IV.	Some of this pressure is lost by friction of the water against inside walls of pipe.	Some of this pressure is lost by the electrical resistance of the conductor. The loss is called *drop of potential.*
V.	This loss by friction is directly proportional to the length of the pipe, and inversely proportional to the diameter of the pipe.	This loss or drop due to resistance is directly proportional to the length of the conductor, and inversely proportional to its area of cross-section.
VI.	No quantity of water can flow through a pipe without suffering some loss in this manner; in other words, there is no such thing as an absolutely frictionless pipe.	No quantity of electricity can flow through a conductor without suffering some loss in this manner ; in other words, there is no such thing as an absolutely resistanceless conductor.

2314. The same action takes place in a current of electricity flowing along a conductor, and can also be graphically shown. In Fig. 914, B represents a voltaic battery with the *negative* electrode connected directly to the earth at E, and the *positive* electrode to a long conductor $A\ L$, which is also connected to the earth at E'. The battery may be regarded as a machine which raises the pressure or potential of electricity from zero (or that of the earth) to a height equal to the distance $a\ a'$; or, in other words, the distance $a\ a'$ represents the total *electromotive force* of the battery. If the circuit is *opened* or *broken* between L and E so that no current flows, then the difference of potential between the conductor and the earth is the same at all points along the conductor, and is represented by the distances between the line $C\ D$ and the conductor $A\ L$.

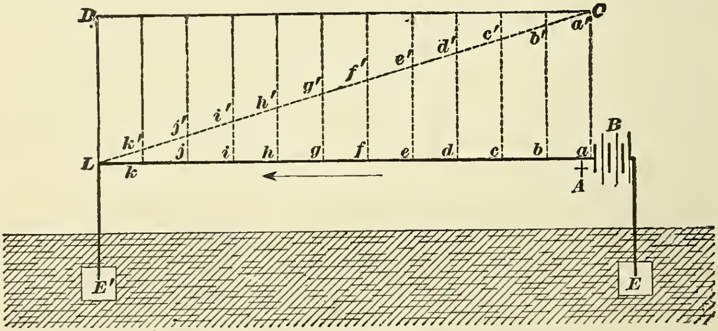


FIG. 914.

But when a current is allowed to flow along the conductor, the difference of potential between the conductor and the earth *decreases in the direction in which the current is flowing*. The vertical distances $b\ b'$, $c\ c'$, $d\ d'$, etc., represent this difference of potential at the points b, c, d, etc., along the conductor. The *loss* or *drop* of potential is represented by the vertical distances between the inclined line $C\ L$ and the horizontal line $C\ D$. This *loss* or *drop* also represents the difference of potential between the point a and any other point along the conductor. For example, at h the difference of potential between the conductor at that point and the earth is represented by the distance $h\,h'$; the *loss* or *drop* of potential is represented by the vertical distance between h' and the horizontal line $C\,D$, which distance also *represents the difference of potential existing between the points a and h.*

2315. The graphical method of determining the difference of potential is seldom used. *Ohm's law* not only gives the strength of the current in a closed circuit, but also the difference of potential in *volts* along that circuit. The difference of potential (E') in volts between any two points along a circuit is equal to the product of the strength of the current (C) in amperes and the resistance (R') in ohms of that part of the circuit between those two points; or $E' = C\,R'$, which is an example of the use of formula **411.** E' also represents the *loss* or *drop* of potential in volts between the two points. If any two of these quantities are known, the third can be readily found; for, by transposing, $C = \frac{E'}{R'}$ and $R' = \frac{E'}{C}$, as already given in formulas **409** and **410.**

EXAMPLE.—Fig. 915 represents part of a circuit in which a current of 2.5 amperes is flowing. The resistance from a to b is 10 ohms; from b to c, 15 ohms, and from c to d, 20 ohms. Find the difference of potential between a and b, b and c, c and d, and a and d.

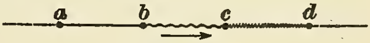


FIG. 915.

SOLUTION.—Since, by formula **411**, $E' = C\,R'$, then

The difference of potential between

a and b is $2.5 \times 10 = 25$ volts;
b and c is $2.5 \times 15 = 37.5$ volts;
c and d is $2.5 \times 20 = 50$ volts;
a and d is $25 + 37.5 + 50 = 112.5$ volts;

or, in other words, the *loss* or *drop* in potential between a and d is 112.5 volts.

2316. In a great many cases, it is desirable to have the current flow from the source a long distance to some electric receptive device, and return without causing an excessive drop or loss of potential in the conductors leading to and from the two places. In such circuits, the greater part of the total generated electromotive force is expended in the receptive device itself, and only a small fraction of it is lost in the rest of the circuit. Under these conditions, it is customary to decide upon a certain drop or loss of potential beforehand, and from that and the current calculate the resistance of the two conductors.

Example.—It is desired to transmit a current of 10 amperes to an electrical device situated 1,000 feet from the source; the total generated E. M. F. is 110 volts, and only 5% of this potential is to be lost in the conductors leading to and from the two plants. Find (*a*) the total resistance of the two conductors, and (*b*) the resistance per foot of the conductors, assuming each to be 1,000 feet long.

Solution.— 5% of 110 volts $= 110 \times .05 = 5.5$ volts, which represents the total drop or loss of potential on the two conductors. Let $E' = 5.5$ volts; $C = 10$ amperes, and $R' =$ the total resistance of the two conductors. Then, by formula **410**, $R' = \frac{E'}{C} = \frac{5.5}{10} = .55$ ohm. (*a*) Ans. The resistance per foot of the conductor is found by formula **406**. In this case, $r_1 = .55$ ohm; $l_1 = 2{,}000$ feet; $l_2 = 1$ foot. Then, the resistance per foot $= r_2 = \frac{.55 \times 1}{2{,}000} = .000275$ ohm. (*b*) Ans.

EXAMPLES FOR PRACTICE.

1. In a part of a closed circuit, the drop or loss of potential caused by the resistance of the conductor is 10 volts. If the current flowing is 4 amperes, what is the resistance of that part of the circuit?

Ans. 2.5 ohms.

2. The total generated electromotive force in a circuit is 220 volts. A current of 10 amperes is transmitted to and from a receptive device situated 110 feet from the source, with a loss of potential of 10%. (*a*) Find the total resistance of the two conductors leading to and from the two places, and (*b*) find the resistance per foot of each conductor, assuming each to be alike and 110 feet long.

Ans. $\begin{cases} (a) \text{ 2.2 ohms.} \\ (b) \text{ .01 ohm per foot.} \end{cases}$

TOTAL AND AVAILABLE E. M. F.

2317. The difference of potential between the two electrodes of a simple voltaic cell when no current is flowing, that is, when the circuit is *open*, is always equal to the total electromotive force developed within the cell; but when a current is flowing, that is, when the circuit is *closed*, a certain amount of potential is expended in forcing the current through the internal resistance of the cell itself. Consequently, the difference of potential between the two electrodes when the circuit is closed is always smaller than when the circuit is open. This difference of potential when the circuit is closed is sometimes called the *available* or *external* electromotive force, to distinguish it from the *internal* or total generated electromotive force.

2318. The available electromotive force is equal to the difference between the total generated electromotive force and the potential expended in forcing the current through the internal resistance when the circuit is closed. From Ohm's law, this loss or drop of potential in the cell itself is equal to the product of the internal resistance and the strength of current flowing.

Let E = total generated E. M. F.;
E' = available E. M. F.;
C = current flowing when the circuit is closed;
r_i = internal resistance of the cell;
r_e = an external resistance.

The drop or loss of potential in the cell $= C r_i$ and $E' = E - C r_i$.

2319. For example, in a voltaic cell the total generated E. M. F. is 2 volts, and the internal resistance is 4 ohms. If the two electrodes are connected to an external resistance of 6 ohms, a current of .2 ampere will flow through the circuit, since $C = \frac{E}{r_i + r_e} = \frac{2}{4 + 6} = .2$ ampere. The loss or drop of potential in the cell $= C r_i = .2 \times 4 = .8$ volt. Then, $E' = E - C r_i = 2 - .8 = 1.2$ volts, which is the electromotive force available to force the current of .2 ampere through the external resistance of 6 ohms, since $C r_e = .2 \times 6 = 1.2$ volts.

OHM'S LAW APPLIED TO DERIVED CIRCUITS.

2320. *A derived or shunt circuit is a branch or additional circuit provided at any part of a circuit through which the current branches or divides, part flowing through the original circuit and part through the new branch.*

A derived circuit is in multiple circuit with the circuit from which it is derived.

In the case of branched circuits, each of the branches acts as a derived circuit to the others. Any number of additional branches may thus be provided.

2321. In treating upon derived or shunt circuits, only that part of the circuit will be considered which is divided into branches and each branch transmitting part of the current; the rest of the circuit is assumed to be closed through some electric source; as, for instance, a voltaic battery.

Before applying Ohm's law to derived circuits, it is necessary that the meaning of **conductivity** should be thoroughly understood. In Art. **2217** it was stated that conductivity is the *inverse* of resistance; or, in other words, it is the *reciprocal* of resistance.

Therefore, since the conductivity is greater the less the resistance, the conductivity may be defined as being equal to $\frac{1}{R}$; that is, the reciprocal of the resistance.

2322. The conductivity of any conductor is, therefore, unity divided by the resistance of the conductor; and, conversely, the resistance of any conductor is unity divided by the conductivity of that conductor. For example, if the resistance of a circuit is 2 ohms, the conductivity is represented by $\frac{1}{R} = \frac{1}{2}$; if the resistance is increased to 4 ohms, the conductivity would be only one-half as much as in the first case, and would now be $\frac{1}{4}$.

There is no established unit of conductivity; it is used merely as a convenience in calculation.

2323. Fig. 916 represents a derived circuit of two branches.

Let r_1 and r_2 = the separate resistances of the branches, respectively;

c_1 and c_2 = the currents in each branch, respectively;

C = the current in the main circuit.

Then, $$c_1 + c_2 = C.$$

When the current flows from a to b, if the resistances r_1 and r_2 are equal, the current will divide equally between the two branches. Thus, if a current of 2 amperes is flowing in the main circuit, 1 ampere will flow through each branch.

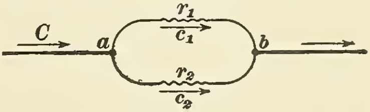


FIG. 916.

When the resistances are unequal, the current will divide inversely as the respective resistances of the two branches; or, since the conductivity is the reciprocal of the resistance, *the current will divide in proportion to their respective conductivities.*

In Fig. 916 the conductivities of the two branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$, respectively.

Therefore, $$c_1 : c_2 :: \frac{1}{r_1} : \frac{1}{r_2}, \text{ or } \frac{c_1}{c_2} = \frac{r_2}{r_1}.$$

EXAMPLE.—Given $C = 60$ amperes; $r_1 = 2$ ohms; $r_2 = 3$ ohms. Find c_1 and c_2.

SOLUTION.— $\frac{c_1}{c_2} = \frac{r_2}{r_1}$, or $\frac{c_1}{c_2} = \frac{3}{2}$, or $c_1 = \frac{3c_2}{2}$. But $c_1 + c_2 = 60$, or $c_1 = 60 - c_2$. Substituting for the value of c_1 gives $60 - c_2 = \frac{3c_2}{2}$. Transposing gives $5c_2 = 120$, or $c_2 = 24$ amperes. Ans. $c_1 = 60 - 24 = 36$ amperes. Ans.

2324. It is clear that two conductors in parallel will conduct an electric current more readily than one alone; that is, their *joint* conductivity is greater than either of their separate conductivities taken alone. This being the case, their resistances must follow the inverse law; viz., the *joint* resistance of two conductors in parallel must be less than either of their separate resistances taken alone Hence:

Rule.—*If the separate resistances of two conductors are equal, their joint resistance when connected in parallel is one-half of their separate resistance.*

2325. When the separate resistances of two conductors in parallel are *unequal*, the determination of their joint resistance when connected in parallel involves some calculation.

In Fig. 916 the conductivities of the branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$, respectively.

Their joint conductivity $= \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2}$;

their joint resistance $R'' = 1 \div \frac{r_2 + r_1}{r_1 r_2} = \frac{r_1 r_2}{r_2 + r_1}$. **(412.)**

Rule.—*The joint resistance of two conductors in parallel is equal to the product of their separate resistances divided by the sum of their separate resistances.*

EXAMPLE.—In Fig. 916, given $r_1 = 4$ ohms ; $r_2 = 6$ ohms, and $C = 30$ amperes. Find c_1 and c_2 in the separate branches and the joint resistance of the branches from a to b.

SOLUTION.— $\frac{c_1}{c_2} = \frac{6}{4}$, or $c_1 = \frac{6 c_2}{4}$. But $c_1 + c_2 = 30$, or $c_1 = 30 - c_2$; substituting, $30 - c_2 = \frac{6 c_2}{4}$. Reducing gives $10 c_2 = 120$, or $c_2 = 12$ amperes. Ans. $c_1 = 30 - 12 = 18$ amperes. Ans.

By formula **412**, the joint resistance $R'' = \frac{r_1 r_2}{r_2 + r_1} = \frac{4 \times 6}{10} = 2.4$ ohms. Ans

2326. Fig. 917 represents a derived circuit of 3 branches.

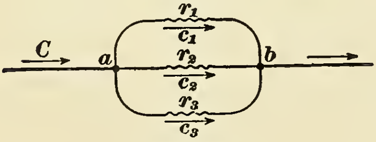


FIG. 917.

Let r_1, r_2, and r_3 = the separate resistances of the three branches, respectively ; then, $\frac{1}{r_1}$, $\frac{1}{r_2}$, and $\frac{1}{r_3}$ represent the separate conductivities of the three branches, respectively.

Their joint conductivity $= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}$.

Since the joint resistance is the reciprocal of the joint conductivity, then

$$R''' = 1 \div \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3} = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}, \qquad (413.)$$

the joint resistance of the three branches in parallel from *a* to *b*. We have, therefore, the following

Rule.—*The joint resistance of three or more conductors in parallel is equal to the reciprocal of their joint conductivity.*

Example.—In Fig. 917, given $r_1 = 5$ ohms, $r_2 = 10$ ohms, and $r_3 = 20$ ohms. Find their joint resistance from *a* to *b*.

Solution.—By formula **413** the joint resistance

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{5 \times 10 \times 20}{(10 \times 20) + (5 \times 20) + (5 \times 10)} = \frac{1{,}000}{350} = \frac{20}{7} = 2\tfrac{6}{7} \text{ ohms. Ans.}$$

2327. In any derived circuit, the **difference of potential** between where the branches divide and where they unite is equal to the product of the sum of the currents in the separate branches and their joint resistance in parallel, as will be apparent from consideration of Ohm's law, Art. **2310.** For example, if the currents in the three branches, Fig. 917, are 16, 8, and 4 amperes, respectively, and the joint resistance from *a* to *b* is $2\frac{6}{7}$ ohms, then the difference of potential between *a* and *b* is $(16 + 8 + 4) \times 2\frac{6}{7} = 28 \times \frac{20}{7} = 80$ volts.

2328. The **separate currents in the branches** of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate resistance of each branch. For example, in Fig. 917 assume that the difference of potential between *a* and *b* is 80 volts, and that the separate resistances of the three branches are, respectively, 5, 10, and 20 ohms. Then the current in the first branch is $\frac{80}{5} = 16$ amperes; in the second, $\frac{80}{10} = 8$ amperes, and in the third, $\frac{80}{20} = 4$ amperes.

2329. The **separate resistances of the branches** of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate currents in each branch. For example, in Fig. 917 assume the difference of potential between *a* and *b* to be 80 volts, and the currents in the separate branches to be 16, 8, and 4 amperes, respectively; then, the resistance of the first branch is $\frac{80}{16} = 5$ ohms; of the second, $\frac{80}{8} = 10$ ohms, and of the third, $\frac{80}{4} = 20$ ohms.

EXAMPLES FOR PRACTICE.

1. The separate resistances of two branches X and Y of a derived circuit are 13 and 29 ohms, respectively. Find their joint resistance in parallel. Ans. 8.9762 ohms.

2. The sum of the currents in two branches X and Y of a derived circuit is 28 amperes. If the separate resistance of X is 7 ohms and the separate resistance of Y is 4 ohms, what is the separate current in each branch?

Ans. { Current in branch X is 10.1818 amperes.
Current in branch Y is 17.8182 amperes.

3. The separate resistances of three branches of a derived circuit are, respectively, 36, 45, and 64 ohms. Find their joint resistance in parallel. Ans. 15.2381 ohms.

4. The joint resistances of three conductors X, Y, and Z, connected in parallel, is 2.5 ohms. If the separate currents in the branches are, respectively, .6, .7, and .8 ampere, what is the separate resistance of each branch?

Ans. { Resistance of branch $X = 8.75$ ohms.
Resistance of branch $Y = 7.5$ ohms.
Resistance of branch $Z = 6.5625$ ohms.

5. The separate resistances of three branches X, Y, and Z of a derived circuit are 2, 3, and 4 ohms. If the sum of the currents in the three branches is 26 amperes, what is the separate current in each branch?

Ans. { 12 amperes in branch X.
8 amperes in branch Y.
6 amperes in branch Z.

THE JOULE.

2330. The practical unit of electric energy or work is the **joule.**

The joule is greater than the absolute unit of energy or work, the erg (Arts. **2263** and **2264**).

1 absolute unit or erg equals one-ten-millionth $\left(\frac{1}{10,000,000}\right)$ part of a joule.

One joule equals ten million (10,000,000) absolute units or ergs.

ELECTRICAL WORK.

2331. *The joule may be further defined as being that amount of energy which is expended during the time of one second, by one ampere in overcoming the resistance of one ohm.*

2332. But 1 ampere flowing for 1 second = 1 coulomb (Art. **2280**); and 1 ampere flowing through 1 ohm = 1 volt (Art. **2304**); therefore, 1 joule may be defined as being that amount of energy expended when 1 volt propels 1 coulomb, or when 1 coulomb is carried through a distance between which the difference of potential is 1 volt.

The work done, therefore, may be said to be **one volt-coulomb,** just as in mechanics the work done by raising 1 pound through 1 foot is equal to the **foot-pound.**

2333. This **volt-coulomb,** however, which is called the joule, is not as great as the foot-pound, the relation being

1 joule = .7373 foot-pound.
1 foot-pound = 1.356 joules.

We may now state the rule for the determination of electrical work as follows:

2334. Rule.—*To find the amount of electrical work accomplished in joules during a given time, multiply the quantity of electricity in coulombs which has passed in the circuit during that time by the loss or drop of potential.*

2335. This rule may be expressed by the following formulas, for the three cases occurring in practical work:

Let J = electrical work in joules;
C = current in amperes;
t = time in seconds during which current flows;
E = E. M. F. of circuit;
R = resistance of circuit.

Then, according to Art. **2334,**

$$J = \text{coulombs} \times \text{drop}.$$

But, according to Art. **2280,** ampere-seconds = coulombs; so that

$$J = \text{amperes} \times \text{seconds} \times \text{drop}.$$

But, according to Art. **2315,** drop = current × resistance; hence,

$$J = \text{amperes} \times \text{seconds} \times \text{amperes} \times \text{resistance},$$

which can be written, by utilizing the notation given above, as

$$J = C \times t \times C \times R;$$

or,

$$J = C^2 R\,t. \qquad \textbf{(414.)}$$

This formula, then, gives the electrical work in joules when the *current and resistance* are known.

Example.—Find the amount of work done in joules when a current of 15 amperes flows for $\frac{1}{2}$ hour against a resistance of 2 ohms.

Solution.— $\frac{1}{2}$ hour = 1,800 seconds. By formula **414,** the electrical work done

$$J = C^2 R\,t = 15 \times 15 \times 2 \times 1{,}800 = 810{,}000 \text{ joules. Ans.}$$

2336. When the *current and electromotive force* are known, we derive the formula for the electrical work as follows:

According to Art. **2334,**

$$J = \text{coulombs} \times \text{drop}.$$

But drop $= E$ and coulombs (as in Art. **2335**) equal $C\,t$; hence,

$$J = C\,E\,t. \qquad \textbf{(415.)}$$

This formula expresses the amount of the electrical work in terms of current and drop.

Example.—Find the amount of work in joules done in 1 hour by a current of 25 amperes under an electromotive force of 20 volts.

Solution.— 1 hour = 3,600 seconds. By formula **415,** the electrical work

$$J = C\,E\,t = 25 \times 20 \times 3{,}600 = 1{,}800{,}000 \text{ joules. Ans.}$$

2337. When the *electromotive force and resistance* only are known, we proceed in a similar manner.

Again, according to Art. **2334,**

$$J = \text{coulombs} \times \text{drop}.$$

But coulombs (Art. **2335**) $= C\,t$ and drop $= E$; hence,

$$J = C\,t\,E.$$

But, according to Ohm's law (Art. **2310,** formula **409**), $C = \frac{E}{R}$, and inserting this value of C, we have

$$J = \frac{E}{R} \cdot t \cdot E;$$

or,

$$J = \frac{E^2 t}{R}. \qquad \textbf{(416.)}$$

This formula expresses the amount of the electrical work in terms of the E. M. F. and resistance.

Example.—What is the amount of work done in joules in 45 minutes in a circuit having 200 ohms resistance, the electromotive force being 110 volts?

Solution.— 45 minutes = 2,700 seconds. By formula **416,** the electrical work done

$$J = \frac{E^2 t}{R} = \frac{110 \times 110 \times 2{,}700}{200} = 163{,}350 \text{ joules. Ans.}$$

2338. As stated in Art. **2333,** the joule = .7373 foot-pound; therefore, when the work in joules is known, the work in foot-pounds is

$$\text{F. P.} = .7373\, J, \qquad \textbf{(417.)}$$

which may be expressed by the

Rule.—*The equivalent work done in foot-pounds, when the work in joules is known, is obtained by multiplying the number of joules by .7373.*

Example.—Express the work accomplished in foot-pounds in a circuit where a current of 8 amperes flows for 2 hours, the electromotive force being 10 volts.

Solution.— 2 hours = 7,200 seconds = t. By formula **415,** the electrical work done = $J = 8 \times 10 \times 7{,}200 = 576{,}000$ joules. Expressed in foot-pounds, this will be by formula **417,**

$$\text{F. P.} = .7373 \times 576{,}000 = 424{,}684.8 \text{ foot-pounds. Ans.}$$

Example.—Find the amount of work done in foot-pounds by a current of 4 amperes flowing for 15 seconds against a resistance of 3 ohms.

Solution.—By formula **414,** the electrical work done = $J = 4 \times 4 \times 3 \times 15 = 720$ joules. The mechanical work done, by formula **417,** in foot-pounds is F. P. = .7373 × 720 = 530.856 foot-pounds. Ans.

RELATIONS OF MECHANICAL, ELECTRICAL, AND HEAT ENERGY.

2339. When an electric current flows from a higher to a lower potential, electrical energy is expended and work is done.

This energy is expended in overcoming the resistance of the conductor constituting the circuit.

In the case of analogy IV., Art. **2313,** the friction of the water against the walls of the pipe produces heat, in an exactly similar manner as heat is produced, for instance, by rubbing sandpaper over a wooden surface. In the latter case, however, the friction is very great, and the heat produced is hence quickly felt by the hand, while, in the case of water against metal pipes, the friction is comparatively very small, and the heat produced thereby is not perceptible to our sense of touch. Nevertheless, the heat is there, as the principle of the **conservation of energy** proves (see Art. **960**). This heat is dissipated into the surrounding atmosphere; it is, therefore, not destroyed, but merely exists in another form, having gone to increase the temperature of the air.

2340. Exactly so is it with the energy expended by an electric current in overcoming the resistance of a conductor ; that is to say, when a quantity of electricity flows against the resistance of a conductor, a certain amount of electrical energy is transformed into *heat* energy. This fact becomes very noticeable at times, for the conductor may become exceedingly hot—so hot, indeed, that unless due care is exercised the wire carrying the current may be melted by the great heat produced.

2341. The actual amount of heat developed is an exact equivalent of the work done in overcoming the resistance of the conductor, and varies directly as that resistance. For example, take two wires, the resistance of one being twice that of the other, and send currents of equal strengths through each. The amount of heat developed in the wire of higher resistance will be twice that developed in the wire offering the lower resistance. The distinguished scientist *Joule*, after whom the practical unit of energy is named, made elaborate experiments to determine exactly what relation existed between the mechanical or electrical work done and the heat thereby generated.

2342. Mechanical Equivalent of Heat.—Joule found, as shown in Art. **1148,** that the heat which is generated by doing 778 foot-pounds of work is exactly equal to the amount of heat required to raise the temperature of 1 pound of pure water 1° F., at or near 39° F., the temperature of its maximum density. This amount of heat is called **one British Thermal Unit** (written **B. T. U.**).

Therefore, we have the relation

$$\begin{aligned} 778 \text{ foot-pounds} &= 1 \text{ B. T. U.} \\ 1 \text{ foot-pound} &= .001285 \text{ B. T. U.} \end{aligned}$$

This relation is called the **mechanical equivalent of heat.**

2343. Electrical Equivalent of Heat; Joule's Law.—Upon investigating the amount of heat generated by an electrical current when overcoming the resistance of a conductor, Joule found that one ampere of current flowing through one ohm of resistance during the time of one second always developed .0009477 British Thermal Unit.

He found furthermore that the development of heat was proportional,

1. To the resistance of the conductor ;
2. To the square of the current strength ;
3. To the time during which the current flows ;

so that if

H = B. T. U. developed in the circuit ;
C = current in amperes ;
R = resistance in ohms ;
t = time in seconds,

then the general formula for the development of heat in any electrical circuit is given by what is called **Joule's Law.**

$$H = .0009477\, C^2 R\, t. \qquad \textbf{(418.)}$$

Example.—Determine how many B. T. U. are developed in an electrical circuit having a resistance of 180 ohms, through which a current of 2 amperes flows for 1 minute.

Solution.— $t = 60$ seconds ; $C = 2$; $R = 180$; hence, by formula **418,** the heat units developed are

$$H = .0009477 \times 2 \times 2 \times 180 \times 60 = 40.94 \text{ B. T. U.} \quad \text{Ans.}$$

2344. Referring back to Art. **2335,** we find that the work in joules performed in an electrical circuit is given by a formula similar to formula **418**; in fact, we find that the work in joules is proportional to the same factors as the heat development. This relation is best made clear by solving the following

Example.—Given an electrical circuit having a resistance of 2 ohms, in which a current of 2 amperes flows for 2 seconds, determine (*a*) the work in joules done in this circuit, and (*b*) the number of B. T. U. developed in the circuit.

Solution.— $t = 2$; $C = 2$; $R = 2$; then, by formula **414,** the work in joules, (*a*) $J = C^2 R t = 2 \times 2 \times 2 \times 2 = 16$ joules. Ans. And by formula **418,** (*b*) $H = .0009477\ C^2 R t = .0009477 \times 2 \times 2 \times 2 \times 2 = .0151632$ B. T. U. Ans.

2345. We therefore see that the circuit of the preceding example develops .0151632 heat-unit when 16 joules of work are done.

Consequently, we have the relation

$$16 \text{ joules} = .0151632 \text{ B. T. U.};$$

or,

$$1 \text{ joule} = .0009477 \text{ B. T. U.};$$

and, conversely, 1 B.T.U. = 1,055.20 joules.

Now, since we know that .7373 foot-pound of mechanical work is equivalent to 1 joule of electrical, and since 1 joule of electrical work equals .0009477 heat-unit, it is clear that we have established a complete relation between mechanical work, electrical work, and heat energy, so that any one of these three energies can be mathematically expressed in terms of the others. These relations are expressed clearly by Table 78.

2346. The following table will be found very useful for all examples involving transformations of energy :

TABLE 78.

ENERGY EQUIVALENTS.

Heat Energy.		Mechanical Energy.		Electrical Energy.
1.000000 B.T.U.	=	778.0000 foot-pounds	=	1,055.2000 joules.
.001285 B.T.U.	=	1.0000 foot-pound	=	1.3563 joules.
.0009477 B.T.U.	=	.7373 foot-pound	=	1.0000 joule.

Example.—Given an electrical circuit having a resistance of 3 ohms, through which a current of 5 amperes flows for 1 hour, determine (*a*) the work done in joules; (*b*) how many foot-pounds this work is equivalent to; (*c*) the number of heat-units developed.

Solution.— $t = 3{,}600$ seconds; $C = 5$ amperes; $R = 3$ ohms; then, by formula **414,** the work in joules

$$(a)\ J = C^2 R t = 5 \times 5 \times 3 \times 3{,}600 = 270{,}000 \text{ joules. Ans.}$$

According to Table 78, 1 joule = .7373 foot-pound; hence,

$$(b)\ 270{,}000 \times .7373 = 199{,}071 \text{ foot-pounds. Ans.}$$

According to Table 78, 1 foot-pound is equivalent to a heat development of .001285 B. T. U.; hence,

$$(c)\ 199{,}071 \times .001285 = 255.81 \text{ B. T. U. Ans.}$$

ELECTRICAL POWER.

2347. The total amount of work done is independent of time (see Art. **954**); that is to say, the total work is the same whether it is done in one minute or in one year. But when various amounts of work, done in different times, are to be compared to a common standard of power, the element of time must be considered.

Similarly in the electrical circuit; the total number of joules of work done is independent of the time, but when there are several circuits, the work of each of which is to be compared to a standard, the element of time in which this work is done must be considered.

2348. In **practical mechanical work** the unit of time is always one minute, and the unit which measures the work performed in a given time is the **foot-pound per**

minute. This unit is called the **unit of mechanical power.**

Power is, therefore, rate of doing work, and hence the power exerted can always be determined by dividing the work done in foot-pounds by the time in minutes required to do it.

2349. In **practical electrical work** the unit of time is the second, and the unit which measures the work performed in a given time is the **joule per second.** This unit is called the **unit of electrical power,** and has been named the **watt.**

Hence, if in a certain electrical circuit, say 1,000 joules of work are done in 10 seconds, the power exerted is 1,000 ÷ 10 = 100 joules per second, or 100 watts. If in another circuit the same work is done in 5 seconds, the power there exerted is 1,000 ÷ 5 = 200 joules per second, or 200 watts—just twice as much. Hence, we say that the power exerted in the second circuit is twice that exerted in the first ; and we understand thereby that if in both circuits work is done for the same length of time, the second circuit will do twice as much work as the first.

2350. Equation of Power for Electrical Circuit.— The equation or formula expressing the power exerted in any electrical circuit is determined as follows :

According to Art. **2349,** electrical power is expressed by watts = joules per second.

But, according to Art. **2333,**

$$\text{joules} = \text{volt-coulombs, and hence}$$
$$\text{joules per second} = \text{volt-coulombs per second.}$$

Therefore, also,

$$\text{watts} = \text{volt-coulombs per second.}$$

Now, according to Art. **2280,**

$$\text{coulombs per second} = \text{amperes.}$$

Inserting this value in the next before the last equation above, we have, finally,

$$\text{watts} = \text{volts} \times \text{amperes} \, ;$$

or, if $W =$ total watts exerted in the circuit;
$E =$ volts of electromotive force;
$C =$ current in amperes,

then, $$W = E\,C, \qquad \textbf{(419.)}$$

which may be expressed by the following

2351. Rule.—*In every electrical circuit the power in watts is equal to the product obtained by multiplying the current in amperes by the electromotive force in volts.*

EXAMPLE.—What is the power in watts in an electrical circuit in which .6 ampere flows under a pressure of 110 volts?

SOLUTION.— $C = .6$; $E = 110$; hence, by formula **419,**
$W = E\,C = .6 \times 110 = 66$ watts. Ans.

2352. When the *power* is to be expressed by the *current and resistance*, the formula is obtained as follows: According to formula **419,** we have $W = E\,C$, and according to formula **411,** $E = C\,R$; substituting this value of $E = C\,R$ in formula **419,** we have

$$W = C \times C \times R = C^2 R, \qquad \textbf{(420.)}$$

which may be expressed by the following

Rule.—*In every electrical circuit the power in watts is equal to the product obtained by multiplying the square of the current strength in amperes by the resistance of the circuit in ohms.*

EXAMPLE.—Determine the power expended in watts in an electrical circuit having a resistance of 183.3 ohms, through which a current of .6 ampere is flowing.

SOLUTION.— $C = .6$ ampere; $R = 183.3$ ohms; hence, by formula **420,** $W = C^2 R = .6 \times .6 \times 183.3 = 65.99$ watts. Ans.

NOTE.—It will be observed that this result is the same, within decimal limits, as that obtained from the example in Art. **2351.** It is, in fact, the same circuit.

2353. When the *power* is to be expressed by the *electromotive force and resistance*, the formula is obtained as follows: According to formula **419,** we have $W = E\,C$, and, according to formula **409,** $C = \frac{E}{R}$; substituting this value of $C = \frac{E}{R}$ in formula **419,** we have

$$W = \frac{E}{R} E = \frac{E^2}{R}, \qquad \textbf{(421.)}$$

which may be expressed by the following

Rule.—*In every electrical circuit the power in watts is equal to the quotient obtained by dividing the square of the electromotive force in volts by the resistance in ohms.*

Example.—Determine the power in watts of an electrical circuit having a resistance of 183.3 ohms and an electromotive force of 110 volts.

Solution.— $E = 110$ volts ; $R = 183.3$ ohms ; hence, by formula **421,**

$$W = \frac{E^2}{R} = \frac{110 \times 110}{183.3} = 66.0 \text{ watts. Ans.}$$

Note.—Observe that this is again exactly the same as the results obtained from the examples in Arts. **2351** and **2352.** It is, in fact, the same example in all three cases.

ELECTRICAL HORSEPOWER.

2354. In mechanical calculations the **foot-pound per minute** is found too small a unit for practical use ; therefore a unit has been adopted having the value of 33,000 foot-pounds per minute, which is about equivalent to the power a strong horse can exert. This unit is, therefore, named the **horsepower.** (See Art. **955.**)

2355. Similarly in electrical calculations the **joule per second,** that is, the **watt,** is found too small a unit for practical use ; therefore a unit has been adopted having a value exactly equivalent to the value of the mechanical horsepower. This unit is obtained by transforming 1 horsepower into watts as follows :

1 mechanical horsepower = 33,000 foot-pounds per minute.

But 33,000 foot-pounds per minute $= \frac{33,000}{60} = 550$ foot-pounds per second. Hence, 1 horsepower = 550 foot-pounds per second, or 1 foot-pound per second $= \frac{1 \text{ horsepower}}{550}$. And, according to Table 78, 1 joule = .7373 foot-pound; hence, 1 joule per second or 1 watt = .7373 foot-pound per second, and, hence, 1 foot-pound per second $= \frac{1 \text{ watt}}{.7373}$.

We have, therefore, found the value of the foot-pound per second expressed both in horsepower and in watts; so that

$$1 \text{ foot-pound per second} = \frac{1 \text{ horsepower}}{550} = \frac{1 \text{ watt}}{.7373},$$

from which we find the value of

$$1 \text{ mechanical horsepower} = \frac{550}{.7373} \text{ watts} = 746 \text{ watts.} \qquad \textbf{(422.)}$$

This value, 746 watts, is termed **one electrical horsepower.**

2356. The power exerted in any electrical circuit may now be expressed in horsepower units by the following

Rule.—*To express the rate of doing electrical work in horsepower units, find the number of watts and divide the result by 746.*

If H. P. = horsepower;
W = watts,

$$\text{H. P.} = \frac{W}{746}. \qquad \textbf{(423.)}$$

Since W has the various values given by formulas **419, 420,** and **421,** the horsepower may also be expressed by three other equations:

$$\text{H. P.} = \frac{EC}{746}. \qquad \textbf{(424.)}$$

$$\text{H. P.} = \frac{C^2 R}{746}. \qquad \textbf{(425.)}$$

$$\text{H. P.} = \frac{E^2}{746 R}. \qquad \textbf{(426.)}$$

2357. Before giving examples on the application of the foregoing formulas, it must be mentioned that a practical unit of electrical power in extended use is the **kilowatt,** having the value of 1000 watts. This unit is usually written K. W., and is related to the electrical horsepower by the following equations:

$$1 \text{ K. W.} = 1{,}000 \text{ watts} = 1.34 \text{ H. P.}$$
$$1 \text{ H. P.} = 746 \text{ watts} = .746 \text{ K. W.}$$

EXAMPLE.—The common incandescent electric light consists of a glass bulb containing a simple carbon conductor, the two free ends of which are connected to the source of the electric current. When the current flows through this conductor, it heats it to such a degree that it becomes white hot, or, as such a state is called, incandescent. If this conductor has a resistance of 189.06 ohms and the lamp is supplied with an electromotive force of 110 volts, determine the following points of interest: (*a*) What current does the lamp take? (*b*) How many watts does it consume? (*c*) How many B. T. U. are developed per second? (*d*) How many such lamps would one electrical horsepower keep burning? (*e*) What is the mechanical equivalent of the heat developed per second in the lamp? (*f*) For how many such lamps would 10 K.W. suffice?

NOTE.—Regard the lamp as a simple conductor of the stated resistance in solving all problems relating to it.

SOLUTION.—(*a*) $E = 110$; $R = 189.06$; hence, by formula **409,** $C = \frac{E}{R} = \frac{110}{189.06} = .582$ ampere. Ans.

(*b*) By solution (*a*), $C = .582$; $E = 110$; hence, by formula **419,** $W = C\,E = .582 \times 110 = 64.02$ watts. Ans.

(*c*) By solution (*a*), $C = .582$; $R = 189.06$; $t = 1$ second; hence, by formula **418,** the number of British Thermal Units,

$$H = .0009477\,C^2\,R\,t = .0009477 \times .582 \times .582 \times 189.06 \times 1 = .0607 \text{ B. T. U.} \quad \text{Ans.}$$

(*d*) By solution (*b*), the lamp consumes 64.02 watts. According to formula **422,** 1 horsepower = 746 watts; hence, 1 horsepower will supply $\frac{746}{64.02}$ = about 12 such lamps. Ans.

(*e*) By solution (*c*), the number of B. T. U. developed per second = .060718. By Table 78, 1 B. T. U. = 778 foot-pounds; hence, .060718 B. T. U. = $.0607 \times 778 = 47.22$ foot-pounds per second. Ans.

(*f*) According to Art. **2357,** 1 K. W. = 1,000 watts; hence, 10 K. W. = $10 \times 1{,}000 = 10{,}000$ watts. But by solution (*b*), 1 lamp requires 64.02 watts; hence, 10 K. W. will suffice for $\frac{10{,}000}{64.02}$ = about 156 such lamps. Ans.

EXAMPLES FOR PRACTICE.

1. Find the rate of doing work in watts when a current of 40 amperes flows against a resistance of 2½ ohms. Ans. 4,000 watts.

2. Express the rate of doing work in horsepower units when a current of electricity loses a potential of 20 volts in passing through a resistance of 1 ohm. Ans. .5362 horsepower.

3. How many watts in 4.5 horsepower? Ans. 3,357 watts.

4. The power in an electric circuit is equivalent to 4 horsepower. If a current of 30 amperes is flowing, what is the electromotive force developed? Ans. 99.4667 volts.

MAGNETISM.

NATURAL MAGNETS.

2358. Near the town of *Magnesia*, in *Asia Minor*, the ancients found an ore which possessed a remarkable attractive power for iron. This attractive power they named **magnetism,** and a piece of ore having this power was termed a **magnet.** The ore itself has since been named **magnetite,** and has been found to be a chemical combination of about 72 parts of iron and 28 parts of oxygen, by weight.

2359. A still more remarkable discovery was made concerning this ore. It was found that when a piece of the ore was hung from a thread, it invariably swung around to such a position that one of its ends pointed north and the other south. It was also observed that the *same* end always pointed north. Due to this fact, small pieces of the ore so suspended were used in navigation. Ships could be steered in any direction by its aid, because the direction of the north was always shown by one end of the stone. From this fact the name **lodestone** (meaning "*leading stone*") was given to the natural ore.

ARTIFICIAL MAGNETS.

2360. When a bar or needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone, without the latter losing any of its own magnetism. Such bars are called **artificial magnets.**

Artificial magnets which retain their magnetism for a long time are called **permanent magnets.**

The common form of artificial magnets is a bar of steel bent into the shape of a horseshoe and then hardened and magnetized. A piece of soft iron called an **armature,** or **keeper,** is placed across the two free ends, which helps to prevent the magnet from losing its magnetism.

2361. If a bar magnet is dipped into iron filings, the filings are attracted towards the two ends and adhere there in tufts, while towards the center of the bar, half way between the ends, there is no such tendency. (See Fig. 918.)

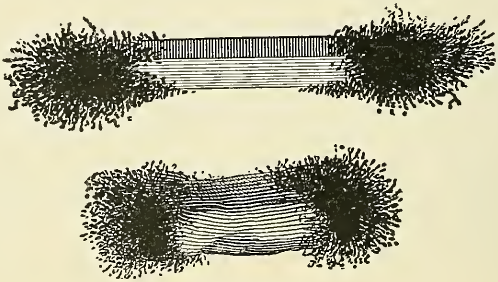


FIG. 918.

That part of the magnet where there is no apparent magnetic attraction is called the **neutral line,** and the parts around the ends where the attraction is greatest are called **poles.** An imaginary line drawn through the center of the magnet from end to end, connecting the two poles together, is termed the **axis of magnetism.**

2362. The **magnetic compass** consists of a magnetized steel needle, Fig. 919, resting upon a fine point, so as to turn freely in a horizontal plane. When not in the vicinity of other magnets or magnetized iron, the needle will always come to rest with one end pointing towards the north and the other towards the south. The end pointing northwards is the north-seeking pole, commonly called the **north**

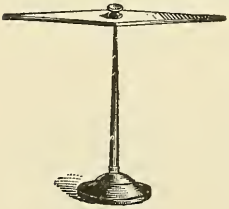


FIG. 919.

pole, and the opposite end is called the **south pole.** This *polarity* applies as well to all magnets.

2363. If the north pole of one magnet is brought near the south pole of another magnet, *attraction* takes place; but if two north poles or two south poles are brought together, they *repel* each other. In general, *like magnetic poles repel one another; unlike poles attract.*

2364. The earth is a great magnet whose magnetic poles coincide nearly but not quite with the true geographical north and south poles. By the laws of attraction and repulsion, given in Art. **2363,** it is seen why a freely suspended magnet, therefore, will always point in a north-south direction.

2365. It is impossible to produce a magnet with only one pole. If a long bar magnet is broken into any number of parts, each part will still be a magnet and have two poles, a north and a south.

2366. Magnetic substances are those substances which are not in themselves magnets, that is, they do not possess poles and neutral lines, but, nevertheless, are capable of being attracted by a magnet. A piece of soft iron will attract either pole of a magnet, or will itself be attracted towards a pole of a magnet, but when not in the vicinity of a magnet it has no defined poles. In addition to iron and its alloys, the following metals are magnetic substances: *nickel*, *cobalt*, *manganese*, *cerium*, and *chromium.* These metals, however, possess magnetic properties in a very inferior degree, compared with iron and its alloys. All other known substances are called *non-magnetic* substances.

2367. The space surrounding a magnet is called a **magnetic field;** or, in other words, a *magnetic field* is a place where a freely suspended magnetic needle will always come to rest pointing in the same direction.

MAGNETIC LINES OF FORCE.

2368. Magnetic attractions and repulsions are assumed to act in a definite direction and along imaginary lines called **lines of magnetic force,** or simply *lines of force.* Their position in any plane may be shown by placing a sheet of paper over a magnet, and sprinkling fine iron filings over the paper. In the case of a bar magnet lying on its side, the iron filings will arrange themselves in curved lines extending from the north to the south poles, as shown in Fig. 920. A view of the magnetic field looking towards either pole of a bar magnet would exhibit merely radial lines, as shown by the iron filings in Fig. 921.

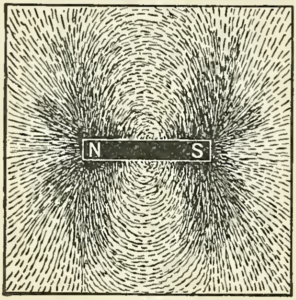


FIG. 920.

Every line of force is assumed to pass *out from* the north pole, make a complete circuit through the surrounding medium, and return *into* the south pole; from thence through the magnet to the north pole again, as shown in Fig. 922.

This is called the **direction of the lines of force,** and the path which they take is called the **magnetic circuit.** Every line of force forms a complete magnetic circuit by itself.

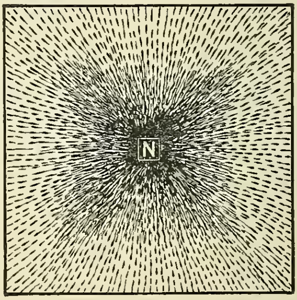


FIG. 921.

The *direction of the lines of force* in any magnetic field can be traced by a small freely suspended magnetic needle, or a small compass such as is shown by *m* in Fig. 922. The north pole of the needle will always point *in the direction of the lines of force*, the length of the needle lying parallel or tangent to the lines of force at that place. If the needle be moved bodily in the direction towards which its north pole points, its center or pivot will describe a path coinciding with the direction of the lines of force along that part of the magnetic field. In Fig. 922 the arrow-heads indicate the direction of the lines of force. It will be noted that in Figs. 920, 921, and 923, the magnetic lines are shown in one plane only, namely, in the plane of the paper. It should be borne in mind, however, that they extend out from the magnet in every direction, above, below, and to both sides.

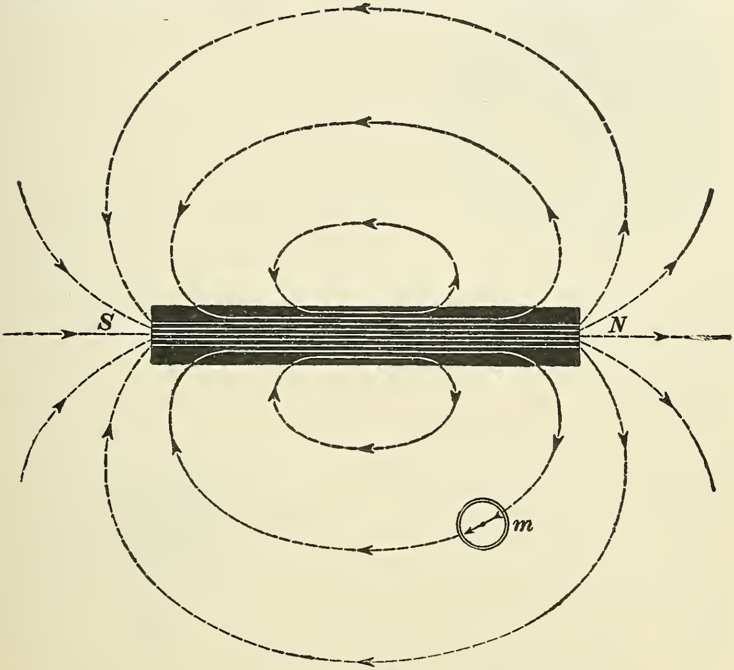


FIG. 922.

MAGNETIC CIRCUITS.

2369. The *length* of a magnetic circuit represents the average lengths of all the lines of force measured from where they pass out from the north pole along their circuit through the surrounding medium to where they enter the south pole, plus their length in the magnet. In a short bar magnet, the length of the magnetic circuit may be exceedingly large and difficult to measure, because a great many of the lines of force will travel a long distance before entering the south pole. In a longer bar, however, bent into the shape of a horseshoe, the lines of force pass out from the north pole and enter the south pole immediately, thus making the average length of the magnetic circuit comparatively short and easy to determine. Lines of force can never intersect each other; when two opposing magnetic fields are brought together, the lines of force from each will be crowded and distorted from their original direction until they coincide in direction with those opposing and form a resultant field, in which the direction of the lines of force will depend upon the relative strengths of the two opposing magnetic fields. The action of the lines of force when opposing each other in direction is shown in Fig. 923 and Fig. 924, by the aid of iron filings.

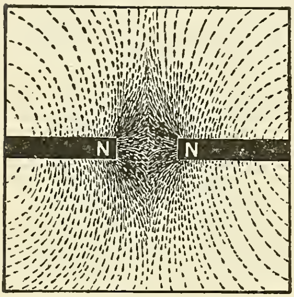


FIG. 923.

The resulting poles thus formed are called **consequent** poles.

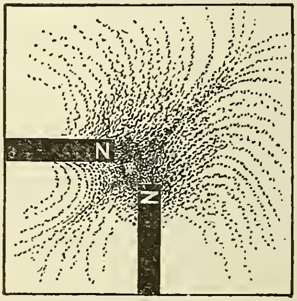


FIG. 924.

2370. In every magnetic field there are certain stresses which produce a tension along the lines of force and a pressure across them; that is, the magnetic lines tend to *shorten* themselves from end to end, and *repel* one another as they lie side by side.

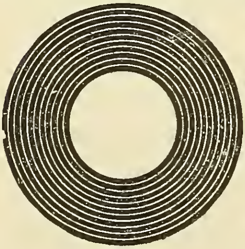


FIG. 925.

2371. A **simple magnetic circuit** is one composed of some magnetic substance having a uniform sectional area throughout its entire length, as shown in Fig. 925, which represents a simple ring.

2372. A **compound magnetic circuit** is a circuit in which the lines of force pass consecutively through several different kinds of magnetic or non-magnetic substances. Fig. 926 represents a compound magnetic circuit in which the lines of force pass through two halves of an iron ring and across two air-gaps.

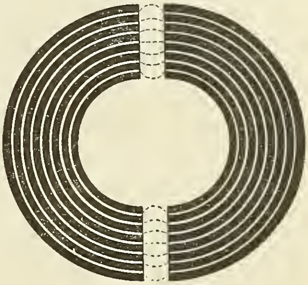


FIG. 926.

2373. A **closed magnetic circuit** is a circuit composed entirely of magnetic substances, and in which the lines of force do not pass across an air-gap. A *closed* magnetic circuit may sometimes be a *compound* one, as would be the case, for instance, in Fig. 926 if the air-gaps there shown were filled, say, with cobalt, or any of the substances mentioned in Art. **2366.**

2374. The **sectional area** of a magnetic circuit at any point is the area of a plane through which the lines of force pass, the plane being taken perpendicularly to their direction at that point. In a rectangular bar magnet, the *sectional area* of the magnetic circuit at the neutral line will be the sectional area of the bar at that line, or the breadth of the magnet multiplied by its thickness.

The *sectional area* of the magnetic circuit outside the magnet would be an indeterminate quantity, because the lines of force spread apart and diverge in all directions before entering the south pole. But where the lines of force have only a small air-gap to pass across, as in Fig. 923, the tendency to spread apart will be less, and the *sectional area* of the magnetic circuit may be taken as the area of the *polar face*.

For example, the sectional area of the magnetic circuit in a bar magnet .5 inch wide by .25 inch thick is $.5 \times .25 = .125$ square inch; that of a round bar magnet 1 inch in diameter is $1^2 \times .7854 = .7854$ square inch, since the area of a circle is equal to its diameter squared multiplied by .7854.

MAGNETIC INDUCTION.

2375. When a magnetic substance is brought into a magnetic field, that is, in the neighborhood of a magnet, so that the lines from the magnet reach it, the substance also immediately becomes magnetic. The lines of force emanating from the magnet and reaching the substance crowd together and tend to pass through the substance. The substance so magnetized is, however, only a *temporary* magnet. When it is again removed from the magnetic field, its magnetism disappears. While under the influence of the magnet, however, it behaves as does any other magnet, and has polarity, but this polarity is so distributed in the substance that its south pole is that pole where the magnetic lines coming from the magnet enter it, while its north pole is in that portion of the substance where the magnetic lines leave it. The production of magnetism in a magnetic substance in this manner is called **magnetic induction.** The production of artificial magnetism in a hardened steel needle or bar by contact with a lodestone is only a special case of magnetic induction.

2376. The **amount** or **quantity of magnetism** is expressed by the total number of magnetic lines of force passing along the magnetic circuit. In a bar magnet, for instance, the quantity of magnetism would be that number of lines which pass through the metal from pole to pole, and which, if the magnet is imagined cut through at the neutral line, would pass through the surfaces thus produced.

2377. If this surface is divided into unit areas, for instance square inches, then the number of magnetic lines passing through each such unit of area is termed the **magnetic density** of the substance.

Magnetic density is, therefore, the number of lines of force passing through a unit area measured perpendicularly to their direction.

The length of the magnetic circuit does not affect the magnetic density in that circuit so long as the total number of lines of force remains unchanged.

To find the magnetic density per square inch when the sectional area of the magnetic circuit and the total number of lines of force are known,

Let N = total number of lines of force;
A = sectional area of magnetic circuit in square inches;
B = magnetic density per square inch.

Then, $$\mathsf{B} = \frac{N}{A}. \qquad \textbf{(427.)}$$

That is to say, *the magnetic density in lines of force per square inch is obtained by dividing the total number of lines of force by the sectional area of the magnetic circuit in square inches.*

For example, after measuring the magnetism in a straight bar magnet $\frac{1}{2}$ inch square and of any length, the total amount of magnetism at the neutral line is found to be 25,000 lines of force. The magnetic density in the bar is, therefore, by formula **427**, $\mathsf{B} = \frac{N}{A} = \frac{25,000}{.5 \times .5} = 100,000$ lines of force per square inch. This is equivalent to saying that 100,000 lines of force would pass through the magnet if its sectional area were increased to 1 square inch and the lines of force were increased in the same proportion.

The total magnetism in a horseshoe magnet made of a bar of iron $1\frac{1}{2}$ inches square is 90,000 lines of force. The magnetic density in the bar is, therefore, by formula **427**, $\mathsf{B} = \frac{N}{A} = \frac{90,000}{1.5 \times 1.5} = 40,000$ lines of force per square inch That is, 40,000 lines of force would pass through the magnet if its sectional area were reduced to 1 square inch, and the lines of force were reduced in the same proportion.

2378. To find the total number of lines of force in a magnetic circuit when the sectional area of the magnetic circuit and the magnetic density at that point are known, use the notation of Art. **2377**, giving

$$N = A\,\mathsf{B}. \qquad \textbf{(428.)}$$

That is to say, *the total number of lines of force in a magnetic circuit is obtained by multiplying the sectional area in square inches by the magnetic density per square inch.*

EXAMPLE.—In a certain part of a magnetic circuit the cross-section is .75 inch × .5 inch, and the magnetic density at that point is 50,000 lines of force per square inch; find the total number of lines of force in the magnetic circuit.

SOLUTION.—The sectional area of the magnetic circuit is $A = .75 \times .5 = .375$ square inch. By formula **428**, the total number of lines of force $= N = A\,\mathbf{B} = .375 \times 50{,}000 = 18{,}750$ lines of force. Ans.

EXAMPLE.—The cross-section of a magnetic circuit is a circle 1.5 inches in diameter, and the magnetic density is 20,000 lines of force per square inch; find the total number of lines of force passing through the circuit.

SOLUTION.—Sectional area $= A = 1.5^2 \times .7854 = 1.76715$ square inches. By formula **428**, the total number of lines of force $= N = 1.76715 \times 20{,}000 = 35{,}343$. Ans.

MAGNETIC UNITS.

2379. To properly define the **strength of a magnet pole,** a unit must be adopted by which this strength can be expressed. By universal agreement a magnet pole having unit strength is defined as a pole which meets the following conditions:

1. *It must, when placed at a distance of 1 centimeter from a similar pole having equal strength repel this pole with a force of 1 dyne.*

2. *It must, when placed in the center of a sphere having a radius of 1 centimeter, send out such a number of lines of force that exactly 1 line of force passes through every square centimeter of the surface of the sphere.*

2380. Number of Magnetic Lines per Unit Pole.—Directly from condition 2, of the preceding article, the number of magnetic lines per unit pole may be calculated. It is there stated that a sphere of 1 centimeter radius receives 1 line of force per square centimeter of surface when a unit pole is situated at its center. This is equivalent to saying that a unit pole has as many magnetic lines as there are square centimeters on the surface of a sphere having a radius of 1 centimeter. If a sphere has a radius = 1 cm., its diameter = 2 cm. By the rule of Art. **817**, area of surface = diameter squared × 3.1416; hence, area of surface of our sphere = $2^2 \times 3.1416 = 12.5664$ square centimeters. But, as stated before, number of square centimeters of surface equal number of magnetic lines, whereby we have the

Rule.—*Every magnet pole of unit strength has 12.5664 magnetic lines.*

Note.—In this result, fractions of magnetic lines appear. Such fractions of magnetic lines are often obtained in magnetic calculations. They are treated in the same manner as other fractions are. Their significance may be made clear by the following consideration: Suppose we have a piece of cloth 1 inch wide and 1 inch long, that is, 1 inch square. Let us further suppose that, say, 13 pins were stuck vertically into this cloth. We could then say there are 13 pins per square inch. Assume now that one of these pins was removed, split lengthwise in half, and the one half again stuck into the cloth. Now we would say that there were only 12½, that is, 12.5 pins per square inch of cloth. Similarly, in the rule above, when we speak of 12.5664 magnetic lines, we mean that a little over 12½ magnetic lines are sent out from every magnet pole of unit strength.

2381. Unit Density of Magnetism.—In Art. **2377** density of magnetism was defined as being the number of lines of force passing through unit area. To express the magnetic density definitely, however, we must have a unit whereby to measure it. This unit is derived from condition 2, in Art. **2379**, where it is stated that a unit magnet pole sends 1 line of force through every square centimeter of the surface of the sphere there mentioned. In accordance with this, unit density of magnetism is a density of 1 line of force per square centimeter. Since 1 square inch equals 6.452 square centimeters, this is equivalent to a density of 6.452 lines of force per square inch, so that we have the

Rule.—*Unit density of magnetism is a density of 6.452 lines of force per square inch.*

When every square inch cross-section of a magnetized substance has exactly the same number of lines of force passing through it, the magnetic density of the substance is said to be **uniform.**

When this is not the case, the density is said to be **non-uniform.**

2382. Relation Between Electrical and Magnetic Units.—In Art. **2379** a magnet pole of unit strength is defined as exerting, under the condition stated, a force of 1 dyne. In Art. **2262,** however, the dyne is given as the fundamental unit of force in general. This fact makes it possible to compare magnetic forces to both electrical and mechanical forces; for

1 dyne = unit of magnetic force;

1 dyne = unit of force in general (see Art. **2262**).

1 dyne exerted through 1 centimeter = 1 dyne cm. = 1 erg = unit of work (Art. **2263**).

10,000,000 ergs = 1 joule = unit of electrical work (Art. **2330**).

1.356 joules = 1 foot-pound = unit of mechanical work (Arts. **2333** and **2348**).

We thus have given the relation between dynes, ergs, joules, and foot-pounds, or, in other words, the relation between force and work for magnetic, electrical, and mechanical quantities.

Example.—Two similar magnet poles 3 centimeters apart repel each other with a force of 4 dynes. (*a*) How many ergs of work must be expended to bring the one pole up to the other one against this repulsion? (*b*) How many foot-pounds of work is this equivalent to?

Solution.—(*a*) To bring one pole up to the other through a distance of 3 centimeters, a force of 4 dynes must be overcome through a distance of 3 centimeters. By Art. **2263,** the work done equals 4 × 3 = 12 dyne centimeters, or 12 ergs. Ans.

(*b*) By Art. **2330,** 1 joule = 10,000,000 ergs; hence, 1 erg = .0000001 joule. By solution (*a*) the work done is 12 ergs = 12 × .0000001 = .0000012 joule. By Art. **2346,** Table 78, 1 joule = .7373 foot-pound. Hence, the work done in foot-pounds equals .0000012 × .7373 = .00000088476 foot-pound. Ans.

ELECTROMAGNETISM.

2383. If a conductor conveying a current of electricity be brought near a freely suspended magnetic needle, the needle will tend to place itself at right angles to the conductor, as indicated by the arrows in Fig. 927; or, in general, *an electric current and a magnet exert a mutual force upon each other.* From the definition given in Art. **2367,** the

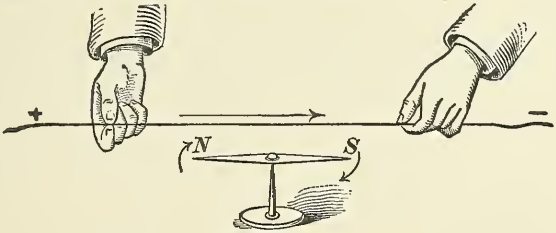


FIG. 927.

space surrounding the conductor is a *magnetic field.* If the conductor is threaded up through a piece of cardboard, and iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles around the wire, as shown in Fig. 928. This effect will be observed throughout the whole length of the conductor, and is caused entirely by the current. In fact, every conductor conveying a current of electricity can be imagined as completely surrounded by a sort of *magnetic whirl*, as shown in Fig. 929, the magnetic density decreasing as the distance from the conductor increases.

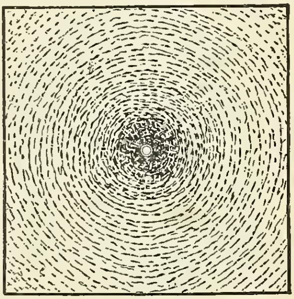


FIG. 928.

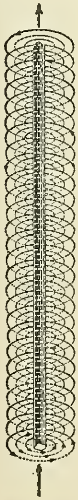


FIG. 929.

2384. If the current in a horizontal conductor is flowing *towards the north* and a compass is placed *under* the wire, the north pole of the needle will be deflected towards the *west;* by placing the compass *over* the wire, the north pole of the needle will be deflected towards the *east.* (See Fig. 930.) Reverse

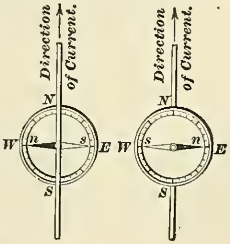


FIG. 930.

the direction of the current in the conductor, and the needle will point in the opposite direction in each case respectively.

If the conductor is placed *over* the needle and then bent back *under* it, forming a loop, as shown in Fig. 931, the tendency of the current in both top and bottom portions of the wire is to deflect the north pole of the needle in the same direction. From these experiments, knowing the direction of current in the conductor, the following rule is deduced for the *direction of the lines of force around the conductor:*

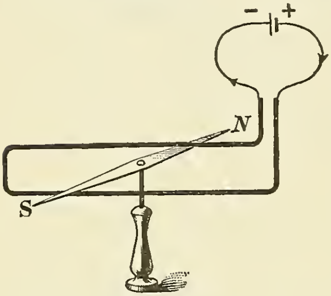


FIG. 931.

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.*

The direction of the lines of force around a conductor is shown by the arrow-heads and compass needles in Fig. 932, where the current is assumed to be flowing downwards, or away from the observer.

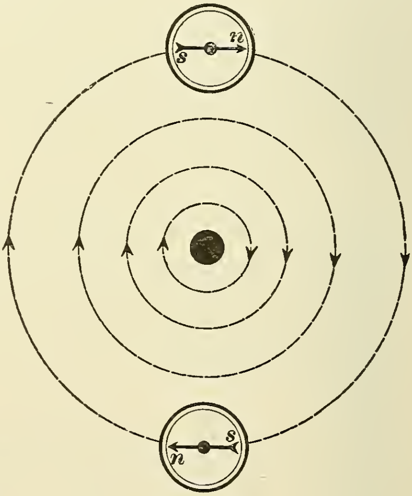


FIG. 932.

ELECTRICAL APPARATUS.

2385. The following is a description of the free electrical apparatus with which the student is furnished in connection with this Course. Directions are given for performing certain experiments calculated to help the student to a better understanding of the subject treated. Unless the student has had previous instruction of a like nature, he is earnestly requested (though not required) to make all the experiments mentioned, and to keep a record of his results by answering the questions under the heading Experiments with Electrical Apparatus, which follows this description of the same. He may forward his record to the School, if he so desires, for correction and approval, when he sends his answers to the questions on this subject, but he will be marked and his work computed in connection with his work on the questions above referred to.

DESCRIPTION OF APPARATUS.

2386. The **cell** illustrated in Fig. 933 is of an improved *Leclanche* type. It is quite similar to the simple cell described in Art. **2240.** The electrolyte is a solution of ammonium chloride (sal ammoniac); the positive element is a piece of rolled zinc Z, and the negative element is a block of carbon C. To prevent the formation of hydrogen that occurs in the simple cell, the carbon block is enclosed in a cup P, made of porous clay, and the space between the cup and the carbon is filled with a substance (peroxide, or black oxide, of manganese) with which the hydrogen formed when the cell is in action combines. Directions for setting up the cell accompany each one; the proper amount of sal ammoniac for one charge is also sent with each cell. The cell is connected to the external circuit by clamping the bared ends of the connecting wires, one under the brass thumb-nut B, the other in the hole in the zinc electrode by means of the thumb-screw B_1.

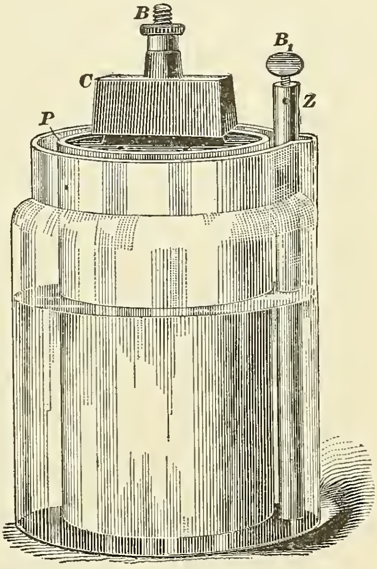


FIG. 933.

Before setting up the cell, unscrew the brass thumb-nut B from the top of the carbon electrode, and scrape away whatever paraffin or black wax there may be on the thumb-nut, its screw, or the surface upon which the nut bears when screwed up tight.

When properly set up, this cell will give an E. M. F. of about 1.5 volts.

A full description of the principles upon which this cell acts will be given in the section on "Batteries."

The **compass** consists of a brass case with a thick glass top, in which a strongly magnetized steel needle is suspended on a steel pivot; beneath the needle is a scale which shows the eight principal points of the compass, and is also divided around the edge into 180 divisions, each division, therefore, having a value of two degrees; by these divisions the angle between any two positions of the needle may be read.

In estimating such an angle, care should be taken to look down on the needle *vertically*, so that the point of the needle will appear directly over the proper degree mark. With a little practice, the position of the needle should be read within one degree, or even less.

The **bar magnet** is a piece of strongly magnetized hardened steel, $5\frac{1}{2}$ in. × $\frac{3}{8}$ in. In one end is a tapped hole, in which is a screw; the use of this little attachment will be explained later in the Course; hence, it should not be mislaid.

The **horseshoe magnet** is also a piece of hardened steel, which was bent to a horseshoe form before being hardened and magnetized. One pole has a mark across it near the end.

The small, soft wrought-iron keeper supplied with the magnet should be kept across the poles when the magnet is not in use, as this tends to keep the strength of the magnet more permanent.

Striking the magnets with any hard substance, dropping them, or heating them to more than about 570° F., should be avoided, as the result would be that they would lose some or all of their magnetic force.

The **iron filings** need no description; their use in connection with the magnets will be given in connection with the description of the experiments.

The **wire** supplied is No. 18 B. & S. gauge (.040″ diam.) copper wire, insulated with two layers of cotton soaked in paraffin. The length of wire in the coil is about 75 feet.

With the above-named apparatus the student may perform certain experiments, as described farther on.

2387. Many of the experiments will require that the circuit be opened and closed, or that the direction of the current in the circuit be reversed. This may be done by changing over the connections at the battery terminals to reverse the current, or by simply disconnecting one wire to break the circuit.

A switch which may be used for such purposes is much more convenient, and should be prepared, if possible.

A cheap and simple switch, which will answer the above requirements, may be easily made with the following materials:

1 piece of pine board about 3 in. × 5 in. × $\frac{7}{8}$ in.
1 piece of wood about $\frac{3}{4}$ in. × $\frac{3}{4}$ in. × 3 in.
9 round-head brass wood screws, $\frac{3}{4}$ in., No. 8.
2 round-head brass wood screws, $\frac{1}{2}$ in., No. 6.
12 copper washers with $\frac{3}{16}$-in. hole.
2 strips of brass about $\frac{1}{2}$ in. wide, $\frac{1}{16}$ in. thick, and $4\frac{1}{2}$ in. long, each with a $\frac{3}{16}$-in. hole $\frac{3}{8}$ in. from each end.
Some short pieces of the insulated wire.

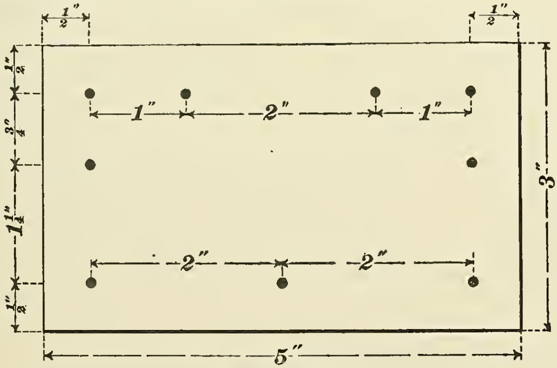


FIG. 934.

Fig. 934 shows the position of the screw holes for the $\frac{3}{4}$ in screws on the board.

Fig. 935 represents the switch with the brass strips in position. The small block of wood $\frac{3}{4}$ in. × $\frac{3}{4}$ in. × 3 in. is shown at

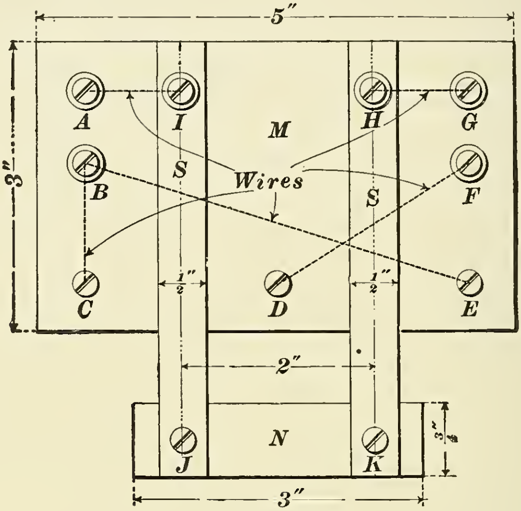


Fig. 935.

N. The two brass strips *S* and *S* are fastened to it by the two $\frac{1}{2}$-in. No. 6 screws *J* and *K*. This block *N* serves to keep the two brass strips equidistant, and also as a handle by which the switch may be moved. The other ends of the strips are held in place by the screws *I* and *H*. These screws should be screwed in tightly enough to press the brass strips down on the screws *C*, *D*, and *E*, when they are swung to one side or the other. These two screws *I* and *H* should be put in as follows (see Fig. 936): Slip the screws through one of the washers *B*; then through the hole in one end of one of the brass strips *A*; then through another washer B_1; then twist the end of a piece of wire *W*, from which the insulation has been scraped, loosely around the screw, and put the whole in place.

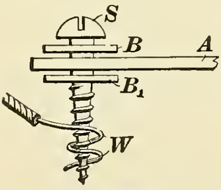


Fig. 936.

The other end of the piece of wire attached to *I* should be twisted around screw *A* before that screw is put in, and the other end of the piece attached to *H* should be twisted around *G*. These screws should also have two washers each. Screws *C*, *D*, and *E* are simply screwed in until their heads are down on the wood. A wire should connect *C* and *E* with *B* and *D* with *F*, care being taken that where the wires cross (see dotted lines in Fig. 935) they are well insulated from each other.

Screws *B* and *F* should each have two washers. Now, on connecting *A* and *G* each to one pole of the battery and connecting any other circuit to *B* and *F* (by loosening the screws and putting the bare ends of the wires between the two washers and screwing all up tight again), the current will flow through this circuit only when the brass strips are resting either on *C* and *D*, or on *D* and *E*, which they should do with a good pressure, and when the brass strips are changed over from *C* and *D* to *D* and *E* the current in this circuit will be reversed in direction, as will be seen by following out the path of the current in either case. In using the switch, be careful that the brass strips do not swing over to either side far enough to touch screws *B* or *F*.

In the middle position of the strips, as shown in Fig. 935, the circuit is open, so this switch may be used either for a reversing switch or a simple circuit-breaker. The screws *A*, *G*, *B*, and *F* may be replaced by small binding-posts, which are more convenient and cost little.

It would be well to fasten this switch to a table or other convenient place, and make permanent connection between *A* and *G* and the battery; then any apparatus that is to be used may be connected to the terminals *B* and *F* of the switch without disturbing the battery.

In the following pages, wherever it is stated that apparatus is to be connected to the battery, it is to be understood that the switch, if made, is to be included in the circuit.

2388. When taking readings of the angle of deflection of the compass needle, after the needle has come to rest the glass should be tapped lightly. The needle will then vibrate a little, and when it again comes to rest it will usually be in a slightly different position than before. This is due to the fact that when the needle is deflected, the slight friction on the point on which the needle rests prevents the needle from swinging as far as it would if there were absolutely no friction.

Tapping the case overcomes this slight friction by causing the needle to jump a little, and thus allows it to come to rest in its proper position.

As the resistance of the No. 18 wire is low, if the battery be left long with the circuit closed it is liable to be weakened; so, after making the experiments, the circuit should be opened at the switch.

In making all experiments, note on a piece of paper or in a note-book the apparatus used and how; if necessary, draw a diagram of connections, etc. Write down each result as soon as each part of the experiment is completed; *do not trust to memory for results.* Make all experiments twice, if possible, thus checking the first results. By taking the above precautions and exercising care in taking the readings, reliable and instructive results may be obtained with this simple apparatus by performing the experiments mentioned in the succeeding pages.

EXPERIMENTS WITH ELECTRICAL APPARATUS.

Experiment 1.—(Arts. **2361** and **2368.**) Spread some of the iron filings on a piece of paper and lay the bar magnet lengthwise on the filings. (*a*) Do the filings change their position? (*b*) If so, how? (*c*) If so, make a sketch showing roughly the positions assumed by the filings.

Experiment 2.—(Art. **2361.**) Take the magnet up from the filings. What happens to the filings?

Experiment 3.—(Art. **2366.**) Cut off five or six pieces of the copper wire, each about an inch long. Take five or six steel pens, needles, tacks, or other small iron or steel objects, and mix them up in a heap with the bits of wire. Touch the end of the bar magnet to the heap and note the result. (*a*) Is the copper wire attracted by the magnet?

(*b*) Are the pens or tacks? (*c*) What sort of a substance should you then call copper? (*d*) steel or iron?

Experiment 4.—(Art. **2366.**) Perform the same experiment with other substances, and name the kind of a substance (as regards its magnetic qualities) you find each to be.

Experiment 5.—(Art. **2368.**) Lay the horseshoe and the bar magnets on the table in the position shown in Fig. 937, making the distance between the two magnets about ½ inch. Lay a sheet of stiff paper over the two and sprinkle a *few* iron filings over them, just enough to show the direction of the lines of force, tapping the paper lightly as the filings are spread over it.

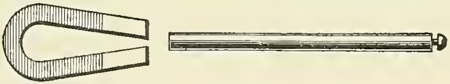


FIG. 937.

(*a*) Make a sketch showing the forms assumed by the iron filings. (*b*) What effect does the bar magnet have on the field of the horseshoe?

Experiment 6.—(Arts. **2362** and **2363.**) How can you determine the polarity (*a*) of the bar magnet? (*b*) of the horseshoe magnet?

Experiment 7.—(Art. **2368.**) Lay the horseshoe magnet on a table and the bar magnet beside it, as shown in Fig. 938, leaving about 1 inch between them. Have the north pole of both the bar and the horseshoe magnet on the same side; spread over them a piece of stiff paper and sprinkle on some iron filings as before. (*a*) Make a sketch showing the direction of the lines of force of the field resulting from the two magnets, as shown by the forms assumed by the iron filings. (*b*) Reverse the bar magnet, i. e., place its north pole where its south pole is in Fig. 938 and repeat the experiment, making a sketch as in (*a*).

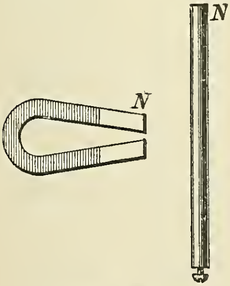


FIG. 938.

These formations of iron filings in magnetic fields may be preserved as follows : Take a piece of window-glass of suitable size and heat it over a stove or flame until a small piece of paraffin placed upon it melts. Let the melted paraffin spread evenly in a thin coat over the glass plate, and then let the plate cool in a horizontal position, so that the paraffin will harden in a layer of even thickness. When the paraffin is hard, place the plate over the magnets, suitably arranged, and on the paraffin side of the glass sprinkle the iron filings. Tap the plate lightly to settle the filings in their places, then lift the plate *vertically* off the magnets until beyond their influence. Again hold the glass plate over the stove or flame until the paraffin again melts, taking pains to hold the plate *horizontally*. The paraffin layer being thin, it will not then run, and the filings will retain their regular positions. When the paraffin is melted, carefully remove the plate to a cool spot, and again let the paraffin harden. If these operations have been carefully gone through with, the filings will be fastened in place by the paraffin and their graceful and instructive forms preserved, making a permanent record. These glass plates may be used as negatives and blue-prints, or photographs may be printed from them.

Experiment 8.—(Art. **2368.**) In the two sketches above, point out (by marking with a letter C) the principal *consequent poles* formed.

Experiment 9.—(Art. **2374.**) What is the sectional area in square inches of (*a*) the bar magnet ? (*b*) the keeper of the horseshoe magnet ?

Experiment 10.—(Art. **2374.**) What kind of a magnetic circuit is that of the horseshoe magnet with the keeper in place, (*a*) simple or compound ? (*b*) closed or not ?

Experiment 11.—(Arts. **2237** and **2239.**) Having set up the battery and let it stand for ten or twelve hours, according to directions, connect the two electrodes together with a piece of the wire five or six feet long, taking good care that the bared ends of the wire make good contact with the binding-posts on the electrodes. (*a*) Which way will the current flow in the wire; i. e., from what to what element of the battery? (*b*) Which is the positive *element?* (*c*) Which is the positive *electrode?*

Experiment 12.—(Arts. **2362** and **2364.**) Set the compass on some level space, with no iron or magnets near. It will soon settle into one position. (*a*) Why? (*b*) What is that position?

Experiment 13.—(Art. **2383.**) Now take about a foot of the wire connected to the battery, between the two hands, in such a way that the current flows *from the right hand towards the left hand*, and stand so that the wire between the hands points north and south, with the right hand towards the north. Still keeping the wire in the same direction, move it over and close to the compass. Now, if the compass needle does not move, it will point in the same direction that the current is flowing in the wire. (*a*) Does it move? (*b*) If so, how?

Experiment 14.—(Art. **2383.**) Cut the piece of wire in the middle and bare the ends. Then, taking one end in each hand, touch the bare ends of the wire to sides of the brass case of the compass, one end opposite the north pole of the needle and one opposite the south. This completes the circuit through the brass case of the instrument. (*a*) Is the needle affected? (*b*) If so, why?

Note.—As the brass case is lacquered, it will probably be necessary to scrape a little bright spot at each point where it is desired to make contact.

Experiment 15.—In Art. **2384** it is stated that if the current in a horizontal conductor is flowing towards the *north* and a compass is placed *under* the wire, the north pole of the needle will be deflected towards the west, and *vice versa.*

Verify this by several experiments, and state the methods and apparatus used, as well as the results.

Experiment 16.—(Art. **2384.**) In this experiment it will be necessary to wind coils of wire around the compass. In order to do this well, it would be very convenient to make a small box of thin wood, of the form and dimensions shown in Fig. 939.

The ends support the body of the box above the table or other surface on which it is placed. The compass should be placed in the center of the box, with the north and south line of the divisions on its scale in line with the center line of the box across, that is, the line *a b*, Fig. 939, and should be fastened there by a drop of mucilage or other means. Now, if the north and south lines of the scale divisions be brought into an actual north and south position, as indicated by the position of the needle, any coil of wire wound around the box on the line *a b* will have its plane in a north and south position, parallel to and coinciding with the axis of magnetism of the needle, and any deflection of the needle caused by a current in such a coil may be read directly in degrees on the scale of the instrument. If desired, two pins may be driven into each edge of the box on the line *a b*, so that the wire may be wound in between the pins, insuring that the coil will be located correctly with respect to the compass.

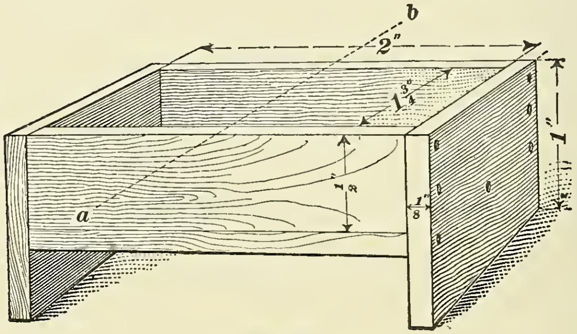


FIG. 939.

If the reversing switch described in Art. **2387** has been made, it will be found very convenient in performing the following experiments. Wind one single turn of wire around the compass and send the current from the battery around it. (*a*) Is the needle deflected? (*b*) If so, how many degrees?

Reverse the current in the wire. (*c*) What is the effect? Make the coil of two turns. (*d*) Is the deflection changed? How much? (*e*) Why is the needle deflected?

ELECTROMAGNETIC REACTION.

2389. Two parallel conductors, both transmitting currents of electricity, are either mutually attractive or repellent, depending upon the relative direction of their currents.

If the currents are flowing in the *same* direction in both conductors as represented in Fig. 940, the lines of force will tend to surround both conductors and contract, thus *attracting* the conductors. If, however, the currents are flowing in *opposite* directions, as represented in Fig. 941, the lines of force lying between the conductors will have the same direction, and therefore *repel* the conductors.

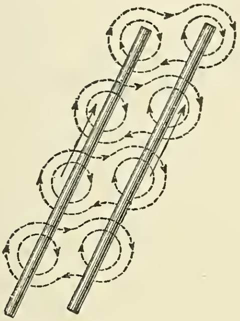


FIG. 940.

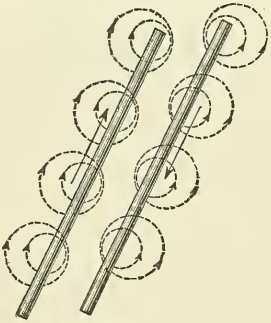


FIG. 941.

THE SOLENOID.

2390. If the conductor carrying the current is bent into the form of a loop, as shown in Fig. 942, then all the lines of force around the conductor will thread through the loop in the same direction.

Any magnetic substance, therefore, such as *m*, when placed in front of the loop, would tend to place itself with its longest axis projecting into the loop, that is, in the direction of the lines of force.

By bending the conductor into a long *helix* of several loops, the lines of force around each loop will coincide with those around the adjacent loops, forming several long lines of force which thread through the entire helix, entering at one end and passing out through the other. The same conditions now exist in the helix as exist in a bar magnet; namely, the lines of force *pass out* from one end and *enter* the other. In fact, the helix possesses a north and south pole, a neutral line, and all the properties of attraction and repulsion of a magnet. If it is suspended in a horizontal position and free to turn, it will come to rest pointing in a north-south direction.

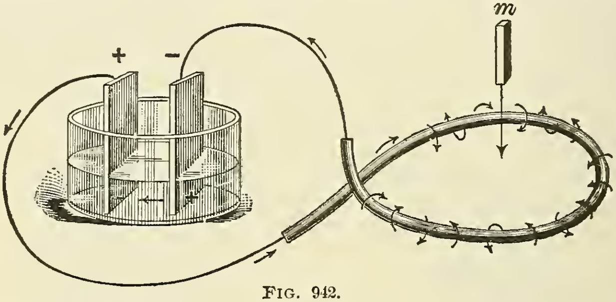


FIG. 942.

A helix made in this manner around which a current of electricity is flowing is called a **solenoid.**

The *polarity* of a solenoid or the *direction of the lines of force* which thread through it depends upon the direction in which the conductor is coiled and the direction of the current in the conductor.

To determine the polarity of a solenoid, knowing the direction of current:

Rule.—*In looking at the end of the helix, if it is so wound that the current flows around in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.*

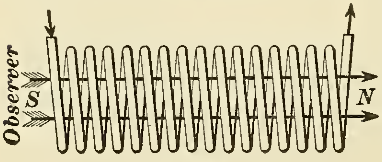


FIG. 943.

Fig. 943 represents a conductor coiled in a right-handed helix. If the current starts to flow from the end where the observer stands, that end will be a south pole, and the observer will be looking through the helix *in the direction of the lines of force.*

The polarity of a solenoid can be changed by reversing the direction of the current in the conductor.

MAGNETOMOTIVE FORCE.

2391. It has been found by experiment that the lines of force produced in a solenoid depend upon the number of turns of the solenoid and on the current circulating therein. The current and the turns together hence act as a **magnetizing force.** This magnetizing force is, therefore, equal to the product of current and turns. When the current strength is given in amperes, this product is called **ampere-turns.** It is found furthermore that the magnetizing force is independent of the size of the wire, and that 20 amperes circulating around 5 turns and producing 100 ampere-turns exert precisely the same magnetizing force as 1 ampere circulating in 100 turns or 50 amperes in 2 turns, all of which produce 100 ampere-turns.

The magnetizing force given by the product of current and turns must, however, be expressed in terms of the standard units. To so express this force, the following calculation is made:

In Art. **2380** it was shown that a unit magnet pole sends out 12.5664, or say 12.57 lines of force. It was also shown that a force of 1 dyne was exerted along each one of these lines; therefore, the total force exerted by a unit magnet pole is 12.57 dynes. Now it can be shown that the magnetic field produced by 1 turn of wire in which 1 absolute unit of current is circulating is the same as this; namely, also 12.57 dynes. Therefore, any greater number of turns and current units will exert a proportionately greater effect. The total effect or magnetizing force is, therefore, equal to 12.57 × current × turns. To express this current in absolute units, it is necessary to divide the value in amperes by 10. (See Art. **2272.**) But when the force is expressed in this manner, it is termed **magnetomotive force;** so that we have the formula

$$\text{Magnetomotive force} = \frac{12.57 \times a\text{-}t}{10} = 1.257 \times a\text{-}t,$$

where a = current in amperes;
t = number of turns.

The formula above is, however, to be applied only in cases where the dimensions of the magnetic circuit are given in metric measure. When given in English measure, as they are throughout the following, the value of the constant is changed, so that the final useful formula is

$$\text{Magnetomotive force} = 3.192 \times a\text{-}t, \qquad \textbf{(429.)}$$

where a = current in amperes;
t = number of turns.

2392. Intensity of Magnetomotive Force.—The magnetomotive force as given by formula **429** represents simply the total magnetizing effect produced by the solenoid. It does not give any information regarding the intensity of this effect at any one point; that is, it gives no information regarding the effect produced by a unit length of such a circuit. This is easily found, however, by dividing the total magnetomotive force by the total length. The quotient is termed the **intensity of magnetomotive force,** and is represented by the letter H. If l represents the length of the magnetic circuit in inches and a-t represents the ampere-turns, then

$$\mathsf{H} = \frac{3.192 \times a\text{-}t}{l}. \qquad \textbf{(430.)}$$

This formula shows that *the intensity of magnetomotive force* H *will produce a uniform magnetic field, in which the density will be* H *lines of force per square inch of sectional area of the magnetic circuit.*

This is equivalent to saying that the induction produced in the circuit is directly proportional to the magnetizing force applied. It should be particularly noted, however, that this is only true for a solenoid in air or in other non-magnetic substances. When a magnetic substance is brought near such a solenoid, the effect is immediately altered, as shall presently be shown.

2393. Total Magnetic Lines in a Solenoid.—The total number of lines of force is found by multiplying the sectional area of the magnetic circuit in square inches by the value of **H**. For example, imagine a coiled conductor of 20 turns bent into a circular shape so that there are no free poles, as represented in Fig. 944. Each line of force will form a complete ring inside the solenoid, and, therefore, the length of the magnetic circuit can easily be measured. Twenty amperes flowing through the conductor will give a magnetizing force of 400 *ampere-turns*. If the mean length of the magnetic circuit is 5 inches, then by formula **430** the magnetomotive force $\mathbf{H} = 3.192 \times \frac{400}{5} = 255.36$, which means that a uniform magnetic field is produced in the solenoid in which the density is 255.36 lines of force per square inch of sectional area. Now, if the sectional area of the magnetic circuit is .5 square inch, there are $.5 \times 255.36 = 127.68$ lines of force produced in the coil. Or, if the sectional area is 1.5 square inches, there are $1.5 \times 255.36 = 383.04$ lines of force produced in the coil.

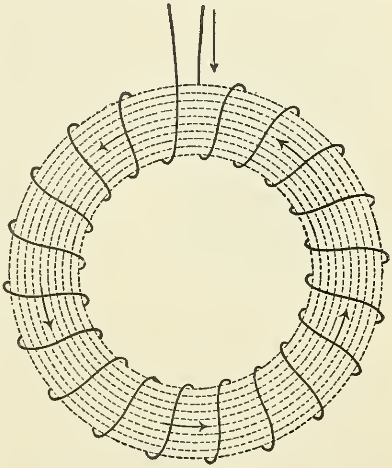


FIG. 944.

MAGNETIC PERMEABILITY.

2394. In Art. **2375** it was stated that when a magnetic substance is brought into a magnetic field, the lines of force in the field crowd together, and all try to pass through the substance; in fact, they will alter their circular shape and extend to a considerable distance from their original position in order to pass through it. A magnetic substance, therefore, offers a better path for the lines of force than air or other non-magnetic substance.

The facility afforded by any substance to the passage through it of lines of force is called **magnetic permeability,** or, simply, *permeability.*

The permeability of all non-magnetic substances, such as air, copper, wood, etc., is taken as 1, or unity. The permeability of soft iron may be as high as 2,000 times that of air. If, therefore, a piece of soft iron be inserted into the magnetic circuit of a solenoid, the number of lines of force will be greatly increased, and the iron will become highly magnetized.

THE ELECTROMAGNET.

2395. A magnet produced by inserting a magnetic substance in the magnetic circuit of a solenoid is an **electromagnet,** and the magnetic substance around which the current circulates is called the **core,** as shown in Fig. 945.

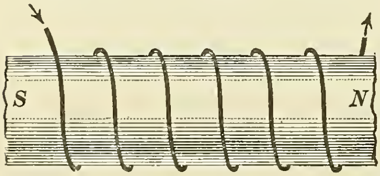


FIG. 945.

In the ordinary form of electromagnet, the magnetizing coil consists of a large number of turns of *insulated* wire; that is, wire covered with a layer or coating of some non-conducting or insulating material, usually silk or cotton; otherwise the current would take a shorter and easier circuit from one coil to the adjacent one, or from the first to the last coil, through the iron core, without circulating around the magnet.

FORMS OF ELECTROMAGNETS.

2396. The simplest form of an electromagnet is the **bar** magnet. As usually constructed, it consists of a straight bar of iron or steel B fitted into a spool or bobbin C made of hard vulcanized rubber or some other inflexible insulating material. The magnetizing coil of fine insulated copper wire *w* is wound in layers in the bobbin, as shown in Fig. 946.

The rule for determining the polarity of a solenoid (Art. **2390**) is the same for an electromagnet. It makes no difference whether the wire is wound in one layer or in any number of layers, or whether it is wound towards one end and then wound back over the previous layer towards the other; so long as the current circulates continually in the same direction around the core, the polarity of the magnet will remain unchanged.

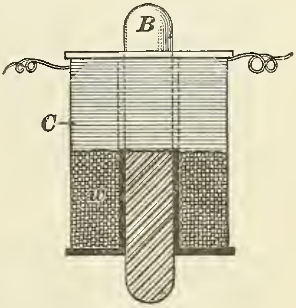


FIG. 946.

The most convenient form of electromagnet for a great variety of uses is the **horseshoe, or U-shaped** electromagnet. It consists of a bar of iron bent into the shape of a horseshoe, with straight ends, and provided with two magnetizing coils, one on each end of the magnet; the two ends which are surrounded by the magnetizing coils are the *cores* of the magnet, and the arc-shaped piece of iron joining them together is known as the **yoke** of the magnet. The ordinary U-shaped magnet, Fig. 947, is made in three parts, namely, two iron cores *M* wound with the magnetizing coils *c* and a straight bar of iron *b* for a yoke joining the two cores together. In looking at the face of the two cores, Fig. 948, the current should circulate around one core in an opposite direction to that around the other. If the current circulates around both cores in the same direction, the lines of force produced in the two cores, respectively, oppose one another, forming two like poles

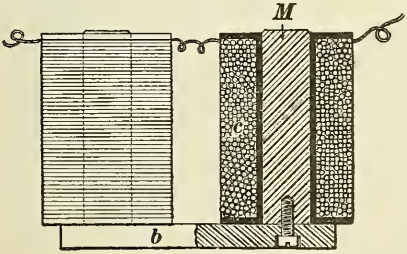


FIG. 947.

at their free ends and a consequent pole in the yoke. The total number of useful lines of force produced by both coils would, under these conditions, be greatly diminished, and the magnet would exhibit only a small amount of magnetic attraction.

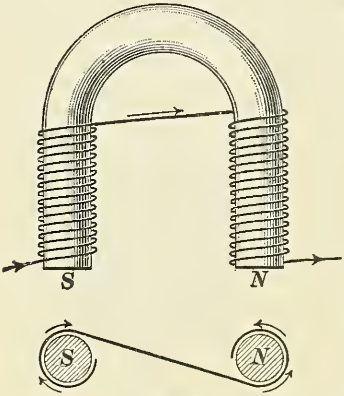


FIG. 948.

2397. Another common form of electromagnet is known as the **iron-clad** electromagnet. In its simplest form, Fig. 949, it contains only one magnetizing coil and one core. The core *M* is fastened to a disk-shaped yoke, and the magnetic circuit is completed through an iron shell *S*, which rises up from the yoke and completely surrounds and protects the coil.

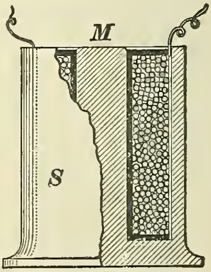


FIG. 949.

2398. Electromagnets may be divided into three general classes, according to their application, viz.:

1. Those for lifting weights and loads by adhesion.

2. Those for producing mechanical motion in an armature or a keeper; that is, for attracting an armature or a keeper through a distance.

3. Those for producing a magnetic field for dynamo-electric machines, and called field magnets.

EXPERIMENTS WITH ELECTRICAL APPARATUS.

2399. The following experiments with the apparatus furnished to the student will assist in making clear the explanation given of the solenoid:

Experiment 17.—(Art. **2390.**) Coil the wire into a helix of about $\frac{1}{2}$ in. diameter and of about 16 turns. (*a*) Send a current from the battery through this helix; will the end of the helix attract the compass needle if held near it? (*b*) Why? (*c*) How can you determine beforehand which pole of the compass needle will be attracted by either end of the helix?

Experiment 18.—(Arts. **2375** and **2395.**) Place the helix on some support, in such a position that its axis will be at right angles to the north and south line. As near as possible to one end of the helix place the compass, as shown in Fig. 950. Send a current through the coil and note the deflection of the compass needle in degrees.

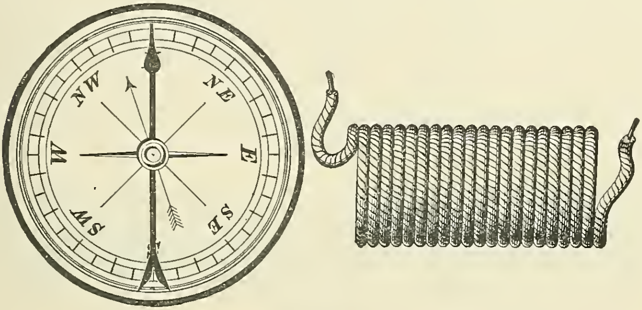


FIG. 950.

Inside the coil place the following substances and note the effect on the needle, the current still circulating in the helix: (*a*) a piece of wood, as a pencil or a few matches; (*b*) three or four wire nails; (*c*) the blade of a knife; (*d*) some brass screws.

Experiment 19.—(Art. **2393.**) Fasten one end of the helix; place the compass at the fixed end of the helix, and on sending a current through the coil, note the deflection of the needle in degrees: (1) with the coils of the helix as close together as possible, making the helix as short as it can be; (2) with the helix pulled out to twice its original length. (*a*) Is there any difference between the deflections in the two cases? (*b*) How much? (*c*) Why?

Experiment 20.—(Arts. **2383** and **2393.**) Make a helix of about 20 turns and about the same diameter as before. Stretch it out until it is twice its minimum length. Place it with its axis east and west, and put the compass near the center of the length of the helix. The compass needle will then point at right angles to the axis of the helix, as shown in Fig. 951. Send a current through the helix and note the deflection in degrees. Wind another helix of the same diameter and length as the one just used, but of twice the number of turns, and place the new helix in the same relative position with the compass. On sending a current through the new helix, the compass needle will be deflected to a greater angle than before. (*a*) Why? Move the compass away from the helix, along a line at right angles to the axis of the helix. The deflection will grow less and less. (*b*) Why?

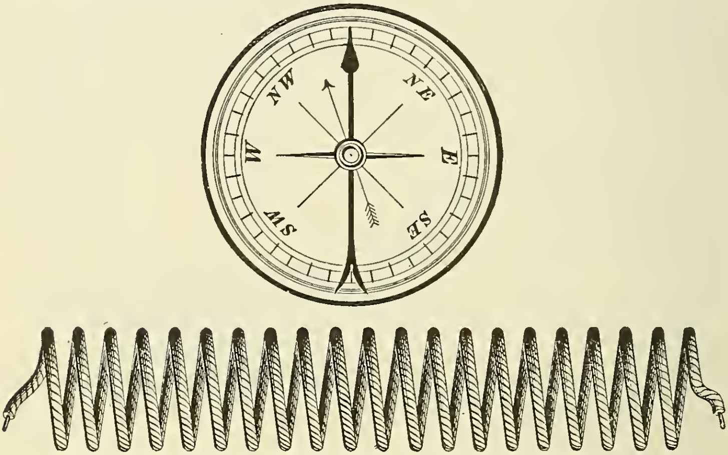


FIG. 951.

MAGNETIZING FORCE AND MAGNETIC DENSITY.

2400. The relation between the magnetizing force and the actual amount of magnetism produced in the core of an electromagnet should be thoroughly understood before studying the special designs of electromagnets and their uses. It has been shown that the magnetic density produced in air by a solenoid depends entirely upon the *intensity of magnetomotive force.* The magnetic density, however, which is produced in a magnetic substance when placed in a solenoid depends upon one other quantity, namely, the *permeability* of the substance. The permeability of a magnetic substance at any stage of magnetization is a ratio between the intensity of the magnetomotive force acting upon the substance and the resulting magnetic density in the substance. Let **H** represent the intensity of the magnetomotive force acting upon a magnetic substance, and let **B** represent the magnetic density produced in the substance, owing to its superior magnetic qualities. The permeability is the quotient arising from dividing **B** by **H**. If μ—pronounced *mu*—represents the permeability, then $\mu = \frac{\mathbf{B}}{\mathbf{H}}$. When any two of these quantities are known, the third can be readily found; for, transposing, $\mathbf{B} = \mu\,\mathbf{H}$ and $\mathbf{H} = \frac{\mathbf{B}}{\mu}$. If there is no magnetic substance in the core of the solenoid, the permeability is 1 and $\mathbf{H} = \frac{3.192 \times \text{ampere-turns}}{l}$; then,

$$\mathbf{B} = \mu\,\mathbf{H} = \frac{1 \times 3.192 \times \text{ampere-turns}}{l} = \frac{3.192 \times \text{ampere-turns}}{l}.$$

Hence, **H** may be expressed by saying that in air it will produce a density of **H** lines of force per square inch. In another case, an iron ring is wound with 100 turns of wire and a current of 10 amperes is flowing through the wire. If the mean length of the magnetic circuit in the ring is 10 inches, by formula **430**, $= \frac{3.192 \times a\text{-}t}{l} = \frac{3.192 \times 10 \times 100}{10} =$ 319.2. The magnetic density produced in the ring by this magnetizing force depends upon the permeability of the iron at that stage of magnetization. By the aid of certain electrical instruments, which will be described in the section on Electrical Measurements, the magnetic density in the iron ring can be determined directly. Suppose, for an illustration, the magnetic density is found to be 63,840 lines of force per square inch. Then, $\mu = \frac{\mathbf{B}}{\mathbf{H}} = \frac{63{,}840}{319.2} = 200$, which represents the permeability of the iron when the density is 63,840 lines of force per square inch. The permeability, however, of a given magnetic substance changes with every stage of magnetization. In all kinds of magnetic substances, the permeability decreases when the magnetism is increased beyond a certain limit. This tendency of the substance to become less permeable is called **magnetic saturation;** that is, the substance becomes *saturated* with magnetism. A limit is never reached where actual saturation takes place, but there is a limit beyond which it becomes impracticable to magnetize the substance. The practical saturation point in wrought iron, soft annealed sheet iron, and cast steel is when the density is between 120,000 and 130,000 lines of force per square inch. Hence, in these metals, **B** may have any value from 0 to 130,000.

In gray cast iron the practical saturation limit is from 60,000 to 70,000 lines of force per square inch.

The intensity of magnetomotive force **H** is very seldom carried beyond 1,500, and, therefore, **H** may have any value between 0 and 1,500.

2401. Before designing an electromagnet for any purpose, it is first necessary to know the magnetic properties of the particular quality of iron to be used in the core—to find its permeability at different stages of magnetization and its saturation limit. Tests are taken upon small samples of the metal by electrical instruments, and the values of **B**, **H**, and μ are calculated from the readings of the instruments. As these tests require exceedingly delicate instruments and a large number of careful measurements, it is customary to consult the results taken in some laboratory on an average quality of iron and its alloys. The results given in Tables 79, 80, 81, and 82 have been found to agree very closely with iron and steel ordinarily used in foundries and machine-shops.

Table 79 is a list which gives seven values of **H** and the corresponding values of **B** and μ, taken on a piece of ordinary *gray cast iron* of average quality.

TABLE 79.

Gray Cast Iron.		
B	**H**	μ
10,000	64	156.3
20,000	105	190.5
30,000	164	182.9
40,000	262	152.9
50,000	430	116.3
60,000	718	83.6
65.000	1,030	63.1

TABLE 80.

Cast Steel—Unannealed.		
B	**H**	μ
10,000	18	555.5
20,000	28	714.3
30,000	35	857.1
40,000	43	930.2
50,000	54	925.9
60,000	72	833.3
70,000	99	707.1
80,000	146	547.3
90,000	225	400.0
100,000	375	266.6
110,000	730	150.7
115,000	1,015	113.3

Table 80 gives the results of a test on an average quality of *cast steel* when unannealed. The effect of annealing metals is to increase their permeability at low stages of magnetization. In practice, however, it is found most economical to magnetize cast steel above 75,000 lines of force per square inch, and at such stages of magnetization annealing has practically no effect upon its permeability.

Table 81 gives the results of a test taken on sheets .014 in. thick, of *soft annealed charcoal iron* of average quality.

Table 82 gives the results of a test taken on an average quality of wrought-iron forgings.

The peculiarities of these tests should be carefully noted. For example, it will be seen that at all stages of magnetization, cast iron is vastly inferior to any one of the other three metals. To produce a density of 40,000 lines of force per square inch in cast iron requires that **H** = 262; whereas, in cast steel at the same density, **H** = 43, which indicates that at this density cast iron would require 262 ÷ 43, or 6.093 times as much magnetizing force as would be required for cast steel. Therefore, other things being equal, it would be more economical to use cast steel rather than cast iron for magnetic purposes.

TABLE 81.

Sheet Iron—Annealed.

B	**H**	μ
10,000	16	625.0
20,000	23	869.6
30,000	28	1,071.4
40,000	33	1,212.1
50,000	42	1,190.4
60,000	53	1,132.0
70,000	68	1,029.4
80,000	94	851.0
90,000	138	652.2
100,000	214	467.3
110,000	374	294.1
120,000	725	165.5
125,000	1,075	116.3

TABLE 82.

Wrought-Iron Forgings.

B	**H**	μ
10,000	12.0	833.3
20,000	15.0	1,333.3
30,000	18.8	1,595.7
40,000	23.0	1,739.1
50,000	30.0	1,666.6
60,000	44.0	1,363.6
70,000	65.0	1,076.9
80,000	104.0	769.2
90,000	200.0	450.0
100,000	430.0	232.6
105,000	630.0	166.6
110,000	1,035.0	106.3

CURVES OF MAGNETIZATION.

2402. The most convenient mode of representing the magnetic qualities of iron and other magnetic substances is to plot the *curves of magnetization* on two sheets of cross-section paper. On one sheet are plotted **saturation curves** which indicate the relation of the intensity of the magnetomotive force **H** to the magnetic density **B**; on the other sheet are plotted the resulting **permeability curves** which indicate the relation of the permeability μ to the magnetic density **B**.

The cross-section paper should be divided into squares of equal dimensions of about $\frac{1}{2}$ inch on a side, although it will be more accurate if these squares are still further divided into smaller ones $\frac{1}{10}$ inch on a side. The sheets should be at least 11 inches wide by 14 inches high.

The horizontal divisions are called **abscissas,** and are indicated by numbers placed in the margin either above or below the chart. The vertical divisions are called **ordinates,** and are represented by marginal numbers on the right or left hand of the chart. The terms *abscissa* and *ordinate*, therefore, express clearly which set of divisions, the horizontal or vertical, is referred to, instead of designating the rows of figures with reference to relative position.

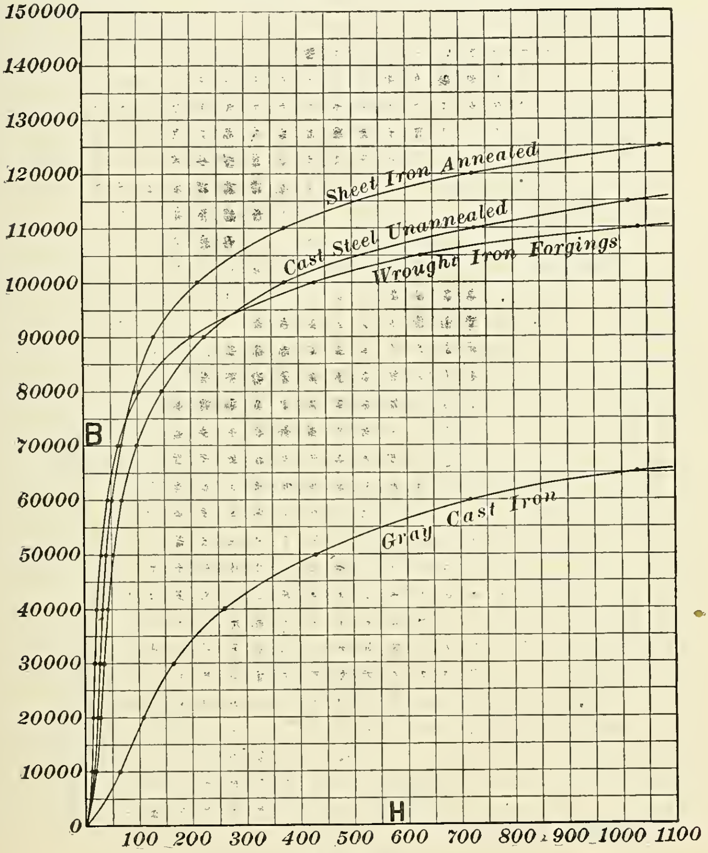


Fig. 952.

2403. On the sheet for the saturation curves, Fig. 952 (reduced), the divisions of the abscissas represent the different values of **H**, and each $\frac{1}{2}$-inch division represents 50 **H**.

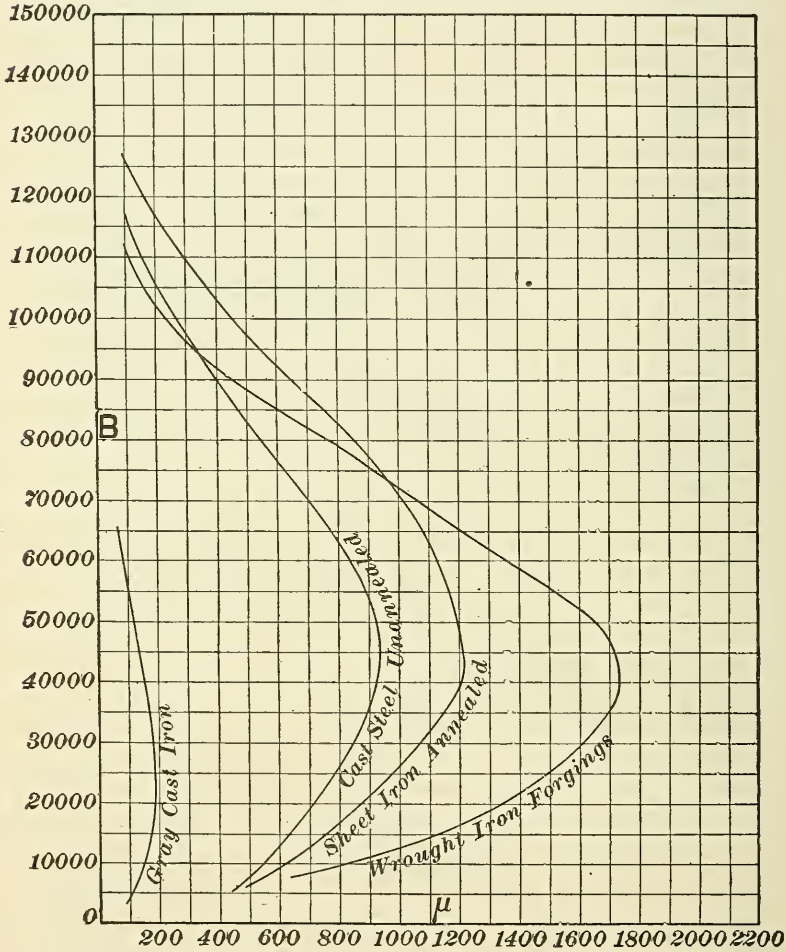


FIG. 953.

Starting with the extreme lower left-hand line as zero, the remaining lines are numbered consecutively in units of 50. The ordinates represent the different values of the magnetic density **B**, and each $\frac{1}{2}$-inch division represents 5,000 **B**.

Starting with the bottom line as zero, the remaining lines are numbered consecutively in units of 5,000.

2404. On the sheet for the permeability curves, Fig. 953 (reduced), the divisions of the abscissas represent the different values of μ and each $\frac{1}{2}$-inch division represents 100 μ. Starting with the extreme left-hand line as zero, the remaining lines are numbered consecutively in units of 100. The ordinates represent the different values of **B**, and are numbered as described for **B** on the sheet for saturation curves.

METHOD OF PLOTTING CURVES.

2405. In the first set of readings on cast iron, Table 79, **H** = 64 and **B** = 10,000. A dot is placed on the bottom line, Fig. 952, representing 64 **H**. The value **B** = 10,000, when pointed off on the extreme left-hand vertical line, is represented by two divisions, and the point falls on the line marked 10,000. This line is followed along horizontally until a point is reached which is directly over the dot on the bottom line. A heavy dot placed here will indicate the combined values of **B** and **H** at the first readings. The remaining readings in Table 79 are plotted in a similar manner, and afterwards all the heavy dots are joined together by one long curve. All the intermediate values of **H** and the corresponding values of **B** are now indicated by the curved line. For example, in the saturation curve for cast iron, where **H** is 350, the corresponding value of **B** is about 46,000 lines of force per square inch. The same method is used for plotting the rest of the saturation curves in Fig. 952 and the permeability curves in Fig. 953.

ACCURACY OF CURVES.

2406. If cross-section paper with $\frac{1}{2}$-inch divisions is used, the curves should be plotted and read with the help of a scale divided into tenths of an inch. Under these conditions, points plotted within $\frac{1}{20}$ of an inch of their correct position on the sheet will be considered as accurate.

All magnetic calculations in which readings are used that are taken from the saturation and permeability curve sheets will be considered accurate when within 2.5% of the correct figures.

CALCULATION OF THE MAGNETIC CIRCUIT.

2407. The calculation of a magnetic circuit is a more complicated problem than that of the electric circuit, but the operation is much simplified by treating the magnetic circuit in the same manner as an electric one and applying the principle of Ohm's law; it must be understood, however, that it is only the *principle* of Ohm's law that is applied, and not any of the actual electrical quantities.

The magnetomotive force has been described as that which *produces* the magnetism, but it will now be considered as that *which tends to drive the lines of force along the magnetic circuit against a resistance.*

The resistance, or that which opposes the lines of force, is called *reluctance*, to distinguish it from electrical resistance.

2408. The quantity of magnetism or the total number of lines of force which are driven along the magnetic circuit is called the *induction*, and is found by dividing the magnetomotive force by the reluctance. Or, expressed algebraically, it will give the formula

$$\text{Induction} = \frac{\text{magnetomotive force}}{\text{reluctance}}.$$

The numerical value for the magnetomotive force is always 3.192 × *ampere-turns.*

2409. The reluctance of the magnetic circuit depends upon three quantities: (1) the length of the circuit, (2) the sectional area of the circuit, and (3) the permeability of the substances which form the circuit.

The reluctance:

Increases as the length of the magnetic circuit increases.

Decreases as the sectional area increases.

Decreases as the permeability increases.

If l represents the length of a magnetic circuit in inches,

A its sectional area in square inches, and μ its permeability, the reluctance of the circuit can be expressed by the formula

$$\text{Reluctance, } \mathbf{R} = \frac{l}{A \times \mu}. \qquad \textbf{(431.)}$$

Writing N for the induction, $a\text{-}t$ for the ampere-turns, and substituting the values of the magnetomotive force and reluctance, the formula for the magnetic circuit, given in Art. **2408**, becomes

$$N = \frac{3.192 \times a\text{-}t}{\mathbf{R}}. \qquad \textbf{(432.)}$$

In practice, the *induction*, or the total number of lines of force, is established in the beginning by the requirements of the magnet, and, therefore, it is necessary to find the number of ampere-turns required to drive that induction along the magnetic circuit. By transposing, the ampere-turns

$$a\text{-}t = \frac{N}{3.192} \times \mathbf{R}.$$

The magnetic circuit, however, is generally a compound one; that is, it is composed of two or more substances. The total reluctance of the circuit would then be the sum of the separate reluctances of each substance. Let $\frac{l_1}{A_1 \times \mu_1} = \mathbf{R}_1$ be the reluctance of the first substance, $\frac{l_2}{A_2 \times \mu_2} = \mathbf{R}_2$ be the reluctance of the second, and so on. Then, the sum of the separate reluctances is $\mathbf{R}_1 + \mathbf{R}_2 +$ etc. Therefore, the ampere-turns $a\text{-}t$ are given by the formula

$$\text{ampere-turns } a\text{-}t = \frac{N}{3.192} \times (\mathbf{R}_1 + \mathbf{R}_2 + \text{etc.}). \qquad \textbf{(433.)}$$

2410. After the dimensions and induction of a magnet have been established by the requirements, it is necessary to know the permeabilities μ_1, μ_2, etc., before the ampere-turns can be calculated. The permeability depends not only upon the kind and quality of the magnetic substance, but also upon the density of the lines of force. The density is found (see formula **427**) by dividing the total number of lines of force which pass through a circuit by its sectional area. Consequently, the densities in the different substances which compose the magnetic circuit will be $\frac{N}{A_1}$, $\frac{N}{A_2}$, etc. Then, referring to the curves in Fig. 953, the permeability of any of the different metals, corresponding to their densities, can be found. *The permeability of all non-magnetic substances is always 1, irrespective of the density of the lines of force.*

Example.—Find the ampere-turns required to drive an induction of 55,000 lines of force through the circuit of a horseshoe magnet made of cast iron, when a bar of wrought iron is placed across its two ends, but separated from them by an air-gap of $\frac{1}{4}$ inch. The dimensions of the magnet and bar are shown in Fig. 954.

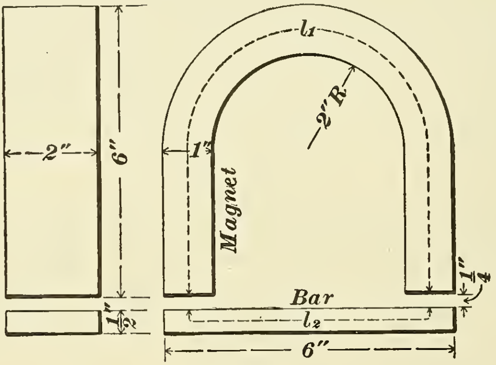


Fig. 954.

Solution.—This magnetic circuit is a compound one, composed of three different substances: (1) the cast-iron magnet, (2) the wrought-iron bar, and (3) the two air-gaps.

Let $N =$ total induction ;
l_1, l_2, and $l_3 =$ the average lengths of circuit in magnet, bar, and total air-gap, respectively ;
A_1, A_2, and $A_3 =$ the sectional areas, respectively ;
$\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3 =$ the magnetic densities, respectively ;
$\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3 =$ the reluctances, respectively ;
μ_1, μ_2, and $\mu_3 =$ the permeabilities, when the densities are $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{B}_3$, respectively.

By formula **433,** the ampere-turns $a\text{-}t = \frac{N}{3.192} \times (\mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3)$.

By formula **431,** the reluctance of the circuit in the cast-iron magnet is $\mathbf{R}_1 = \frac{l_1}{A_1 \times \mu_1}$. The length of the circuit $= l_1 = \frac{5 \times 3.1416}{2} + 6 = 13.854$ inches. The sectional area $= A_1 = 2 \times 1 = 2$ square inches. By formula **427,** the density $\mathbf{B}_1 = \frac{N}{A_1} = \frac{55{,}000}{2} = 27{,}500$ lines of force per square inch. From Fig, 953, μ is about 180, when $\mathbf{B} = 27{,}500$ in cast iron. Then the reluctance

$$\mathbf{R}_1 = \frac{l_1}{A_1 \times \mu_1} = \frac{13.854}{2 \times 180} = .03848.$$

The reluctance of the circuit in the wrought-iron bar is $\mathbf{R}_2 = \frac{l_2}{A_2 \times \mu_2}$. The length of the circuit $= l_2 = 5 + .25 + .25 = 5.5$ inches. The sectional area $= A_2 = 2 \times .5 = 1$ square inch. $\mathbf{B}_2 = \frac{N}{A_2} = \frac{55{,}000}{1} = 55{,}000$ lines of force per square inch. From Fig. 953, μ is about 1,520 when $\mathbf{B} = 55{,}000$ in wrought iron. Then, by formula **431,** the reluctance,

$$\mathbf{R}_2 = \frac{l_2}{A_2 \times \mu_2} = \frac{5.5}{1 \times 1{,}520} = .00362.$$

Since one magnetizing coil is used for the whole magnetic circuit, the two air-gaps are added together, and in the calculations a single air-gap of double length, that is, $2 \times \frac{1}{4} = \frac{1}{2}$ inch, is considered. The reluctance of the circuit in the air-gap is $\mathbf{R}_3 = \frac{l_3}{A_3 \times \mu_3}$. The length of the circuit $= l_3 = .5$ inch. The sectional area $= A_3 = 2 \times 1 = 2$ square inches. In the case of air, the permeability $\mu_3 = 1$.

$$\text{The reluctance is then} = \frac{l_3}{A_3 \times \mu_3} = \frac{.5}{2 \times 1} = .25.$$

By formula **433,** the necessary ampere-turns =

$$\frac{55{,}000}{3.192} \times (.03848 + .00362 + .25) = \frac{55{,}000}{3.192} \times .2921 = 5{,}033.05,$$

which means that a magnetizing force of 5,033.05 *ampere-turns* will have to circulate around the magnet arms to force 55,000 lines of force through the magnetic circuit. Ans.

RESIDUAL MAGNETISM.

2411. Residual magnetism is the magnetism which a magnetic substance retains after being removed from a magnetic field. In general, soft iron and annealed steel retain only a small amount of magnetism, and in some cases the residual magnetism is imperceptible. A closed magnetic circuit of soft iron, that is, a magnetic circuit which consists of soft iron throughout its entire length, will exhibit a large amount of residual magnetism so long as the circuit remains unbroken. This tendency can be shown by a **U**-shaped electromagnet of soft iron, across the two ends of which is placed a well-fitted *keeper*. If the circuit is magnetized by a current of electricity which is suddenly turned off, the keeper will still adhere to the ends, and may even require considerable force to detach it. But when once it is detached and the circuit broken, the keeper will not adhere again without the aid of the current.

Chilled iron and hardened steel retain residual magnetism in large quantities. Artificial or permanent magnets are made by placing a piece of hardened steel in a dense magnetic field or in contact with another magnet. Lodestone is the result of a natural residual magnetism.

HYSTERESIS.

2412. When the magnetism of an electromagnet is rapidly reversed, that is, when the direction of the lines of force is suddenly changed several times in rapid succession by changing the direction of the magnetizing current, the iron or steel becomes heated, and a certain amount of energy will be expended. This effect is due to a kind of internal *magnetic friction*, by reason of which the rapid changes of magnetism cause the iron to grow hot. This effect is called **hysteresis** (histeree′-sis).

2413. The energy expended by hysteresis is furnished by the force which causes the change in the magnetism; in the case of an electromagnet, where the magnetism is reversed by the magnetizing force, the energy is supplied by the magnetizing current.

The complete operation of magnetizing and demagnetizing an electromagnet in one direction, then magnetizing and demagnetizing in the opposite direction by reversing the magnetizing current, is called a **cycle of magnetism.**

One *cycle* is made by two reversals of magnetism. For example, reversing the magnetism 40 times in one second will make 20 cycles in one second.

The loss of energy by hysteresis depends (1) upon the hardness and quality of the magnetic substance in the core; (2) upon the amount of metal magnetized; (3) upon the number of cycles per second, and (4) upon the density in the substance when the magnetizing force is not changing.

2414. Table 83 gives the power in watts expended by hysteresis in soft sheet iron when subjected to a rapid succession of cycles of magnetism at different magnetic densities. The watts expended are directly proportional to the number of cycles per second and to the number of cubic inches of iron magnetized.

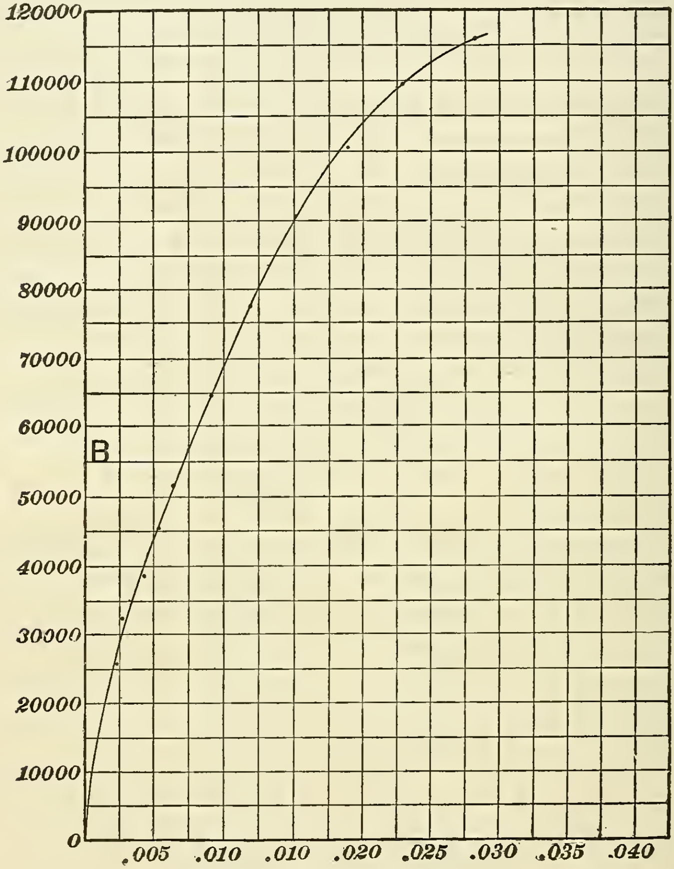
TABLE 83.

B	Watts Expended per Cubic Inch, 1 Cycle per Sec.
25,800	.002320
32,250	.002715
38,700	.004340
45,150	.005320
51,600	.006370
64,500	.009040
77,400	.011920
90,300	.015180
103,200	.018780
109,650	.022850
116,100	.028150

Let w = power in watts expended per cubic inch per cycle;
v = volume in cubic inches;
n = cycles per second;
W = total watts expended.

Then, $$W = w\,v\,n. \qquad \textbf{(434.)}$$

Rule.—*To find the power expended by hysteresis in sheet iron at a given stage of magnetization, multiply the watts expended at that stage, as given in Table 83, or Fig. 955, by the number of cubic inches of iron in the magnet and the number of cycles per second.*



Watts per cubic inch for one cycle.

Fig. 955.

The readings given in Table 83 are plotted on a sheet of cross-section paper in Fig. 955, and the various points are connected by a curved line. The ordinates represent the different densities **B**, and the abscissas the corresponding number of watts expended in one cubic inch of iron for one cycle per second. By referring to the curve, all the intermediate values of **B** and the corresponding watts expended can be determined.

Example.—In an electromagnet, made with sheets of soft iron, there are 18 cubic inches of iron. Find the power in watts expended when the magnetizing current is reversed 70 times per second and the magnetism reaches a density of 90,000 lines of force per square inch.

Solution.— 70 reversals are equivalent to 35 cycles $= n$. From Fig. 955, the watts expended per cubic inch for one cycle, at a density of 90,000, are equal to .015. Then, by formula **434**, the total power expended,

$$W = .015 \times 18 \times 35 = 9.45 \text{ watts. Ans.}$$

LEAKAGE.

2415. All the lines of force produced by the magnetomotive force can not be confined along one path; a certain number in every magnetic circuit will stray from the main circuit and take shorter cuts. This tendency is called **magnetic leakage.**

2416. The magnetic leakage becomes greater when the reluctance along the main circuit is not uniform at all points. The nature of magnetic leakage may be better understood by remembering that air is really a magnetic conductor, although its reluctance is much greater than that of iron or other magnetic substance. Consequently, when the reluctance of the main circuit becomes large at any point, some of the lines of force find a shorter and easier path for themselves through the surrounding air.

Fig. 956 represents a **U**-shaped electromagnet made of iron with a keeper of the same metal and sectional area. By placing the keeper tightly against the two ends, the reluctance becomes practically uniform throughout the entire magnetic circuit, and there is no perceptible leakage at any place. But if the reluctance of the circuit is changed by separating the keeper from the ends of the magnet by a small air-gap, as in Fig. 957, the conditions are altered.

In the first place, the total number of lines of force will be reduced in all parts of the circuit, and, secondly, a certain number of the lines of force will leak across from end to end of the magnet without passing through the keeper. The larger the air-gap between the keeper and the magnet, the greater will be the magnetic leakage. An approximate idea of the magnetic leakage is shown in Fig. 958, where the keeper is placed at a considerable distance from the ends of the magnet, and Fig. 959 shows the state of the lines of force when the keeper is removed entirely.

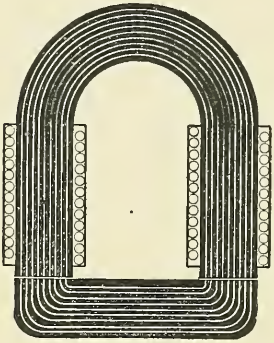


FIG. 956.

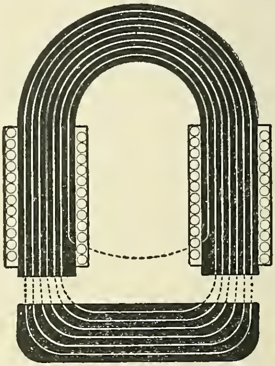


FIG. 957.

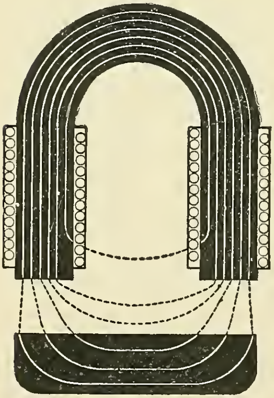


FIG. 958.

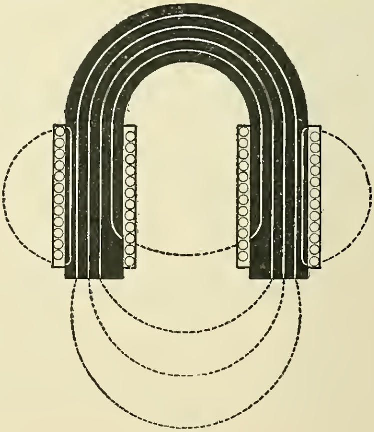


FIG. 959.

2417. Magnetic leakage may be also defined as the difference between the number of lines of force produced by the magnetomotive force and the number that are *useful* in attracting or lifting a given weight.

There are no definite laws governing magnetic leakage, and it is almost impossible to calculate the number of stray lines of force in any compound magnetic circuit. After a magnet is built, the leakage can be determined with the proper instruments and under certain conditions.

In general, if the magnetic circuit is composed of magnetic substances whose permeabilities are high and there are no large air-gaps to be crossed, the magnetic leakage will be but a small factor.

2418. If the total number of lines of force produced by the magnetizing coils and the useful number are known, the magnetic leakage can be expressed by a per cent. of the total number produced. Thus,

Let l = total number of lines of force;
l_u = useful number of lines of force;
l_s = stray lines of force;
p = per cent. leakage.

Then,

$$l_s = l - l_u. \qquad \textbf{(435.)}$$

For example, assuming that 60,000 lines of force are produced by the magnetizing coils of an electromagnet, and that only 42,000 are useful in attracting an armature or lifting a weight, then by formula **435** the number of stray lines of force $l_s = 60{,}000 - 42{,}000 = 18{,}000$.

2419. The percentage of leakage is found from the formula

$$p = \frac{100\, l_s}{l}. \qquad \textbf{(436.)}$$

That is to say, *the percentage of leakage is found by dividing the stray number of lines of force by the total* *number produced and multiplying the quotient by 100.* In the above case

$$p = \frac{100 \times 18{,}000}{60{,}000} = 30\% \text{ leakage.}$$

2420. To find the total number of lines of force when the percentage of leakage and the number of useful lines of force are known, use the following formula:

$$l = \frac{100\, l_u}{100 - p}. \qquad \textbf{(437.)}$$

Here we *divide the useful lines of force by 100 minus the per cent. leakage and multiply the quotient by 100.*

EXAMPLE.—Assuming that the magnetic leakage in an electromagnet is 25% and that there are 75,000 useful lines of force, how many lines of force are produced by the magnetizing coils?

SOLUTION.—By formula **437,** the total lines of force

$$l = \frac{100 \times 75{,}000}{100 - 25} = \frac{7{,}500{,}000}{75} = 100{,}000$$

total lines of force produced by the magnetizing coils. Ans.

EXAMPLES FOR PRACTICE.

2421. 1. 100,000 lines of force are produced by the magnetizing coils of an electromagnet and only 40,000 are useful. What is the % leakage? Ans. 60% leakage.

2. In an electromagnet there are 27,000 stray lines of force and 63,000 useful; find the % leakage. Ans. 30% leakage.

3. The magnetic leakage in an electromagnet is 45% and there are 110,000 useful lines of force; find the total number of lines produced by the magnetizing coils. Ans. 200,000 lines of force.

4. If the magnetic leakage in an electromagnet is 35% and there are 60,000 lines of force produced by the magnetizing coils, how many lines of force are useful? Ans. 39,000 useful lines of force.

LIFTING MAGNETS.

2422. The lifting power or adhesive force of a magnet is called its **tractive force,** or, simply, *traction.* The common form of electromagnet for traction is a stumpy *horseshoe* magnet M with two magnetizing coils c, c, as shown in Fig. 960. The magnet is generally provided with an armature of soft iron a, which is placed across the two poles. When the current is flowing in the magnetizing coils, the full tractive force of the magnet is exerted between the armature and the two polar surfaces. The maximum tractive force is found by hanging known weights W of any material upon the armature in a suitable manner and observing the heaviest load it will sustain without separating from the magnet. The total tractive force of the magnet will be the weight of the armature plus the load sustained.

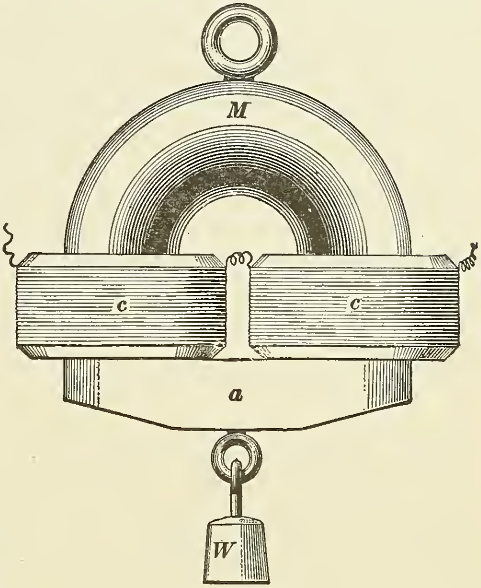


FIG. 960.

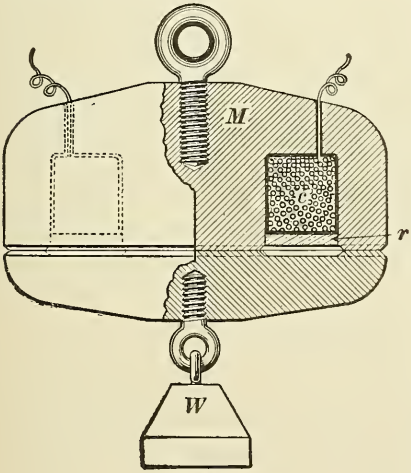


FIG. 961.

2423. Another economical form of electromagnet for traction is made in the shape of a *diving-bell*, as shown in Fig. 961. This magnet is iron-clad; that is, the magnetizing coil is completely surrounded and protected by the return magnetic circuit, and requires only one magnetizing coil to excite it.

If the magnet proper M is made in one casting, the coil c is wound independently in some suitable shape; afterwards it is thoroughly insulated by wrappings of cloth, mica, or tape, then placed around the inside core of the magnet and held in position by a ring of brass or other non-magnetic metal r wedged between the core and the outside shell. The connections to the coil from an outside source are made to *leads* (pronounced *leeds*) passing from the coil up through holes in the top of the magnet. By designing the magnet low and large in diameter, the magnetic circuit can be made exceedingly short in proportion to its sectional area, thus realizing one of the conditions of an economical design.

2424. In determining the tractive force of a magnet, let

A = total area of contact surface;
$\mathbf{B}$ = density in lines of force per square inch;
P = total tractive force in pounds.

Then,
$$P = \frac{\mathbf{B}^2 A}{72{,}134{,}000}. \qquad \textbf{(438.)}$$

That is, *the tractive force of a magnet increases directly as the total area of the surface in contact with the armature, and as the square of the density of the lines of force in the magnetic circuit where it passes across that surface.* Formula **438** is deduced from the force exerted upon a unit pole placed in a unit magnetic field, and assumes that the distribution of the lines of force is uniform throughout the entire contact surface. In actual practice it is impossible to obtain this result on account of magnetic leakage and other causes. The calculated load and the actual load lifted will generally differ—the actual being somewhat less than the calculated, due to the fact that some of the magnetic lines leak away from the attracting surfaces.

In all electromagnets designed for traction there will be two contact surfaces, one at the north pole of the magnet and the other at the south pole; or, in other words, the total lines of force developed in the magnetic circuit are used twice in producing the traction of the magnet. If the two contact surfaces are symmetrical and equal in area, the total tractive force of the magnet will be twice the result obtained by considering one contact surface alone; but if the contact surfaces are unlike, the tractive force exerted by each surface should be calculated separately, and the two results thus obtained added together.

2425. The most economical electromagnet designed for traction is one that will lift the greatest load in proportion to its *own weight*. To accomplish this result, the following facts must be considered:

The magnetic circuit in the magnet and keeper should be as short as possible.

The sectional area of the magnetic circuit should be uniform and large in proportion to the over-all dimensions.

The iron or steel used in the magnet and keeper should have a high permeability.

The magnetic density of the contact surface should be about 110,000 lines of force per square inch, for, if the magnetism is pushed higher than this density, the reluctance of the magnetic circuit will be increased, which increases the weight of the copper used in the magnetizing coils.

CALCULATION FOR LIFTING MAGNET.

2426. To find the magnetic density at the contact surface required to produce a given tractive force when the area of the contact surface is known:

Let A = area of contact surface in square inches;
P = tractive force in pounds;
$\mathbf{B}$ = magnetic density of lines of force at contact surface.

Then.
$$\mathbf{B} = 8{,}493\sqrt{\frac{P}{A}}. \qquad \textbf{(439.)}$$

Rule.—*In an electromagnet the density of lines of force at the contact surface is equal to 8,493 times the square root of the tractive force in pounds divided by the area in square inches.*

2427. To find the total number of lines of force in the magnetic circuit when the tractive force and the magnetic density at the contact surface are known:

Let N = the induction, or the total number of lines of force.

Then, $$N = 72{,}134{,}000 \frac{P}{\mathbf{B}}. \qquad \textbf{(440.)}$$

Rule.—*The total number of lines of force in an electromagnet is found by dividing the tractive force in pounds by the magnetic density at the contact surface and multiplying the quotient by 72,134,000.*

2428. To find the tractive force in pounds per square inch when the area of the contact surface and the total number of lines of force are known:

Let p = tractive force in pounds per square inch.

Then, $$p = \frac{N^2}{72{,}134{,}000\, A^2}. \qquad \textbf{(441.)}$$

Rule.—*The tractive force of an electromagnet in pounds per square inch is equal to the square of the number of lines of force divided by 72,134,000 times the square of the area of contact surface in square inches.*

2429. To find the tractive force in pounds per square inch when the density at the contact surface is known:

$$p = \frac{\mathbf{B}^2}{72{,}134{,}000}. \qquad \textbf{(442.)}$$

Rule.—*The tractive force of an electromagnet in pounds per square inch is equal to the square of the magnetic density at the contact surface divided by 72,134,000.*

2430. To find the area of the contact surface when the total number of lines of force and the tractive force are known:

$$A = \frac{N^2}{72{,}134{,}000\, P}. \qquad \textbf{(443.)}$$

Rule.—*The total area of contact surface of an electromagnet is found by dividing the square of the total number of lines of force by 72,134,000 times the tractive force in pounds.*

2431. To find the number of ampere-turns required to energize a magnet for a given traction when the permeability of the iron or steel used is known and the dimensions of the armature and magnet have been established:

Let $P =$ tractive force of one contact surface; then, $2\,P$ is the total tractive force of the magnet;
l_1 and $l_2 =$ the lengths of the magnetic circuit in magnet and armature, respectively;
A_1 and $A_2 =$ sectional areas of magnetic circuit in magnet and armature, respectively;
μ_1 and $\mu_2 =$ permeabilities of the iron or steel used in the magnet and armature, respectively;
$\mathsf{B} =$ magnetic density at contact surface.

Then, the ampere-turns

$$a\text{-}t = 22{,}598{,}370 \times \frac{P}{\mathsf{B}} \times \left(\frac{l_1}{A_1 \times \mu_1} + \frac{l_2}{A_2 \times \mu_2}\right). \qquad \textbf{(444.)}$$

Rule.—*In the case of an electromagnet intended to develop a given tractive power, the ampere-turns are equal to the tractive force of one contact surface multiplied by the reluctance of the circuit and by 22,598,370, and divided by the magnetic density at the contact surface.*

2432. To find the ampere-turns required to energize a magnet for a given tractive force when the armature and magnet are made of the same quality of iron or steel and the sectional area of the magnetic circuit is the same in the armature, magnet, and contact surfaces:

Let $l =$ total length of magnetic circuit in inches;
$P =$ tractive force at one surface;
$\mu =$ permeability of iron or steel used;
$A =$ cross-sectional area of magnetic circuit;
$N =$ total number of lines of force in the magnetic circuit.

The ampere-turns necessary, $a\text{-}t = 2{,}661 \times \frac{l}{\mu} \times \sqrt{\frac{P}{A}}$. **(445.)**

Rule.—*To determine the ampere-turns for an electromagnet of uniform sectional area and material when the tractive force at one surface is given, find the square root of the tractive force divided by the area, multiply this value by 2,661 times the length of circuit in inches and divide by the permeability.*

As showing the relation between formulas **439** and **445**, the latter may be written:

$$a\text{-}t = \frac{8{,}493\sqrt{\frac{P}{A}}}{3.192} \times \frac{l}{\mu} = \frac{\mathbf{B}}{3.192} \times \frac{l}{\mu}. \qquad \textbf{(446.)}$$

2433. In designing an electromagnet for a certain tractive force, several assumptions have to be made in the beginning. The first assumption is the magnetic density in the armature, magnet, and contact surface. If wrought iron, cast steel, or soft annealed sheet iron is used, the density in the armature and magnet should be between 100,000 and 120,000 lines of force per square inch. If, however, the metal is gray cast iron, the density should be between 50,000 and 70,000 lines of force per square inch. As already stated, the density of the contact surface in any coil should be about 110,000 lines of force per square inch. If the magnet is made of cast iron in which the density is low, the edges of the pole-pieces should be chamfered off to increase the density of the contact surface. This chamfering will slightly increase the reluctance of the magnetic circuit at that point, but the amount will be small and can be neglected. The next assumptions are the over-all dimensions of the magnet. The relation between the tractive force for which the magnet is to be designed and the magnetic densities determines the sectional areas of the armature and magnet, but does not give any information regarding the over-all dimensions. Several trials may be necessary to determine the most economical dimensions. In the first trial, ample space should be left for the magnetizing coils, and if this space is found to be too small or larger than necessary, the over-all dimensions should be changed and the magnet recalculated.

Example.—Design an electromagnet for a maximum tractive force of 672 pounds.

Solution.—From formula **442** the tractive force in pounds per square inch $p = \frac{\mathsf{B}^2}{72,134,000}$. Using a density of 110,000 lines of force at the contact surface gives $p = \frac{110,000^2}{72,134,000} = 167.74$, or about 168 pounds per square inch. The total tractive force divided by the tractive force per square inch gives the total area of the contact surfaces. Therefore, $\frac{672}{168} = 4$ square inches for the area of the two contact surfaces, or 2 square inches for the area of one contact surface. The total lines of force in the circuit are $110,000 \times 2 = 220,000$.

In the first trial, imagine a bar of wrought iron 8 in. long, 2 in. wide, and 1 in. thick, bent in the direction of its least dimension into the form of a horseshoe with straight sides, so that the distance between the centers of the poles is 3 in. The armature may be a bar of wrought iron 4 in. long, 2 in. wide, and 1 in. thick. The sectional area of the magnetic circuit is 2 square inches in magnet, armature, and contact surface, and the density is 110,000 lines of force per square inch. From Table 82, when $\mathsf{B} = 110,000$ the permeability in wrought iron is 106.3. The mean length of the magnetic circuit in the magnet is 8 in., and in the armature it is $3 + \frac{1}{2} + \frac{1}{2} = 4$ in.; hence, the total length l is $8 + 4 = 12$ in. By formula **445**, the ampere-turns $a\text{-}t = 2,661 \times \frac{12}{106.3} \times \sqrt{\frac{336}{2}} = 2,661 \times \frac{12}{106.3} \times 12.961 = 3,893.42$, or about 3,893 ampere-turns required to magnetize the magnetic circuit under these conditions.

Assuming the current to be 10 amperes, then $3,893 \div 10 = 389.3$, or say 389 turns of a conductor to be wound around the magnet. A copper wire covered with two layers of cotton thread can be used for the conductor. A size of wire must be used which will not heat excessively when a current of 10 amperes is flowing through it. From experiment, it is found that a copper wire .091 in. in diameter will carry 10 amperes with safety. After the wire has been covered with two layers of cotton, the diameter will be about .1 in. The wire should be wound in two coils, one on each pole of the magnet. If each coil is wound in layers extending 2 in. from the polar surfaces, there will be $2 \div .1 = 20$ turns of wire lying side by side, or 20 turns in each layer in each coil. The total number of turns in each coil should be $\frac{389}{2} = 194.5$, or say 195.

The number of turns divided by the turns in one layer will give the number of layers; therefore, $\frac{195}{20} = 9.75$ layers in each coil. The maximum depth of wire will be 10 layers or 1 in. on each coil, which exactly fills up the space between the two poles after both coils have been wound. It is better practice, however, to design the magnet with some space between the two coils; in the preceding example a space of from $\frac{1}{4}$ inch to $\frac{3}{4}$ inch might have been allowed between the two coils.

MAGNETS FOR ATTRACTION.

SHORT-RANGE MAGNETS.

2434. Electromagnets designed for attracting their armatures through a distance can be divided into two subclasses, namely, *short* and *long range* magnets. **Short-range magnets** are used in places where the armature is required to move rapidly through a short distance, exerting comparatively little force; as, for example, in telegraph apparatus, electric bells, arc lights, etc. Such magnets are usually of the horseshoe type, as shown in Fig. 962, which represents an electromagnet for a telegraph relay. In this particular magnet the cores are made of two round bars of soft iron *M*, $\frac{3}{8}$ in. in diameter and 2 in. long. The cores are screwed into a yoke of soft iron *b*, $\frac{3}{8}$ in. wide by $\frac{1}{4}$ in. thick and 2 in. long. The magnetizing coils are wound over vulcanized rubber bobbins or spools, and contain, all told, about 8,500 convolutions, or turns, of insulated copper wire .009 in. in diameter. The total resistance of the wire in the two magnetizing coils is about 150 ohms. A vulcanized rubber shell or cover *c* is slipped over each coil when wound, to protect it from dust and bruises.

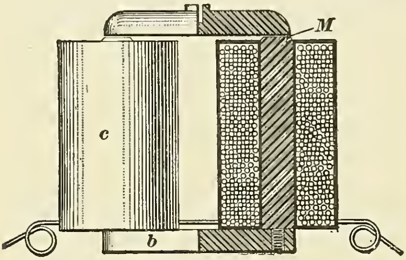


FIG. 962.

2435. Fig. 963 represents another form of magnet used for rapid vibrations of the armature. The cheapness of winding only one coil instead of two and its simplicity of construction recommend it for a large variety of practical uses. The principal disadvan-

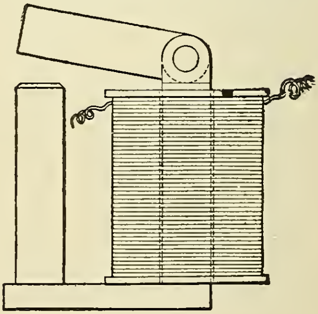


FIG. 963.

tage is the large amount of magnetic leakage caused by an unbalanced magnetic field. There is a large variety of short-range electromagnets adapted to special uses, but all the various types are modifications of the same general principle.

The magnitude of the force which short-range electromagnets are usually required to exert is comparatively small; in most cases the armature moves only a fraction of an inch against the tension of a light helical spring. Consequently, it is unnecessary to calculate the magnetic circuit and the force of attraction. The size and amount of wire to be used for the magnetizing coils depend upon the local conditions, and the most satisfactory results are obtained by experimental trials in each particular case.

LONG-RANGE MAGNETS.

2436. The most economical form of *long-range* magnets is the **coil-and-plunger** magnet; that is, a magnet in which a part or the whole of the armature moves inside the magnetizing coils. The simplest, although the most inefficient, type of such magnets is a straight bar of iron moving freely into one magnetizing coil or solenoid. The bar will always be attracted towards the center of the solenoid, with its neutral line coinciding with that of the solenoid.

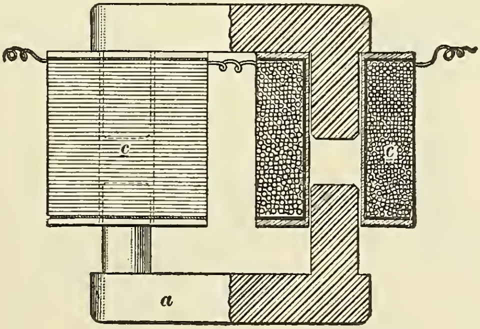


FIG. 964.

The range of action is long, but the force exerted is comparatively weak.

Fig. 964 represents an effective type of *coil-and-plunger* magnet, and one capable of exerting heavy pulls through long ranges. The magnetic circuit is divided at about the center of the coils *c*, *c*, and half of each core is attached to the armature *a*. The advantage thus gained consists in causing the greatest reluctance to take place where the magnetizing force is the strongest, and, hence, the tendency to magnetic leakage is reduced. A coil-and-plunger magnet of this type weighing about 65 pounds will give an *initial* pull of approximately 50 pounds when the air-gap between the cores of the armature and the cores of the yoke is 3 inches. As soon, however, as the armature starts to move into the coils, the reluctance of the magnetic circuit and the magnetic leakage are reduced; consequently, the density of the magnetic field increases, which in turn increases the attractive force. If the magnetizing force remains unchanged, the attractive force when the armature has moved through only part of the distance will be several times the initial attractive force.

2437. A combination of the coil-and-plunger and ironclad types with one magnetizing coil gives an efficient magnet for powerful pulls over short ranges. The inside core *m*, instead of protruding above the top of the magnetizing coil as in ordinary short-ranged ironclads, rises to only about half the height of the coil, as shown in Fig. 965. The other half of the core *n* is attached to the armature *a*, and moves inside the magnetizing coil *c*. This is wound in a metal spool or bobbin, which is rigid enough to serve as a guide for the armature. The range of action is limited on account of the enormous magnetic leakage taking place across the top of the coil when the air-gap becomes large.

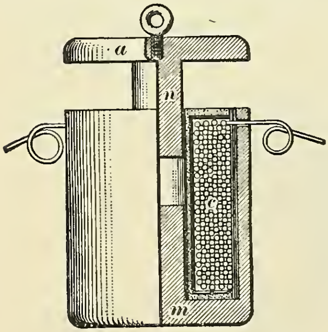


Fig. 965.

ELECTROMAGNETIC INDUCTION.

2438. It has been shown that a magnet and a conductor carrying a current of electricity exert a mutual force upon each other ; or, in other words, each tends to produce motion in the other. In general, when a conductor carrying a current of electricity is placed in a magnetic field, the conductor will tend to move in a definite direction and with a certain force, depending upon the strength and direction of the current and upon the direction and density of the lines of force.

2439. To determine the direction of motion of a conductor carrying a current of electricity when placed in a magnetic field :

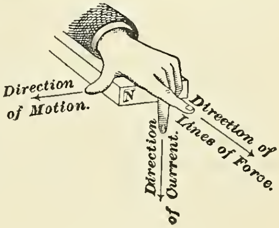


FIG. 966.

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two, as shown in Fig. 966 ; if the forefinger shows the direction of the lines of force and the middle finger shows the direction of the current, then the thumb will show the direction of motion given to the conductor.*

The direction of motion produced in the conductor can also be graphically shown. The diagram, Fig. 967, indicates a cross-section of a magnetic field; the dots represent an end view of the lines of force, and the heavy line a conductor conveying a current of electricity. If the direction of the lines of force is *downwards*, that is, piercing the paper, and if the current flows in the direction indicated by the arrow-heads,

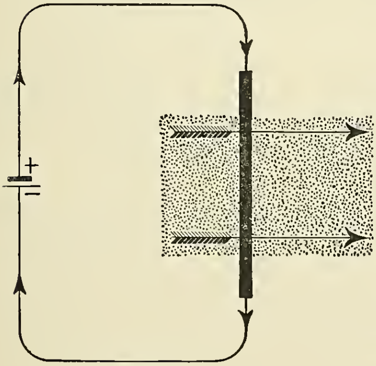


FIG. 967.

then the conductor will be moved bodily to the *right*, as indicated by the two arrows.

2440. This action is also true of an *electric arc* passing through a magnetic field, that is, a current of electricity passing or jumping in the form of a continuous spark between two electrodes across an air-space which is traversed by lines of force, as indicated in Fig. 968. The arc or spark will be impelled to one side in the same direction as the conductor in the previous case. If the electrodes remain in a fixed position relative to the magnetic field, the arc will be *blown out;* that is, the spark will be extinguished and the current will cease to flow in the circuit. In both cases the motion is caused by the mutual action of the lines of force in the magnetic field and those produced by the current itself, as shown in Fig. 969, where the current is assumed to be flowing downwards. The lines of force in the magnetic field tend to coincide in direction with those around the current, and in doing so they exert a *crowding* effect on the current, which, in the first case, produces motion in the conductor, and in the second a tension upon the arc.

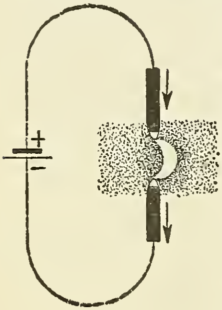


FIG. 968.

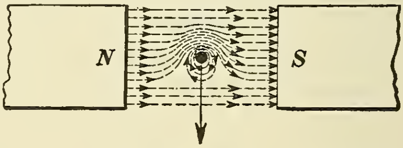


FIG. 969.

2441. The converse of this effect is also true, namely, *when a conductor forming a closed circuit is moved across a magnetic field at right angles to the lines of force, a current is induced in the conductor.*

This statement will be better understood by comparing the action in Fig. 967 with that in Fig. 970. In the former case, when a current is flowing in the direction indicated by the arrow-head the conductor will move bodily to the right. In Fig. 970, however, when the conductor is moved to the right by some exterior means a current is induced in it which tends to flow in an opposite direction to the current which produces the same motion in the former case.

This generation of current may be explained by saying that the motion of the conductor across the lines of force from the magnet sets up an *electromotive force* in the conductor, which, when the circuit is completed, causes a current to flow. The direction of the current induced in the conductor will be at right angles to the lines of force and to the direction of motion of the conductor.

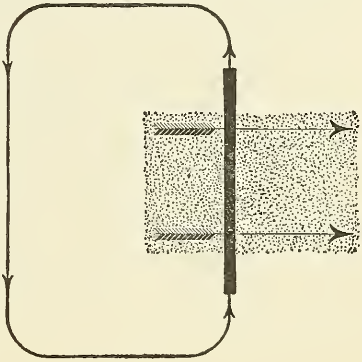


FIG. 970.

2442. To determine the direction of induced currents:

Rule.—*Place thumb, forefinger, and middle finger of the right hand each at right angles to the other two; if the forefinger shows the direction of the lines of force and the thumb shows the direction of motion of conductor, the middle finger will show the direction of the induced current.* (See Fig. 971.)

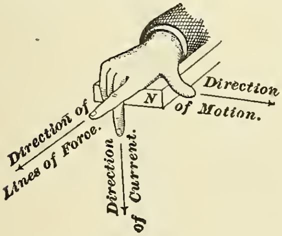


FIG. 971.

2443. The **positive end** of a conductor in which a current is generated by moving across a magnetic field is that end *towards* which the current tends to flow; the **negative end** is that *from* which the current tends to flow.

2444. An electric current will be induced in a *coiled conductor* when a pole of a magnet is suddenly inserted into the coil. The current will be continuous so long as there is a *change in the number of lines of force* passing through the coil, but the current will cease to flow when the number of lines of force becomes constant, that is, when the *lines of force inside the coil do not increase or diminish in number.*

In reality, currents produced in a conductor *cutting* lines of force and currents induced in a *coiled* conductor by a *change* in the number of lines of force which pass through the coil are due to the same motion, for every conductor carrying a current of electricity forms a closed coil, and every line of force is a complete magnetic circuit by itself. Consequently, when any part of a closed coil is cutting lines of force, the lines of force are passing through the coil in a definite direction, and changing at the same rate as the cutting.

In calculations, however, it is more convenient to make a distinction between the two cases, and to consider that the current or, more strictly, the E. M. F., in the first case is *generated* by a conductor of a certain length cutting the lines of force at right angles; while, in the second case, the current in a closed coil is *induced* by a change in the number of lines of force passing through the coil.

2445. The action of induced currents can be shown by a closed coil of any conducting material moving in a magnetic field. If it is moved in a uniform field along the lines of force, as in Fig. 972, so that only the same number of lines of force pass through it, no current will be generated. Or, if the coil be moved across the lines of force in a uniform

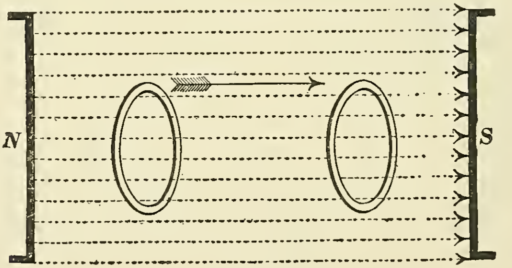


FIG. 972.

field, Fig. 973, as many lines of force are left behind as are gained in advancing, and there will be no current generated in the coil. Rotating the coil on a central axis, like the rim of a pulley, will not generate a current, because there is no change in the number of lines of force passing through the loop.

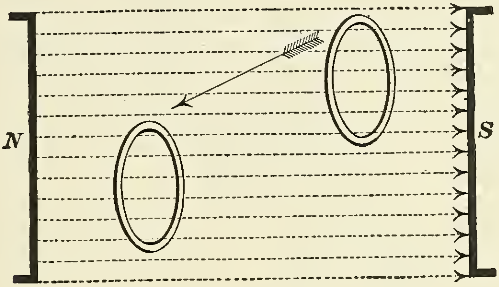


FIG. 973.

But if, as in Fig. 974, the coil be tilted in its motion across the uniform field, or rotated around on any axis in its own plane, then the number of lines of force that pass through it will be altered and a current will be developed. Where the magnetic field is not uniform, the removal of the coil bodily from a place where the lines of force are dense to where they are less dense, as from position *1* to position *2*

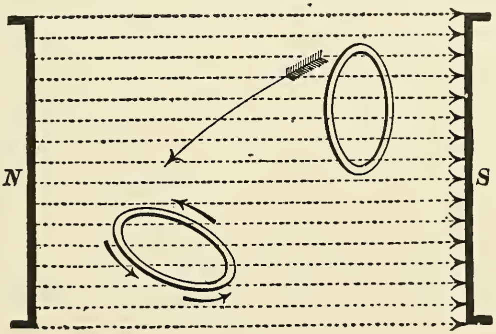


FIG. 974.

in Fig. 975, will cause the generation of a current in the coil ; or if the coil is moved to a place where the direction

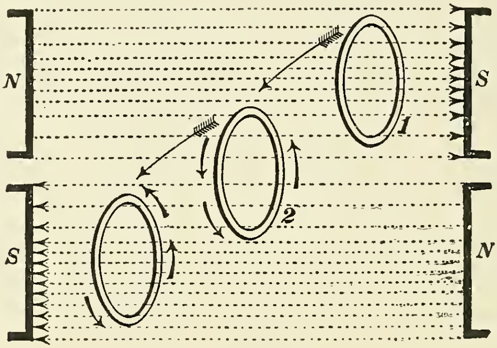


FIG. 975.

of the lines of force is reversed, the effect will be the same.

2446. To determine the direction of induced currents in a closed coil :

Rule.—*If the effect of the movement is to diminish the number of lines of force that pass through the coil, the current will flow around in the conductor in the direction of the hands of a watch as viewed by a person looking along the magnetic field in the direction of the lines of force ; but if the effect is to increase the number of lines of force that pass through the coil, the current will flow around in the opposite direction.*

2447. In the explanations just given, it was stated that *currents* are generated by moving the conductor in a magnetic field. It must be remembered, however, as shown in the beginning, that a current is merely the equalization of a difference of potential. Strictly speaking, therefore, it is not actually *a current*, but *electromotive force*, that is developed by induction in the moving conductor ; for, on opening the circuit, the electromotive force will still exist, but no current can flow. The word *current* is used merely to avoid complication.

EXPERIMENTS WITH ELECTRICAL APPARATUS.

2448. (Art. **2439.**) Take a piece of wire about 12 inches long ; about an inch each side of the center make a right-angle bend ; bare the ends of the wire and bend about an inch of each end into a loop. This will make a sort of *trapeze* of wire, as shown at *A A*, Fig. 976. Bare the ends of two wires leading from the battery (via the reversing switch), scrape them bright to ensure good contact, and support them in the same line about 2 inches apart, so that the bent wire may hang from them, as shown in the figure, where *S* and *S* represent the supports of the wires.

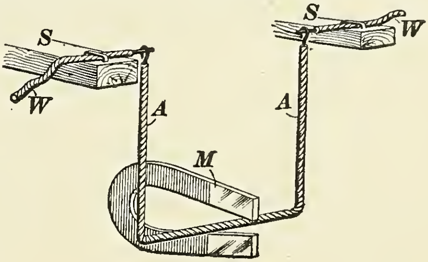


FIG. 976.

Now, hold the horseshoe magnet *M*, Fig. 976, in such a position that the bent wire may swing freely between its poles, and with the switch complete the circuit. (*a*) What happens ? (*b*) Reverse the current through the hanging loop ; what happens ? (*c*) How can you foretell which way the wire will swing ?

(Art. **2439.**) Replace the bent wire in the above experiment with a wire bent into a coil of about three turns, large enough to slip freely over one pole of the magnet, and suspend this coil, as before. Repeat the first two experiments, using this coil instead of the wire trapeze. Are the effects noted above altered any ? Why ?

DETERMINATION OF E. M. F.

2449. The electromotive force generated in a conductor cutting lines of force at right angles is proportional to the **rate of cutting.** The rate of cutting is found by dividing the number of lines cut by the time taken to cut them.

One absolute unit of potential is generated in a conductor when it is cutting lines of force at the rate of *one line of force per second.*

By definition, *one volt* is equal to 100,000,000 (10^8) *absolute units* (see Art. **2303**); consequently, in order to generate an electromotive force of one volt, the *rate of cutting* must be 10^8 lines of force per second. This can also be expressed algebraically.

Let E = the electromotive force in volts;
N = the total number of lines of force cut by the conductor;
t = time in seconds taken to cut the lines of force.

Then,
$$E = \frac{N}{10^8 \times t}. \qquad \textbf{(447.)}$$

That is, *the electromotive force in volts generated in a moving conductor is found by dividing the total number of lines of force cut by the conductor by the time taken and by 100,000,000.*

If the total number of lines of force remains unchanged, the electromotive force developed is the same, whether the lines of force proceed from a permanent magnet or electromagnet.

2450. According to *Ohm's law*, the current obtained from conductors cutting lines of force is equal to the quotient arising from dividing the total electromotive force generated by the total resistance of the circuit through which the current passes. In general, the total resistance is the resistance of the conductor cutting the lines of force, or the resistance of the *internal* circuit, plus the resistance of any conductor or conductors which complete the *external* circuit. If E represents the total electromotive force in volts, r_i and r_e the resistance in ohms of the internal and external circuits, respectively, and C the current in amperes, then $C = \frac{E}{r_i + r_e}$.

It will be seen from the above expression that a large or small induced current can be obtained from conductors cutting lines of force by simply changing the combined resistance of the internal and external circuits. There is, however, a maximum limit to the amount of current obtained in this manner. The lines of force which are produced around the conductor by the current itself will always act in opposition to the lines of force producing the electromotive force, and will tend to distort or crowd them away from their original direction. The number of lines of force produced around the conductor by the current is directly proportional to the strength of the current; and, consequently, as the current becomes larger and larger, the lines of force cutting the conductor become more and more distorted and crowded away from their original direction, until the conductor no longer cuts all the lines of force, and, therefore, the electromotive force generated becomes smaller. A general rule to get rid of this effect is to make the density of the magnetic field large in proportion to the current.

PRODUCTION OF INDUCED E. M. F.

2451. There are three ways of producing an electromotive force by induction in a coiled conductor, namely, by *electromagnetic induction, by self-induction, and by mutual induction.*

2452. In **electromagnetic induction** the change in the number of lines of force which pass through the coil is due to some relative motion between the coil and the magnetic field; as, for example, by thrusting a magnet pole into the coil, or by taking the magnet out from the coil, or by suddenly turning the coil in a magnetic field.

2453. In **self-induction** the change in the number of lines of force is caused by sudden changes in a current which is flowing through the conductor itself and supplied from some exterior source. If this exterior current is suddenly increased, it will produce a change in the number of lines of force; the change in turn *induces* an electromotive force in the conductor which opposes the exterior current in the coil and tends to keep it from rising. The exterior current will eventually reach its maximum strength in the coil, but its progress will be greatly retarded by the induced electromotive force. If, on the contrary, the exterior current is suddenly allowed to decrease, it will produce a change in the lines of force; this change induces an electromotive force in the coil which acts in the *same* direction as that of the exterior current, and tends to keep it from decreasing. As in the previous case, the exterior current will eventually decrease to its minimum strength, but it will fall gradually, and a portion of a second will elapse before it becomes constant. In fact, the current flowing through a coiled conductor acts as if possessing *inertia;* any sudden change in the strength of the current will produce a corresponding electromotive force which will tend to oppose that change and keep the current in its original strength.

2454. In **mutual induction,** two separate coiled conductors, one carrying a current of electricity, are placed near each other, so that the magnetic circuit produced by one will be enclosed by the other, as shown in Fig. 977, in which the current is flowing around coil *P*.

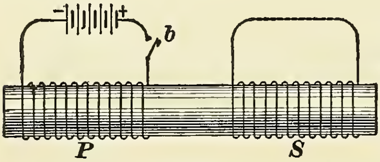


FIG. 977.

The coil (*P*) around which the current is flowing is called the **primary** or **exciting coil;** the other (*S*) is the **secondary coil.**

Any change in the strength of the current flowing around the *primary coil* will produce a corresponding change in the lines of force in the magnetic circuit, and, consequently, an electromotive force will be *induced* in the *secondary coil.* If the current in the primary coil is *increasing*, the electromotive force induced in the secondary coil will cause a current to flow around in the *opposite* direction to the current in the primary coil. If the current in the primary coil is *decreasing*, then the induced electromotive force in the secondary coil will cause a current to flow around in the *same* direction as the current in the primary coil.

2455. An **induction-coil** is an apparatus devised on the principle of mutual induction for producing pulsating currents of electricity of high electromotive force. Induction-coils are sometimes called Ruhmkorff coils, from the name of a celebrated manufacturer of them. They consist, essentially, of two coils, *primary* and *secondary*, wound around a core consisting of a bundle of iron wires. In Fig. 978 the secondary coil is composed of a large number of turns of fine insulated wire, while the primary coil P contains only a few turns of thick insulated wire. The primary circuit is automatically *opened* and *closed* at a and i, in the following manner: t represents a spring which tends to keep the circuit closed between the armature a and the contact pin i. As soon, however, as the circuit is closed by the action of the spring, the current from the battery B begins to circulate around the core m, thereby producing an electromagnet and attracting the armature a away from the contact pin i. Upon opening the circuit between a and i, the magnetism in the core begins to weaken, the spring once more closes the circuit, and the entire operation is again repeated. These actions take place in rapid succession, several times a second, constantly producing a change in the lines of force passing through the core, and thereby inducing a current in the secondary coil.

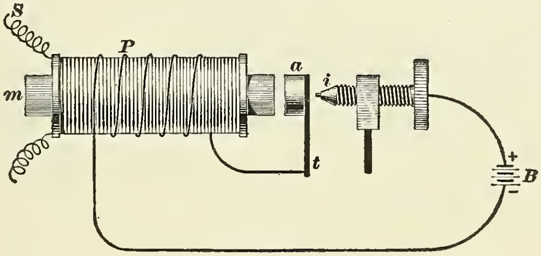


FIG. 978.

2456. Fig. 979 shows the commercial form of Ruhmkorff coil. The primary coil is wound around the core of soft iron wires, and its two ends brought out at p, p'. The secondary coil, consisting of several miles of fine insulated wire, is wound over the primary coil, and its ends attached to the insulated electrodes *s*, *s'*. The current in the primary coil is obtained from a voltaic battery connected to the terminals at *t*, *t'*, and is interrupted by means of a *mercury break* at *A* and *B*. The apparatus is also provided with a *commutator C*, which *commutes* or changes the direction of the current in the primary coil. When a battery which develops an electromotive force of a few volts and comparatively large currents is connected in the primary circuit as described, a torrent of sparks passes between *s* and *s'*, under an electromotive force of several thousand volts.

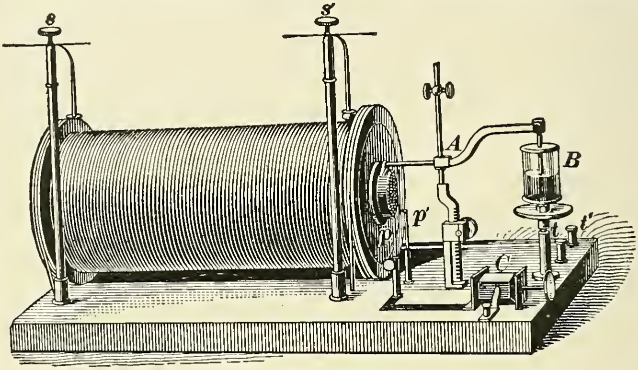


FIG. 979.

ELECTRICAL MEASUREMENTS.

ELECTROMAGNETIC MEASUREMENTS.

2457. A current of electricity is not a material substance, and, therefore, has no *dimensions* (length, area, or weight) by which it might be measured. *A current of electricity must, therefore, be measured by the effects which it produces.*

2458. These effects manifest themselves as follows :

When a current of electricity is flowing in a conductor, the energy expended in overcoming the resistance of the conductor manifests itself as **heat.** The amount of this energy is equal to the square of the current times the resistance (see Art. **2341**); therefore, the heat generated in a circuit will be proportional to the square of the current if the resistance be constant, or to the resistance if the current be constant.

When a current of electricity flows through a conducting liquid, the liquid is decomposed. This decomposition is due to a chemical action of the current, known as **electrolysis,** and is distinct from the heating effect. The decomposition either liberates a certain amount of gas or deposits one or more of the elements of the liquid upon one of the electrodes. *The amount of liquid decomposed is directly proportional to the quantity* (coulombs) *of current;* hence, the rate of decomposition, or the amount of liquid decomposed per unit of time, is proportional to the strength of the current in amperes.

When a current of electricity flows through a conductor, a field of magnetic force is set up around the conductor which *tends to produce a relative motion in any other magnetic field in the vicinity;* as, for instance, that emanating from a magnet pole. The force acting on such a pole will be directly proportional to the strength of the current, to the length of the conductor, to the strength of the magnet pole, and inversely proportional to the square of the distance between the conductor and the magnet pole.

An instrument which measures a current by its electromagnetic effect is called a **galvanometer.**

THEORY OF THE GALVANOMETER.

2459. As the units of electrical measurements are based upon the so-called "absolute" or "C. G. S." system (see Art. **2254**), measurements of current by means of electrolytic effect can be made only when the effect of unit current has been previously determined. By the electromagnetic action the absolute value of a current may be derived as follows :

As stated in Art. **2458,** the force exerted on a unit pole by a neighboring current is proportional to the strength of the current, to the length of the conductor, to the strength of the pole, and inversely proportional to the square of the distance from the conductor to the unit pole.

Then, to exert unit force on the unit pole, it is necessary to employ unit current, and a conductor of unit length, that is, one centimeter long, which must be bent to an arc of unit (one centimeter) radius, in order that each part of the conductor be at unit distance from the unit pole.

Under these conditions, a current of one C. G. S. unit flowing through the conductor will act on a unit pole at the center of the arc to which the conductor is bent with a force of one *dyne.* Thus the absolute value of one C. G. S. unit of current may be determined.

2460. When a magnetic pole is placed near another magnetic pole, the attraction (or repulsion) of the two poles is proportional to the product of the strengths of the two poles, and inversely proportional to the square of the distance between them; so, two equal magnetic poles, which, when placed at a unit distance (one centimeter) apart, exert a force of attraction or repulsion on one another of *one dyne*, are said to be of *unit strength.*

2461. In Art. **2272** it was pointed out that the C. G. S. unit of current is ten times greater than the practical unit, the latter being more convenient to use. Similarly in Arts. **2282** and **2303** the C. G. S. and practical values of the units of resistance and electromotive force were given.

It would be very difficult to construct apparatus that would fulfil the conditions given in Art. **2459.** It is much easier to use a conductor bent into a complete circle, and as the effect of changing various dimensions is known from the relations given in Art. **2459,** a formula may be constructed which will give the effect on a magnet pole of a current flowing through a conductor of any length bent to any radius.

Let r represent the radius in centimeters to which the conductor is bent; now, if the conductor be of sufficient length to be bent into a coil of more than one turn having a radius r, the length of *each turn* of the bent conductor is $\pi d = 2 \pi r$ centimeters, and the *total length* of the conductor $= 2 \pi r t$ centimeters, where t represents the number of turns that the conductor makes when bent into the coil. The distance between the conductor and the center of the coil is obviously equal to r centimeters; then the force that a current of 1 C. G. S. unit flowing through the coil would exert on a unit magnet pole placed at the center of the coil $=$ $\frac{2 \pi r t}{r^2}$ dynes, being directly proportional to the length of the conductor, and inversely proportional to the square of the distance between the magnet pole and the conductor. (Art. **2458.**)

This force being also directly proportional to the strength of the current, a current of A C. G. S. units will exert a force of $A \times \frac{2\pi r t}{r^2}$ dynes; or, representing the force exerted on the magnet pole in dynes by f,

$$f = \frac{2\pi A r t}{r^2}.$$

Dividing both terms of the fraction by r,

$$f = \frac{2\pi A t}{r}, \qquad \textbf{(448.)}$$

which is the formula required.

2462. It is not convenient to directly measure the force exerted on a unit pole by a current circulating in a coiled conductor.

If, however, any magnet pole can be influenced by a known constant force in one direction, then, by exerting upon it another force, due to a current circulating in a coil, but acting in a different direction, the resultant of the two forces may be accurately determined and the value of the second force measured.

This known constant force is furnished by the earth itself, which is a magnet of such enormous size that for short distances the direction of its lines of force may be considered as perfectly parallel. The actual direction of the earth's field is not horizontal, but at an angle to the horizontal, so the actual field may be said to be made up of two components—a horizontal and a vertical component. The horizontal component is most frequently made use of in measurements, as in this case. A small bar magnet placed across the earth's field of force will have equal and opposite forces acting on its poles or ends, since the lines of force act in a parallel direction; this results in turning the magnet about its center, if the magnet is free to move, until the forces act in a direct line with the center, when it can no longer move. This is illustrated by the magnet in the common compass. The force of the earth's field tends to keep the magnet parallel to the lines of force of the earth's field, and, consequently, the magnet points *north* and *south*.

2463. Fig. 980 illustrates this action. The direction of the earth's field of force is represented by the line *a b*. A bar magnet, *N S*, placed across this line at an angle with it will have equal and opposite forces acting upon the poles *N* and *S*, as shown by the arrows. These forces may be considered as parallel to the line *a b*; so, if the magnet be free to turn about its center, these forces will bring it to a state of rest when the line *a b* passes through the magnet from *N* to *S*.

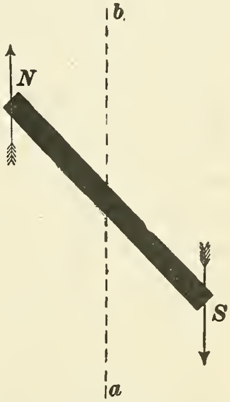


FIG. 980.

If the magnet *N S* be acted upon by another force at an angle with *a b*, the magnet will come to rest at a point where the two forces balance.

In Fig. 981 the magnet *N S* is acted upon by the earth's field along the line *a b*, the direction of the force on the *N* pole of the magnet being along the line *d N*, and that on the *S* pole along the line *c S*, as indicated by the arrow-heads. In addition, another force is acting along the line *x y*, at right angles to *a b*, the direction of the force on the *N* pole being along the line *c N*, and on the *S* pole being along the line *d S*, as indicated by the arrows. Under the influence of these two forces the magnet is deflected into the position shown, where it remains at rest, making the angle *m*° with the line *a b*.

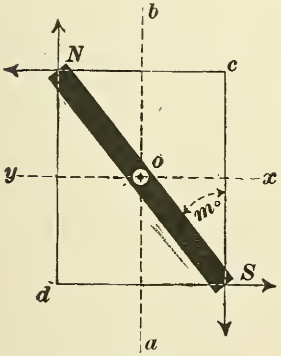


FIG. 981.

Calling the horizontal component of the strength of the earth's field *H*, the strength of the force acting along the line *x y*, *f*, and the strength of each pole of the magnet *N S*, *p*, then the forces acting on the *N* pole of the magnet are equal to $H \times p$ in the direction *d N*, and $f \times p$ in the direction $c N$; the forces acting on the S pole are equal to $H \times p$ in the direction $S c$, and $f \times p$ in the direction $d S$. The force $H \times p$ acting in the opposite directions on the two poles of the magnet form a couple tending to rotate the magnet about its center o. The moment of this couple is equal to one of the forces multiplied by the perpendicular distance between their lines of action. (See Art. **906.**) That is, the moment of the couple produced by the force $H p$ is equal to $H p \times c N$, and its direction is right-handed. Similarly, the force $f p$ produces a couple which tends to produce left-handed rotation of the magnet, and the moment of this couple is $f p \times S c$. Since the magnet is in equilibrium, that is, at rest, these two moments are equal, and

$$f p \times S c = H p \times c N, \text{ or } f \times S c = H \times c N.$$

Since this last equation does not contain p, it follows that the deflection of the magnet *is independent of the strength of the magnet.*

Since $f \times S c = H \times c N$, $f = H \frac{c N}{S c}$.

In Art. **754,** rule **5,** it is stated that the tangent of an angle is equal to the side opposite divided by the side adjacent.

In Fig. 981, $c N$ is the side opposite the angle $m°$, and $S c$ the side adjacent. Therefore, $\frac{c N}{S c}$ is the tangent of the angle $m°$, and the force f is given by the formula

$$f = H \times \tan m°. \qquad \textbf{(449.)}$$

H being constant, f varies as the *tangent of the angle through which the magnet is deflected.* An instrument which measures current on this principle is called a **tangent galvanometer.**

2464. The horizontal component (H) of the earth's field has been accurately measured at various places, and the following table gives the values for some well-known localities:

TABLE 84.

HORIZONTAL COMPONENT OF THE EARTH'S MAGNETISM

Locality.	Value of Component. Lines of Force per Square Centimeter.
London, England	.180
Paris	.188
Berlin	.178
Rome	.240
Montreal	.147
Niagara	.167
Halifax	.159
Boston	.170
New York	.184
Philadelphia	.194
Washington	.200
Chicago	.184
Cleveland	.184
San Francisco	.255

TANGENT GALVANOMETER.

2465. It is necessary that the lines of force that influence the magnet be practically parallel within the range covered by the swing of the magnet. With the earth's field this is the case, as has been pointed out in Art. **2462**; but with a coiled conductor, this only holds true of a very small space relative to the diameter of the coil, at the center of the coil. A tangent galvanometer must, therefore, have a magnet of short length as compared with the diameter of the coil.

A magnet $\frac{3}{4}$ in. long can be used with a coil of 8 in. diameter with accurate results.

The deflections of a magnet as short as this could scarcely be read directly. A very thin light pointer is, therefore, attached to the magnet, usually at right angles to it, which extends out over a scale upon which the deflections may be read.

Fig. 982 gives a top view of a simple tangent galvanometer in which $N S$ is the coil of wire and P is the pointer attached to the permanent magnet M. Two scales are shown, one on each side of the coil. One is divided into degrees, and the divisions on the other are proportional to the tangents of the angles represented by the divisions on the degree scale.

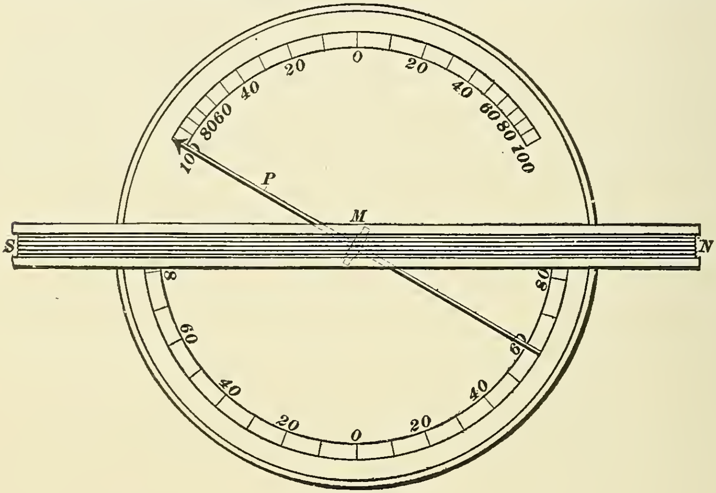


Fig. 982.

2466. In order that a variety of current strengths may be measured with the same instrument, it is customary to wind the coil in two or more parts, of varying number of turns and size of wire. The terminals of these parts of the coil are led out to binding-posts, b, b, b, b, Fig. 983, on the base of the instrument, so that either one or all the parts of the coil may be used. Even this method of winding does not give much range to the instrument. Another way of regulating its indications is to vary the *effective* earth's field, which may be accomplished by placing a permanent bar magnet, called a **controlling magnet,** in the plane of the coil and parallel to it.

Fig. 983 shows a tangent galvanometer, with an adjustable controlling magnet *m*. If this controlling magnet be so placed that its *S* pole corresponds in direction with the *N* pole of the magnet of the instrument, its field will be added to the earth's field, so that a given current will give a smaller deflection than if the controlling magnet were removed. If the polarity of the controlling magnet be reversed, the opposite effect will result, and the instrument will give a deflection with a very small current.

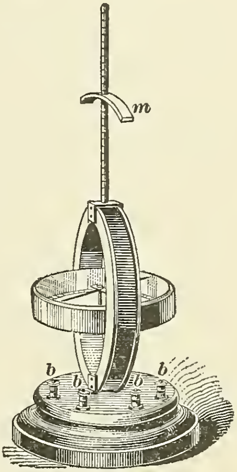


FIG. 983.

2467. Controlling magnets are used on many forms of galvanometers; there is a difficulty, known as **drift,** which attends their use, especially when used to make the galvanometer very sensitive. This difficulty is due to the fact that the direction of the earth's field is continually changing slightly, and its effect is to make the zero-point of the instrument vary from time to time. This effect may be shown by the diagram in Fig. 984. In (*a*), $n\,o$ represents the direction and magnitude of the force due to the earth's field, and $n\,m$ the direction and magnitude of the force due to the controlling magnet. The resultant $n\,s$ is then the direction which the magnet of the instrument would assume. Now if the direction of the earth's field change through a slight angle to the position shown in (*b*), the resultant is then the line $n_1\,s_1$, and its direction is at an angle

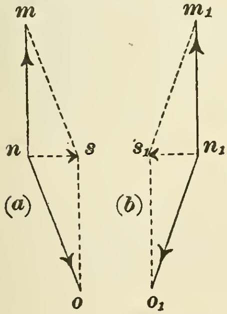


FIG. 984.

of nearly 180° to the resultant $n\ s$. If the controlling magnet had not been used, there would have been a slight "drift," but the use of the controlling magnet to lessen the effective field very much magnifies the effect of any change in the direction of the earth's field.

2468. When a controlling magnet is used, it is necessary to find the deflection that a certain known current will produce, as the actual value of H is no longer known. Knowing the deflection with a given current, other currents may be measured, as the galvanometer is still governed by the same law, and formula **449** may be changed to read

$$C = K \tan m^\circ, \qquad \textbf{(450.)}$$

where $C =$ current in amperes and $K =$ a constant, called the *galvanometer constant*, by which the tangent of the angle of deflection must be multiplied to get the value of the current flowing. This process of finding the constant of a galvanometer or other measuring instrument by comparing it with a known standard is called **calibration.**

The formula for the value of f with this form of tangent galvanometer is the same as before, viz., $f = H \tan m^\circ$, but the value of H is now the intensity of the earth's field *plus* or *minus* (according to its polarity) the intensity of the field due to the controlling magnet. After having found the *galvanometer constant*, this value of H may be calculated.

2469. The following examples illustrate the application of the formulas of the tangent galvanometer:

Example.—What will be the force in dynes exerted on a unit magnet pole placed at the center of a coiled conductor of three turns bent to a circle of 12 cm. radius, by a current of 2 C. G. S. units?

Solution.—Use formula **448,** $f = \dfrac{2\pi A t}{r}$.

$2\pi = 6.2832$; $A = 2$; $t = 3$; $A t = 6$; $r = 12$.

Then, $f = \dfrac{6.2832 \times 6}{12} = \dfrac{37.6992}{12} = 3.1416$ dynes. Ans.

Example.—A tangent galvanometer has the following dimensions: Mean diameter of coil, $7\frac{7}{8}$ in.; number turns first section, 3; number turns second section, 1. If this instrument is set up in Boston, and a current of 2 amperes* is sent through the first section of the coil, what will be the deflection of the magnet in degrees?

SOLUTION.—Use formula **448**, $f = \frac{2\pi A t}{r}$.

Diameter of coil = $7\frac{7}{8}$ in. = 20 cm.; turns = 3; amperes = 2; C. G. S. units = .2.

Therefore, $f = \frac{2 \times 3.1416 \times .2 \times 3}{10} = \frac{3.76992}{10} = .376992$ dyne due to coil.

Also, from formula **449**, $f = H \times \tan m°$.

Transposing, $\tan m° = \frac{f}{H}$. $H = .170$. (Table 84.)

Therefore, $\tan m° = \frac{.376992}{.170} = 2.2176$.

Referring to the table of Natural Tangents, the tangent of the angle 65° 46′ is 2.22164. This is as nearly correct as the deflection could be read on the scale. Ans. 65° 46′, nearly.

2470. EXAMPLE.—Another galvanometer is constructed exactly like that referred to in Art. **2469**, but with a controlling magnet attached to increase its range. The two galvanometers are connected together by wires, so that the *second section* of the coil of the *first galvanometer* (called No. 1) is in series with the *first section* of the galvanometer with the controlling magnet (called No. 2). On sending a current through the two instruments, the deflection of No. 1 is 52°, while the deflection of No. 2 is but 38°. (*a*) What current is passing through the galvanometers, and (*b*) what is the value of the galvanometer constant of No. 2, if the experiment is made in Philadelphia?

SOLUTION.—(*a*) Consider No. 1 only.

From formula **449**, $f = H \tan m°$.

As $H = .194$, $m° = 52°$, and $\tan m° = 1.28$, nearly,

$$f = .194 \times 1.28 = .24832.$$

Also, from formula **448**, $f = \frac{2\pi A t}{r}$, or $\frac{2\pi A t}{r} = .24832$; hence, $\frac{6.2832 \times A \times 1}{10} = .24832$; $.62832\,A = .24832$, and $A = .3952$ C. G. S. units of current.

As A equals C. G. S. units, this result must be multiplied by 10 to give practical units = 3.952 amperes. Ans.

(*b*) In No. 2 the current is 3.952 amperes and the tangent of the angle of deflection = tan 38° = .7813, nearly; substituting in formula

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[*] Whenever the word *ampere* is used alone, the practical unit (one-tenth of the C. G. S. unit) is understood. The C. G. S. unit, when used, is called the *C. G. S. unit*.

450, $C = K \tan m°$; $C = 3.952$, and $\tan m° = .7813$, $3.952 = K \times .7813$, or $\frac{3.952}{.7813} = K = 5.058$. Ans.

REMARK.—This value of K is only good for the first section of the galvanometer coil, which consists of three turns. If the value of H in formula **449,** $f = H \tan m°$, be calculated for this galvanometer, then changes may be made in the number of turns of the coil used, without recalculating a galvanometer constant, if the controlling magnet be unchanged.

In the example above given, the value of f in No. 2 is obviously three times that in No. 1, as the same current passes through three times the length of wire. Therefore, $f = 3 \times .24832 = .74496$, and $\tan m° = \tan 38° = .7813$, nearly. As $\frac{f}{\tan m°} = H$, then, $\frac{.74496}{.7813} = H = .9535$.

This value of H represents the combined value of the field due to the earth and that due to the controlling magnet. As will be seen, the intensity of this field is nearly five times that of the earth alone; so galvanometer No. 2 may be used to measure currents of about five times the strength that No. 1 will measure under the same conditions.

EXAMPLES FOR PRACTICE.

2471. 1. A coil of wire is wound 20 cm. in diameter and consists of 5 turns. Through this coil a current of 12 amperes is passed. What will be the force exerted on a unit magnet pole at the center of the coil? Ans. 3.77 dynes.

2. Using the galvanometer (Art. **2469**), a current of 5 amperes is passed through the second section of the coil. (*a*) What will be the deflection in degrees? (*b*) What would be the deflection if the instrument were in Washington instead of Boston? (*c*) What current would a deflection of 46° indicate, using the first section of the coil and taking the measurement in Chicago?

Ans. (*a*) 61° 35′, nearly.
(*b*) 57° 31′, nearly.
(*c*) 1.01 amperes, nearly.

3. A galvanometer with a coil 12 in. diameter having 12 turns of wire gives a deflection of 42° when a certain current is passed through the instrument being set up in Montreal. What is the value of this current in amperes? Ans. .267 ampere.

SINE GALVANOMETER.

2472. Another form of galvanometer, shown in Fig. 985, employs much the same principle as the tangent galvanometer, except that its coil C is movable about a vertical axis.

This instrument being set up with the plane of its coil in the earth's magnetic meridian, and the pointer (which, as in the tangent galvanometer, is usually fixed at right angles to the magnet) at zero, a current is sent through the coil by means of the wires W, which deflects the magnet. The coil is then turned in the same direction that the magnet is deflected, until in such a position that the magnet comes to rest with the plane of the coil coinciding with the direction of the magnet. This point is usually determined by a mark on a part of the support of the coil, which must be made to register with the pointer attached to the magnet. The angle through which the coil has been turned is read by a vernier from a scale of degrees S attached to the base of the instrument, *and the sine of this angle multiplied by the proper constant gives the current flowing in the coil*, whence the name.

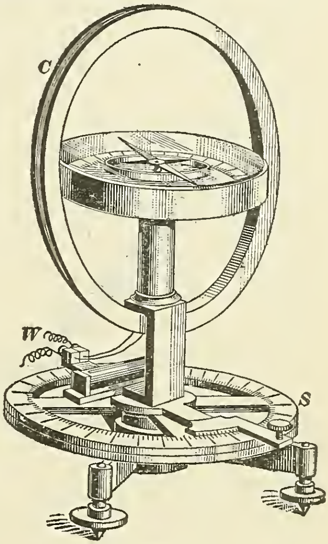


FIG. 985.

2473. The theory of the sine galvanometer may be demonstrated as follows:

In Fig. 986, NS is a magnet which is acted on by the earth's field along the line $a\,b$, the direction of the force on the N pole being represented by the line $N c$. Another force is also acting on the magnet at right angles to its axis NS, along the line $x y$. This force acts on the N pole in the direction represented by the line $N d$. As the forces acting on the S pole are equal to those acting on the N pole, only the latter need be considered.

As before, call the horizontal component of the strength of the earth's field H, the strength of the force acting along the line $x y$, f, and the strength of the pole of the magnet, p.

Let the line Nc represent the amount and direction of the force Hp due to the earth's field, and the line Nd the amount and direction of the other force; then, by completing the parallelogram of forces (see Art. **875**) the resultant of the two, Nc, is found.

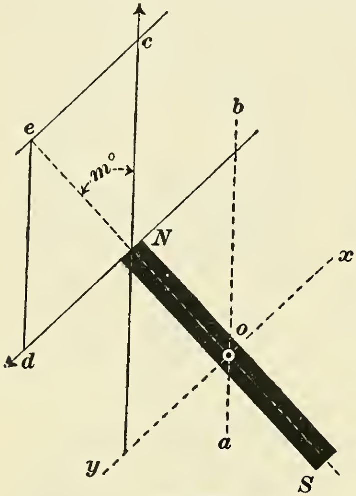


Fig. 986.

This resultant Ne represents the direction and amount of the single force that would deflect the magnet to the position shown, where it makes the angle m° with the line Nc, which is parallel to $a\,b$.

It is evident that the lengths Nc and Nd can represent H and f, respectively, since they have been assumed to represent those amounts each multiplied by the constant p. Then, as $Nd = ce$ and $Nd = f$, $f = ce$. From Art. **754**, rule **2**, $ce = Nc \sin m^\circ$; hence, as $f = ce$ and $H = Nc$,

$$f = H \sin m^\circ. \qquad \textbf{(451.)}$$

Note.—This principle may also be demonstrated by the use of couples, as in the case of the tangent galvanometer, and it is recommended that the student work out that demonstration himself.

The value of f may be obtained in the same manner as for the tangent galvanometer.

If a magnet be used whose length is nearly equal to the inside diameter of the coil, the current will still be proportional to the sine of the angle of deflection of the needle, as the axis of the needle is always at right angles to the lines of force of the coil, but the value of f will no longer be correct if calculated from formula **448**, as the force acting on the magnet poles is not uniform throughout the area enclosed by the coil, but is greater near the coil than towards the center, and formula **448** gives the force at the center only.

The sine galvanometer is not as convenient an instrument to use as the tangent galvanometer, as the coil must be carefully adjusted to the correct position to get accurate results instead of taking the reading directly from the position of the pointer, but it is more accurate than ordinary forms of tangent galvanometers.

REFLECTING TANGENT GALVANOMETER.

2474. If the needle of a tangent galvanometer be suspended by a fiber of raw silk or other similar material without twist, and if a beam of light reflected from a small mirror attached to the needle be used for a pointer, very accurate measurements of the deflection can be obtained.

An instrument so constructed is known as a **reflecting tangent galvanometer.** It is usual in this case to set up a suitably divided *straight* scale some distance from, and parallel to, the normal (zero) position of the mirror. The lamp giving the light is located behind the scale, in which is a small slit or hole through which the necessarily small beam of light passes, being reflected from the mirror back to the scale.

Since the reflected beam of light makes the same angle *with the mirror* that the original beam does, but on the opposite side of a perpendicular to the mirror, the angle between the original beam and the reflected beam will be equal to *twice the angle of deflection of the mirror.* Allowance for this fact, also for the fact that a *straight* scale is used, must be made in calculating the constant of a reflecting tangent galvanometer.

It is usual in this class of instruments to make the magnet of a number of small magnets, made from short bits of steel needles or pieces of watch-spring, and arrange one-half of the magnets with their poles opposing the remainder, which makes the magnet **astatic**; that is, the earth's field has almost no directive force on the magnetic system of the instrument. By using a strong controlling magnet, the instrument is made almost independent of the earth's field, and thus errors or drift due to variations in the horizontal component of the earth's magnetism are rendered of little effect.

2475. When the magnetic system and mirror are thus constructed and suspended by a long fiber, considerable difficulty in reading may be met with, owing to the length of time required for the needle to come to rest after being deflected. This is corrected by **damping** the moving parts of the instrument, which may be effected by suspending from the needle a small fan of very light construction, which, by reason of the friction of the air on the blades of the fan as it moves, causes the needle to swing more slowly and come to rest at once.

This damping effect is an important feature of most measuring instruments. Other methods than that given above are used, one of which is to enclose the moving magnetic needle in a cavity in a block of copper; movement of the needle then sets up little eddy currents in the copper block, which retard the movement of the needle, giving the desired damping effect.

In the D'Arsonval galvanometer, described in the following article, the damping of the moving coil is effected by winding that coil on a bobbin of thin copper or other non-magnetic metal. The movement of this bobbin with the coil through the field generates eddy currents in the bobbin itself, which currents produce the required damping effect.

THE D'ARSONVAL GALVANOMETER.

2476. Another electromagnetic measuring instrument which is quite extensively used is the **D'Arsonval galvanometer,** which derives its name from its inventor, a French physicist. Its principle is slightly different from most of the other forms of galvanometers in that the magnet is large and stationary, and the coil is small and movable. It consists of a large permanent horseshoe magnet, between the poles of which is suspended a coil of wire.

Current is led to the coil by means of the suspension, and this current in the coil causes the coil to rotate about its axis, the tendency of the coil being to place itself at right angles to the lines of force. This tendency is opposed by the suspension, which may be a spring or an elastic wire or fiber. A pointer may be attached to the coil to indicate its deflection, though usually a mirror is used, a reflected beam of light from which forms the pointer, as in the reflecting tangent galvanometer. In many forms of this instrument a soft iron core is supported between the poles of the magnet, a space being left between the core and the magnet in which the coil swings. This core serves to increase the strength of the field in which the coil moves.

By suitably shaping the poles of the magnet, the intensity of the magnetic field in various parts may be so varied that the movement of the beam of light will be directly proportional to the current in the coil. Fig. 987 represents one form of the D'Arsonval galvanometer. In the figure, *PP* is the magnet; *C*, the movable coil; *S*, *S*, fine platinum wires, which suspend the coil *C*; *M*, the mirror, and *I*, the iron cŏre, which is supported from the back in the center of the coil *C*. Connection from the binding-posts *B*, *B* to the coil *C* is made through the platinum suspension wires *S*, *S*. One of the chief advantages of this instrument is the fact that external fields, such as the earth's magnetism, have little effect upon it, so that it requires no controlling magnet or correction for the earth's field, and may be used near dynamos and large masses of iron without being affected.

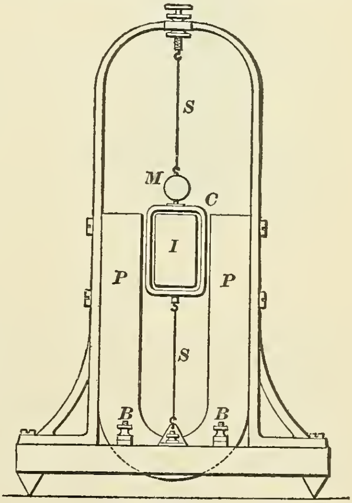


FIG. 987.

Many of the commercial forms of portable instruments are built on the principle of this galvanometer, as will be described.

REFLECTING GALVANOMETER.

2477. It is often desirable to use an instrument for indicating the presence of very small currents without necessarily measuring their value. For this purpose, the reflecting tangent galvanometer is modified by making the coils of considerably less diameter in proportion to the length of the magnet, and by winding the coil with a great many turns of wire, so as to make a very strong field at the center of the coil with a feeble current. The magnetic system is made astatic, but the magnets that point in one direction are hung considerably below those that point in the opposite, and each set of magnets has its own coil; in order that the magnetic system may be suspended in the center of the coils, each coil is wound in two equal parts, and, when mounted, a very small space is left between the parts, through which the fiber which suspends the system passes.

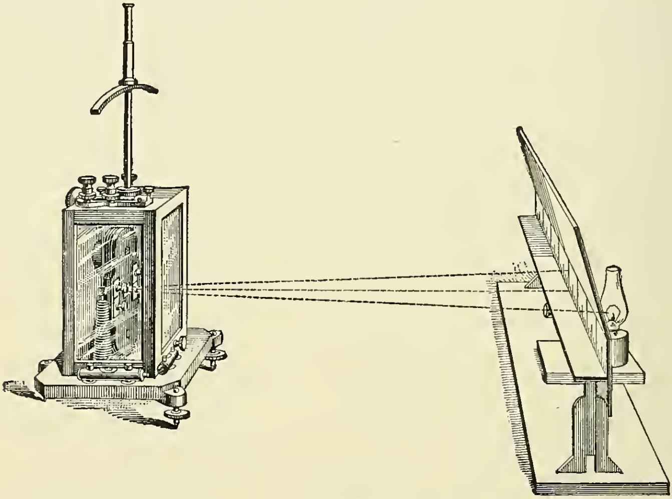


FIG. 988.

This form of instrument is known as a **reflecting galvanometer.** Fig. 988 shows one form of this instrument, with lamp and scale.

BALLISTIC GALVANOMETER.

2478. A special form of reflecting galvanometer known as a **ballistic galvanometer** is used for measuring transient currents, such as are induced in a conductor if a current in a neighboring conductor be started or stopped, or if a magnet be moved in the vicinity. This form of galvanometer has its magnetic system constructed so as to be of considerable weight, and so arranged as to give the least possible damping effect. If a momentary current pass through the coils of the instrument, the impulse given to the needle does not cause appreciable movement of the magnetic system until after the current has ceased, owing to the inertia of the heavy moving parts, which results in a slow swing of the system after the impulse has ceased; the maximum angle of swing may be read by watching the spot of light, reflected from the mirror attached to the magnetic system, move across a suitably divided scale, and noting the point at which the spot of light ceases to move and begins to swing back. The *quantity* of electricity (the number of coulombs) that pass through the coils of the instrument is proportional to *the sine of one-half the angle of deflection of the needle:*

$$Q = K \sin \frac{m^\circ}{2}. \qquad \textbf{(452.)}$$

The deflection being usually small, the quantity of electricity may be regarded as directly proportional to the angle of deflection. As the use of the mirror and ray of light as a pointer merely doubles the angle of deflection, it will introduce no serious error to consider the quantity of electricity proportional to the swing of the spot of light across the scale, and formula **452** may be modified to read

$$Q = K d, \qquad \textbf{(453.)}$$

where $d =$ deflection in scale divisions.

2479. Fig. 989 shows one form of ballistic galvanometer in which C, C_1 are the two parts of the coil, either of which may be swung back, as shown, to examine or remove the magnetic system.

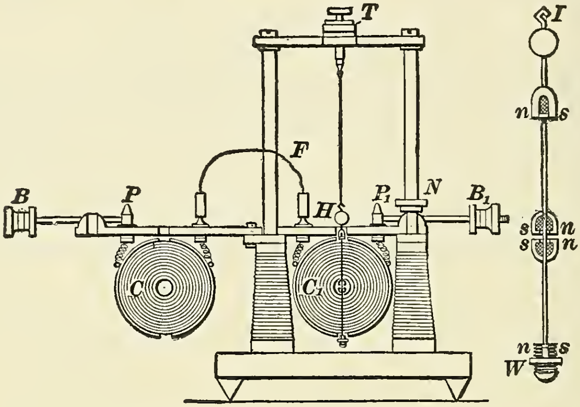


FIG. 989.

Each coil is supported from a brass strip, both of which are clamped in place by the nut N.

Connections to the coils are made from the terminals P, P_1, the coils being connected together through the flexible cable F, which allows either coil to be swung aside without disturbing the connections.

When in use, the instrument is surrounded by a case (not shown) through which the binding-posts B, B_1 project.

The magnetic system, an enlarged section of which is shown at the right, is suspended by a fine quartz fiber from the torsion head T, the magnets and mirror being hooked on to the lower end of the suspension by the hook I.

The magnets are thimble-shaped, and filled with lead to give weight. The system is rendered astatic by the arrangement of polarities as shown, the upper and lower magnets being the stronger, and, therefore, directing the system. No external controlling magnet is used with this system, so drift is very nearly eliminated; the sensibility of the system is varied by screwing the small soft iron ring W

up or down on the lower magnet. If the ring is screwed up, it *short-circuits* some of the lines from that magnet, thus weakening its effect on the system.

2480. One of the principal uses to which the ballistic galvanometer is put is measuring the magnetic qualities of iron.

Samples of the iron to be tested, usually in the form of a ring, are wound throughout their length with insulated wire, so that if a steady current be sent through the wire the ring will be magnetized. If a second coil of wire be wound for a short distance over the first coil, any change in the number of lines of force in the ring will induce an electromotive force in the second coil.

A ring so wound is represented in Fig. 990, where R is the iron ring, $P\,P_1$ the *primary* or magnetizing coil, $S\,S$ the *secondary* or induction coil.

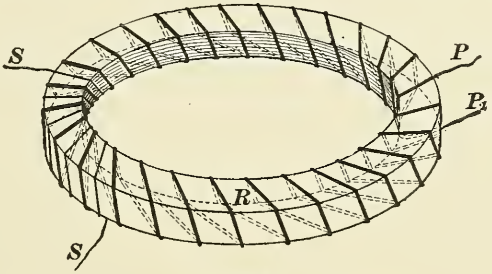


FIG. 990.

If the coil S S be closed through a circuit of fixed resistance, the number of coulombs of electricity that will flow in this secondary circuit when the number of lines of force passing through that coil is changed will be directly proportional to the amount of change in the number of lines of force.

This is independent of the *rate of change;* for, assuming that the number of lines of force changes uniformly for one second, and that the turns of the secondary coil are such that 1 volt is generated in that coil, then, if the resistance of the entire secondary circuit is 1 ohm, 1 ampere will flow for 1 second, or as long as the E. M. F. is being generated;

that is, the *quantity* of electricity will be *one coulomb.* Now, if the number of lines of force be changed by the same amount, but in *two* seconds, only $\frac{1}{2}$ volt will be generated in the secondary coil, and only $\frac{1}{2}$ ampere will flow in the secondary circuit, but it will flow for two seconds, and the quantity of electricity will be the same as before.

The same holds true if the rate of change in the number of lines is not uniform, which is usually the case.

2481. If known currents be sent through the primary coil $P\,P_1$, the magnetizing force **H** may be readily calculated. (See formula **430.**) Any change in this magnetizing current will produce a change in the number of lines of force in the iron ring, which will be indicated by a swing of the galvanometer needle, and the amount of this swing will indicate the relative amount of change in the number of lines of force passing through the secondary coil. For calibrating the ballistic galvanometer for magnetic measurements, it is usual to note the swing when a known number of lines of force is made to cut the turns of a coil in the galvanometer circuit. This may be done by preparing a coil of wire wound on a bobbin of considerable size, and arranging it between supports so that it may be rotated through 180°; by placing the coil with its plane at right angles to the direction of either the vertical or the horizontal component of the earth's magnetism, the rotation of the coil will cause its sides to cut the lines of force of the earth's field. If the value of the component be known, the number of lines enclosed by the coil, and, therefore, the number cut by its rotation, can be calculated, the area of the space enclosed by the coil being known. This method, known as the *earth coil method*, is open to the objection that the components of the earth's magnetism vary slightly from time to time.

2482. Another method for determining the magnetic properties of iron is shown in Fig. 991, where the apparatus and connections are indicated. The iron to be tested is in the shape of a flat iron ring *I*, upon which is evenly wound a certain known number of turns of insulated wire, the terminals being *p* and *p*′. This is called the *primary* or chief coil. The same number of turns of insulated wire are wound on a wooden or other non-magnetic core, with the terminals at *t* and *t*′. The coil *C* is called the *calibrating* coil. On the iron ring *I* and also at the center of the length on the coil *C* are wound induction-coils, or secondary coils, consisting of a few turns of insulated wire.

As the dimensions of the coil *C* and the current passing through the coil are measurable, the exact number of lines of force per square inch designated by **H** in air, wood, or other non-magnetic medium, may readily be determined by formula **430.** According to Art. **2478** and formula **453,**

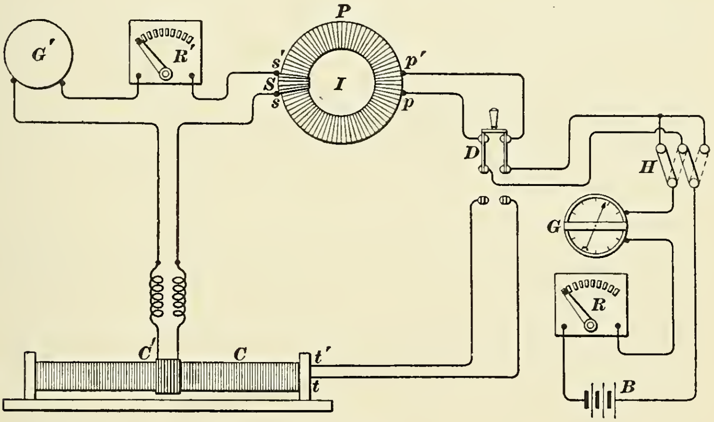


FIG. 991.

the setting up of a certain number of lines of force in the wooden core of *C* will cause the needle of the ballistic galvanometer *G*′ to give a certain kick. By varying the make and break currents, the ballistic galvanometer *G*′ may be calibrated, so that the kick of the galvanometer *G*′ being given, the number of lines of force in the coil *C*′, or in any other similar coil, may be determined.

A succession of currents of different values may be sent through the primary coil *P* of the iron ring *I*, thus producing therein a certain number of lines of force **B**, which number is indicated by the kick of the needle of the ballistic galvanometer, as previously noted. These results may be tabulated, or else laid out in the form of a magnetization curve, as was explained in Art. **2402.**

2483. As the diagram now stands, the calibrating coil is out of circuit, and the primary coil P of the iron ring is being energized by the battery B. The energizing current is regulated by the adjustable resistance R, and is calculated from the dimensions of the primary circuit or measured by the galvanometer, or low reading ammeter, G. The reversing switch H is used to start, stop, or reverse the current in the primary coil P. An adjustable resistance R' is also in the secondary, for varying the range of the ballistic galvanometer G', as the values of $\mathbf{H}$ and $\mathbf{B}$ are widely different.

In order to calibrate the ballistic galvanometer G', the reversing-switch terminals are disconnected from the coil P by means of the double-throw switch D, which then enables connection to be made instead to the terminals t, t' of the calibrating coil C.

2484. The test of the iron may be made in a variety of ways. The two most used are the *step-by-step* and the *reversal* methods.

The **step-by-step** method consists of suddenly increasing or decreasing the magnetizing current in the primary coil by moving the handle of the rheostat R. The swing of the galvanometer G' at each step indicates the amount of change in the lines of force corresponding to a change in the magnetizing force. The total number of lines at any point may be determined by adding together the previous *changes*, as observed by the swing of the galvanometer.

2485. The **reversal** method is to reverse the current in the primary by throwing the reversing switch H. The lines of force will then change from a certain number in one direction down to zero, and then to about the same number in the opposite direction. This change will cause a swing of the galvanometer G', and one-half this swing is taken to represent the number of lines of force in the circuit due to the magnetizing force that has been reversed. By increasing or decreasing this magnetizing force by successive steps, and reversing each time, the *curve of magnetization* may be obtained.

2486. One objection to the step-by-step method is that an error in one of the early observations will be included in the whole series, as they are all added together; but with care in taking the readings, this need not occur. With the method of reversals, however, the *residual magnetism* introduces an error; as, when the magnetizing force is reversed, the lines of force will not also be entirely reversed, so that there will not be as many lines in the circuit after the reversal as before, with the same magnetizing force. In either case, the magnetizing force **H** can be readily calculated from the magnetizing current, and the total induction in the sample of iron may be determined by the galvanometer swings. By this means the value of **B** may be obtained, and the magnetization curve of the particular sample of iron under test may be plotted.

Example.—1. Calibrate the ballistic galvanometer G', shown in Fig. 991, for resistance of rheostat $R' = 0$; 500 ohms; 1,000 ohms, the following information being given:

Data.—The upper terminals of the reversing switch H were disconnected from the primary coil P and connected to the calibrating coil C by the terminals t, t', so that the current from the battery B passed through the calibrating coil C, the primary coil P being out of circuit.

The elements of the present *primary circuit* have resistances as follows:

Resistance of primary calibrating coil $C = 3$ ohms.

Internal resistance of battery $B = 1.2$ ohms.

Resistance of rheostat R, ten steps of 4 ohms each $= 40$ ohms.

Resistance of balance of primary circuit, including connections, $=$ 1.1 ohms.

The parts of the secondary circuit have the following resistances:

Resistance of rheostat R', ten steps of 200 ohms each $= 2{,}000$ ohms.

Resistance of ballistic galvanometer $G' = 500$ ohms.

Resistance of balance of secondary circuit, including both secondary coils, $= 10$ ohms.

The battery B has 6 cells, each furnishing a constant E. M. F. of 1.9 volts.

The secondary coil S consists of 120 turns of No. 22 insulated wire.

The calibrating coil C is wound on a wooden rod 30 inches long and 2 inches in diameter, and consists of 1,200 turns of No. 18 insulated wire, wound evenly in two layers.

The secondary calibrating coil C', wound at the center of the length of the primary calibrating coil C, has 260 turns of No. 22 wire.

The ballistic galvanometer G' is of the type already described in Art. **2477,** with a scale about 4 feet long, and reads from zero at the center to 225 at each end. *The resistance R' all being cut out, the galvanometer G' gives the scale reading 48.*

Solution.—To calibrate the ballistic galvanometer G means to ascertain the deflection of the galvanometer corresponding to one line of force passing through the secondary coil S.

The *current* in the primary calibrating coil C, according to Ohm's law, is

$$C = \frac{E}{R} = \frac{6 \times 1.9}{3 + 1.2 + 1.1} = \frac{11.4}{5.3} = 2.15 \text{ amperes.}$$

The number of lines of force per square inch cross-section of the wooden core, by formula **430,**

$$\mathbf{H} = \frac{3.192 \times a\text{-}t}{l} = \frac{3.192 \times 2.15 \times 1{,}200}{30} = 274.5 \text{ per square inch.}$$

The total number of lines of force in the wooden core $\mathbf{N} = \mathbf{H} \times \text{area} = 274.5 \times 3.1416\, r^2 = 274.5 \times 3.1416 = 862.4$ total lines of force in core.

We have now determined the total number of lines of force that passed through the secondary coil C' of 260 turns when the ballistic galvanometer G' moved 48 scale divisions. If there was only one turn in the secondary coil C' instead of 260, the galvanometer G' would have moved only $\frac{48}{260} = .1846$ of a scale division. If only one line of force had been erected, or dissipated, in the coil C, and consequently C', instead of 862.4 lines of force, the galvanometer would have moved only $\frac{.1846}{862.4}$, or .000214 of a scale division. But the secondary coil S on the iron ring I has 120 turns. Therefore, when $R' = 0$, one line of force passing through the coil S will throw the galvanometer .000214 division $\times$ 120 = .02568 scale division. Ans.

When $R' = 500$, the total resistance of the secondary circuit is increased from 510 ohms to 1,010 ohms, and the current would be proportionally *decreased.* (See Art. **2480.**) Consequently, the scale reading would be decreased, and one line of force passing through the secondary coil S would cause a deflection of $.02568 \times \frac{510}{510 + 500} = .01297$ scale division. Ans.

When $R' = 2{,}000$, the scale reading per line of force $= .02568 \times \frac{510}{510 + 2{,}000} = .0052$ scale division. Ans.

Example.—2. (*a*) From the data and information following, work out the magnetomotive force **H** in the primary coil of the iron ring, and the resulting density of lines of force **B**, *using the step-by-step method;* from these results plot a *magnetization curve* on cross-section paper, showing the magnetic qualities or susceptibility of the iron. (*b*) What kind of iron does the sample seem to be?

Data.—The circuit connections are exactly shown in Fig. 991, the calibrating coil of the previous example having been replaced in the primary circuit by the primary coil *P* of the flat iron ring *I*. The dimensions of the iron ring are: 5 inches inside diameter, 6½ inches outside diameter, and 1 inch thick. The *primary coil P* is wound evenly over the entire ring *I*, and consists of 800 turns of No. 18 insulated wire, affording a resistance of .8 ohm. The *secondary coil* is made up of 120 turns of No. 22 insulated wire. All the resistance of the rheostat *R'*, 2,000 ohms, is in circuit.

The data given and the results obtained in the previous examples are also available.

The manner of performing the experiment is to turn in the whole resistance of rheostat *R* of 40 ohms, and then close the switch *H*. Noting the swing, the spot of light gradually settles down to zero. The rheostat hand of *R* is suddenly thrown back to the second contact. This cuts out 4 ohms resistance, which allows an additional amount of current to flow through the circuit. The addition of this quantity of current sets up additional lines of force, and the additional lines of force set up a current in the ballistic galvanometer *G'*. The rheostat is moved around the successive steps, and the readings noted as follows:

Resistances of *R*.	Deflection of Galvanometer *G'*.
Ohms.	Divisions.
40	220.6
36	11.1
32	12.9
28	7.7
24	14.8
20	13.2
16	16.3
12	19.6
8	26.0
4	30.7
0	40.4

Solution.—(*a*) The calculations should be made in tabular form, for the sake of clearness. The following calculations will have to be made, and a column may properly be assigned for the result of each calculation:

1. The resistance of the primary circuit.
2. The current in the primary circuit.
3. The magnetomotive force **H** of the primary coil (per inch of length of core).
4. The deflection of the ballistic galvanometer G' in scale divisions.
5. The corresponding *change* in the number of lines of force in the iron ring I.
6. The total number of lines of force in the iron ring.
7. The density **B** per square inch in the iron.

Column 1 is found by adding the resistances of the elements of the primary circuit, the several values of the adjustable rheostat R having been given in the example; for illustration, the first quantity equals $.8 + 1.2 + 1.1 + 40 = 43.1$ ohms. The rest are found in the same manner.

1. Resistance of Primary Circuit.	2. Current in Primary Circuit.	3. **H** Magneto-motive Force in Primary Coil.	4. Deflect. of Galvanometer. Divisions.	5. Change in Number of Lines of Force.	6. Total Number of Lines of Force.	7. **B** Lines of Force in Iron, per Square Inch.
43.1	.2645	37.40	220.6	42,420	42,420	56,560
39.1	.2916	41.20	11.1	2,140	44,560	59,410
35.1	.3248	45.90	12.9	2,480	47,040	62,720
31.1	.3666	51.84	7.7	1,480	48,520	64,690
27.1	.4207	59.50	14.8	2,850	51,370	68,490
23.1	.4935	69.78	13.2	2,540	53,910	71,880
19.1	.5968	84.40	16.3	3,140	57,050	76,070
15.1	.7550	106.8	19.6	3,770	60,820	81,090
11.1	1.027	145.2	26.0	5,000	65,820	87,760
7.1	1.606	227.1	30.7	5,900	71,720	95,630
3.1	3.677	520.0	40.4	7,770	79,490	105,990

The readings in column 2 are found by dividing the total electromotive force of the cells, 11.4 volts, by the respective resistances given in column 1.

The values of the magnetomotive force in column 3 can be calculated from formula **430,** $\mathbf{H} = \frac{3.192 \times a\text{-}t}{l}$, where the number of turns $t = 800$, and the current $a = .2645, .2916, .3248$ ampere, etc.; the length of the magnetic circuit l is determined from the dimensions of the ring, as follows: The mean diameter of the ring $= \frac{5 + 6\frac{1}{2}}{2} = 5\frac{3}{4}$ inches. Length $l = 5\frac{3}{4} \times 3.1416 = 18.06$ inches.

The deflections of the galvanometer, column 4, are given in the example.

The *change* in the number of lines of force, that is, the additional number of lines of force due to the increases of primary current, noted in column 2, when the resistance of the rheostat R' equals 2,000 ohms, is found by dividing the respective deflections by .0052; for it was shown in example 1, in the last answer, that when $R' = 2{,}000$ ohms, one line of force causes a deflection of .0052 scale division; therefore, the number of lines of force in the iron is the deflection divided by .0052.

The *total* number of lines of force, column 6, corresponding to the respective magnetic forces tabulated in column 3, are obtained by adding the change in the number of lines of force in column 5 to the total number of lines of force of the reading immediately preceding.

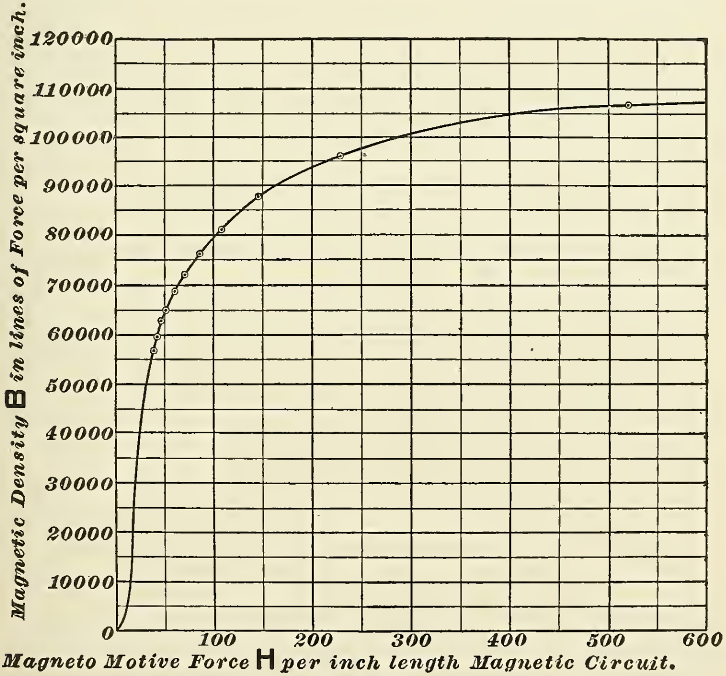


FIG. 992.

The lines of force per square inch **B**, in column 7, are obtained by dividing the total number of lines of force, column 6, by the cross-sectional area of the iron ring. This area is evidently $\frac{3}{4}$ in. × 1 in. = .75 square inch.

NOTE.—The student is advised to perform the computations enumerated, to better comprehend the rules and principles involved.

These values of the magnetomotive force **H** and the corresponding density **B** of the lines in the iron are now plotted on a sheet of cross-section paper, and the points so obtained connected by a line forming the *magnetization curve* of the piece of iron under test. This curve is shown in Fig. 992.

SOLUTION.—(*b*) Wrought iron. This is learned by comparing the magnetization curve obtained with the magnetization curves given in Fig. 952.

2487. The galvanometers thus far described comprise the principal forms of galvanometers in use. The selection of any one instrument for a test depends upon its particular fitness for that work. All galvanometers, however, are merely current measurers, or, in some cases, current indicators only, and certain features of their use and certain apparatus used with them are common to all.

GALVANOMETER SHUNTS.

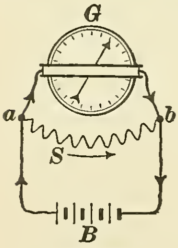


FIG. 993.

2488. If a resistance be connected in parallel with a galvanometer through which a current is flowing, the current will divide between the two branches of the circuit, as shown in Fig. 993, inversely as the respective resistances of the circuits; and the galvanometer is said to be *shunted* by the resistance. (See Art. **2320.**)

The drop in volts in each branch will be the same; that is, $C_s R_s = C_g R_g$, where C_g = current in galvanometer, C_s = current in shunt, R_g = resistance of galvanometer, R_s = resistance of shunt. The total current $C = C_s + C_g$. The fraction of the total current that passes through the galvanometer is found by the formula

$$C_g = \frac{C}{n+1}, \qquad \textbf{(454.)}$$

where $n =$ the resistance of G divided by the resistance of $S = \frac{R_g}{R_s}$.

This results from the equations $C_s\ R_s = C_g\ R_g$ and $C = C_s + C_g$ as follows: $n = \frac{R_g}{R_s}$, or $n\ R_s = R_g$.

Therefore, for $C_s R_s = C_g R_g$, write $C_s R_s = C_g\, n\, R_s$, or $C_s = C_g\, n$; for $C = C_s + C_g$, write $C = C_g\, n + C_g$, or $C = (n+1) C_g$.

Therefore, $$C_g = \frac{C}{n+1}.$$

Thus, by inserting a known resistance in parallel with a galvanometer, also of known resistance, the total current flowing may be calculated from the current flowing in the galvanometer, as measured by it. A resistance arranged for such use with a galvanometer is known as a **galvanometer shunt.**

This affords a convenient means of increasing the range of a galvanometer, as by inserting the proper shunts, currents of any reasonable multiple of the normal range of the galvanometer may be measured.

Galvanometers are often furnished by the makers with shunts of $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of the resistance of the instrument, which increase the range of the instrument 10, 100, or 1,000 times. Applying formula **454,** we obtain for the three different shunts the following currents :

$$\left(C_g = \frac{C}{n+1} = \frac{C}{9+1=10}\right).$$

$$\left(C_g = \frac{C}{n+1} = \frac{C}{99+1=100}\right).$$

$$\left(C_g = \frac{C}{n+1} = \frac{C}{999+1=1,000}\right).$$

The value of $n+1$ is, therefore, the amount by which any particular shunt will multiply the range of the instrument, and is called the **multiplying power** of that shunt.

In the foregoing cases the multiplying powers of the shunts are obviously 10, 100, and 1,000, respectively.

2489. To find the necessary shunt resistance to make the multiplying power of the shunt any desired amount, *divide the resistance of the galvanometer by the multiplying power less 1;* as the multiplying power $= n + 1$, then, $(n + 1) - 1 = n$, or $R_s = \frac{R_g}{n}$.

It is evident that introducing the shunt into the circuit in parallel with the galvanometer reduces the resistance of that part of the circuit (between a and b, Fig. 993). In some delicate measurements it is desirable that this resistance be not altered, and galvanometer shunts are sometimes mounted in connection with a second resistance, known as a **compensating resistance,** which is introduced into the circuit in *series* with the galvanometer and its shunt. This resistance is given such a value that its resistance, plus the *combined resistance* of the galvanometer and its shunt as connected in parallel, is equal to the resistance of the galvanometer alone. The value of this resistance for any particular case may be readily calculated from the formulas for derived circuits. (See Arts. **2320** to **2329.**)

EXAMPLES.

2490. 1. If a galvanometer whose resistance is 21 ohms gives a deflection of 40 with a current of 2 amperes, what will be the resistance of the shunt that must be used to cause 16 amperes to give the same deflection?

SOLUTION.—The multiplying power of this shunt is evidently 8; therefore, $8 = n + 1$ and $n = 7$. $R_g = 21$ and $\frac{21}{7} = 3$ ohms. Ans.

2. What must be the value of a *compensating resistance* if used with the galvanometer and its shunt in the above example?

SOLUTION.—Let $R_g =$ the resistance of the galvanometer $= 21$ ohms, and $R_s =$ the resistance of the shunt $= 3$ ohms. By formula **412,** their joint resistance in parallel is, then, $R = \frac{R_g R_s}{R_g + R_s}$.

Substituting the values of R_g and R_s, $R = \frac{21 \times 3}{21 + 3} = \frac{63}{24} = 2.625$ ohms.

As the compensating resistance plus the joint resistance of the galvanometer and shunt is equal to the galvanometer resistance, or $R_c + R = R_g$, substituting the values gives $R_c + 2.625 = 21$, or $R_c = 21 - 2.625 = 18.375$ ohms. Ans.

3. What is the resistance of a galvanometer if a shunt of 10 ohms resistance has a multiplying power of 8? Ans. 70 ohms.

PRECISION IN MEASUREMENTS.

2491. Mathematical results can be obtained with absolute accuracy with proper attention, but any *measurements* which can be made are liable to error, that is, it can not be determined that the measurement is absolutely correct. For example, an absolutely rectangular portion of the top of a table 37.5 in. long and 20 in. wide has a surface area of absolutely 750 square inches, no more and no less, but it would be impossible to lay out a surface on a table or anywhere else that would be *known* to have a surface area of *exactly* 750 square inches.

Results from a series of measurements can not be expected to have a greater degree of accuracy than the instruments with which such measurements are made; and, conversely, it is unnecessary labor to use very accurate and sensitive instruments to obtain results which it is only necessary to know approximately.

Again, each of a series of measurements should be made with a degree of precision corresponding to the effect each measurement will have on the final result. For example, if it be desired to find the cubic inches of iron in a bar about 20 feet long and about $\frac{3}{4}$ inch square, by measuring its length and width and thickness, it would be absurd to carefully measure the length to eighths of an inch with a graduated scale, and then to estimate the width and thickness by using the end joint of the thumb as an inch and estimating by the eye the fraction of that distance that would equal the width or thickness of the bar.

In making delicate tests that require a high degree of accuracy, the subject should be carefully studied, and precautions taken to remove as far as possible any source of error; the reading should be repeated several times, and, if possible, repeated with different methods and apparatus. Even then the best that can be said is that the results are as nearly accurate as the apparatus will allow, to the best of one's judgment.

So, in making measurements, electrical or otherwise, care should be taken to make the apparatus, methods of using it, and the necessary calculations as accurate as the required degree of precision of the final result requires. At the same time unnecessary labor in making one part of the work precise beyond a point where the unavoidable errors in another part would neutralize such precision should be avoided.

2492. This leads to the consideration of how many significant figures to retain in the readings, calculations, and results to obtain results within the accuracy of the instruments used. By **significant figures** is meant the number of *digits*, with the exception of the zeros used to indicate the position of the decimal point.

For example, 20467, 28.321, and .00010569 would each have five significant figures. If it were necessary to use but four significant figures, these values would be written 20470, 28.32, or .0001057; that is, if the figure dropped be 5 or greater, the next figure to the left is increased 1; if less than 5, the figure to the left is unchanged. Zeros are sometimes significant figures, as in the example, $25 \times 4 = 10\dot{0}$, which has three significant figures in the answer, 100, as the example has been carried out far enough to show that the value of units and tens is 0 in each case. In the previous example where four significant figures are required, the number 20470 indicates that the actual value of the last figure 0 is known to be within 5 units either way from 0. Whereas, if five significant figures were required, 20470 would indicate that the last number was known to be within .5 unit either way from 0.

The requirements of the calculations and results of observations in this respect are as follows:

(*a*) If any one of the measurements can not be determined within 1%, four significant figures retained in any reading, calculation, or result will give an answer correct within the limits of precision of the measurements.

(*b*) If any one of the measurements can not be determined within less than 0.1%, but can be within 1%, five significant figures are required, and (*c*) if not within 0.01%, but within 0.1%, six significant figures are required.

The degree of precision of the various instruments used in making electrical measurements can be obtained either from careful calibration or from the maker's guarantee, and results obtained from such instruments may be calculated with the allowable degree of accuracy by observing the requirements given.

ELECTROCHEMICAL MEASUREMENTS.

2493. The decomposition of liquids by the electric current affords a means of measuring the current that requires but little apparatus and gives very precise results. This method is chiefly used for determining galvanometer constants, as it is not usually well suited for the measurement of commercial currents, that is, currents used for lighting, power, etc.

The following constants for the decomposition of water have been accurately determined. A current of 1 ampere flowing for 1 second will decompose .00009324 gram (.0014388 grain) of water. The gas resulting from this decomposition is a mixture composed of .00001036 gram of hydrogen and .00008288 gram of oxygen; so, by measuring the amount of water decomposed by a current in a given time, either by measuring the loss of weight of the water or by measuring the volume of either or both gases given off, the value of the current may be calculated. The latter method is called the *volume method.* Since corrections must be made for temperature, pressure, etc., in determining by their volume the weight of the gases collected, this method involves considerable labor and time. It is usually simpler to use the first mentioned, or *weight method.*

2494. In this method the gases resulting from the decomposition of the water are allowed to escape into the air. If they were allowed to pass off directly from the surface of the water, considerable water vapor would pass off with them, and the loss of weight of the water would not be a true measure of the current flowing; these gases are, therefore, made to pass through a certain apparatus known as a **desiccator,** which consists merely of a glass tube, loosely filled with some substance which will absorb the water vapor while allowing the gases to escape unchanged. The whole apparatus, including desiccator and water, should be weighed, and the difference between the weight before and after the passage of the current will represent the value of the current.

Fig. 994 shows two forms of apparatus for this method of measuring current. The tube *T* contains the liquid and the (platinum) electrodes, which pass through a cork which has been boiled in paraffin. The tube *t* is the desiccator, and is loosely filled with asbestos soaked in sulphuric acid. The small tube in the end of the tube *t*, through which the gases pass off, should be closed with a paraffined cork *c* previous to and just after the passage of the current, to prevent the acid in the tube *t* absorbing moisture from the air, which would introduce an error.

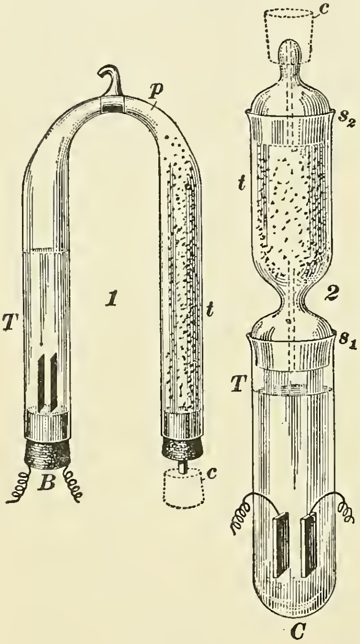


Fig. 994.

The second form of apparatus illustrated is more difficult to make. The joints at s_1 and s_2 are ground to fit, and wires to the electrodes are sealed into the sides of the tube *T*.

As pure water has an extremely high resistance (one authority gives 7 megohms per cubic centimeter), it is necessary to add some substance to increase its conductivity. For these tests sulphuric acid is used; a small proportion of acid is sufficient.

2495. To measure current with this apparatus, the tube T should be filled with acidulated water, and the tube t with asbestos soaked in sulphuric acid. With the cork c in place, carefully weigh the whole apparatus. Then join the terminals of the apparatus to the battery from which the current is to be taken, and removing the cork c, note the exact time at which the final connection is made to the battery. It is better to use some form of switch for closing the circuit after all connections have been made. After allowing the current to pass until sufficient water has been decomposed, break the circuit, noting again the exact instant the current ceases to flow, replace the cork c and reweigh the apparatus.

To find the strength of the current that has been passing in amperes:

Let w_1 = the original weight of apparatus;
w_2 = the weight after the current has passed;
t = time in seconds during which the current flows;
C = strength of current in amperes.

Then, if the weights are taken in *grams*,

$$C = \frac{w_1 - w_2}{.00009324\,t}. \qquad \textbf{(455.)}$$

If the weights are in *grains*,

$$C = \frac{w_1 - w_2}{.0014388\,t}. \qquad \textbf{(456.)}$$

Rule.—*To determine the strength of a current by decomposition of water, subtract from the original weight of the apparatus its weight after the current has passed through; divide this result, expressed in grams or grains, by the length of time the current was passing, in seconds, multiplied by the number of grams or grains of water which can be decomposed by 1 ampere in 1 second.*

EXAMPLE.—The original weight of the apparatus was 980.5 grams; the current was passed through for 38 minutes, and the weight was then found to be 979.6 grams; what was the strength of the current in amperes?

SOLUTION.—In this example, $w_1 = 980.5$ grams; $w_2 = 979.6$ grams 38 minutes = 2,280 seconds = t. Then, by formula **455**,

$$C = \frac{980.5 - 979.6}{.00009324 \times 2{,}280} = \frac{.9}{.2125872} = 4.233+ \text{ amperes. Ans.}$$

It will now be readily seen that if a galvanometer be connected in series with the apparatus for decomposing the water, and the deflection noted, its galvanometer constant may be easily calculated.

EXAMPLES FOR PRACTICE.

1. If the loss in weight of apparatus be 3.462 grains after a current has passed through for 40 minutes, how many amperes have been passing? Ans. 1.0025 amperes.

2. If .756 ampere is passed through the apparatus for 1 hour, what will be the loss of weight of the apparatus (*a*) in grams? (*b*) in grains?

Ans. $\begin{cases} (a)\ .2537+. \\ (b)\ 3.9158+. \end{cases}$

NOTE.—In decomposing water, a battery of sufficient number of cells to give about 2 volts should be used. The cells should give a constant current.

2496. If a solution of copper sulphate (blue vitriol) be used instead of acidulated water, the decomposition of the liquid by the current will cause a deposit of copper on the negative plate. The weight of copper deposited in a given time is proportional to the current flowing, and 1 ampere will deposit .0003286 gram of copper in 1 second. Moderate variations in the proportions of copper sulphate in the solution or the temperature do not affect the result appreciably. The above figure is given for a half saturated solution of copper sulphate [that is, about 1 part (by weight) of copper sulphate to 5 parts of water] at a temperature of 73° F. A reduction of temperature to 54° F. would not alter the figure given by more than .03%.

In making measurements of the amount of copper deposited, electrodes of copper should be used, of such size that there shall be from 8 to 15 square inches of surface to be deposited upon for each ampere of current.

When the copper is deposited from the copper sulphate solution, sulphuric acid is set free, which dissolves a portion of the *positive* plate, forming copper sulphate, thus keeping the amount of copper sulphate in solution practically constant. The positive plate does not lose in weight in direct proportion to the current passing, so in measurements of this description the gain in weight of the negative plate only is measured.

2497. Apparatus prepared according to the following description will afford a means of measuring the current, which requires even less apparatus than the weight method of water decomposition, but the precautions therein noted should be taken to insure reliable results.

Trough.—The vessel or trough used should be of wood or other insulating material, of sufficient size to allow the square part of the plates to hang entirely below the surface of the liquid.

2498. Plates.—It would be best to use three plates, one negative or *gain plate*, suspended between two positive or *loss plates*, which should be of the same shape and material as the gain plate, but somewhat smaller and thicker The gain plate should be of very thin copper, so that its gain in weight will be enough to make considerable difference between its weights before and after the test.

The plates should be cut approximately square and the corners clipped off. It is rather better to make them circular, but this form is often not as convenient to prepare, and is not at all necessary. From one side of the plate a narrow strip should be left projecting, long enough to bend into a hook by which to hang the plate on the scales or in the liquid.

Three pieces of heavy bare copper wire or rod should be provided, long enough to reach across the top of the trough; on resting these on the edges of the trough a short distance apart, the electrodes may be readily hung from them and the necessary connections made to them from the battery.

The positive plates should be rubbed bright on both sides with fine sandpaper. The negative plate should be carefully rubbed smooth and bright with very fine sandpaper or emery, taking great care not to touch the part of the bright surface that will be below the surface of the liquid with the bare fingers or any greasy substance. A piece of clean paper or cloth should be used to handle the plate with. After carefully brightening the plate, it should be washed and dried carefully several times, and then accurately weighed.

This preparation of the gain plate should not be made until all the rest of the apparatus is ready, as a long exposure to the air will oxidize the bright surface of the copper.

2499. Liquid.—Make the liquid by dissolving 1 part (by weight) of crystals of copper sulphate in 5 parts (by weight) of water, and adding about 1 per cent. of strong sulphuric acid. (One per cent. is about 3 teaspoonfuls to the quart.) This excess of acid serves to dissolve such impurities as may exist in the copper sulphate.

A conducting liquid thus prepared for electrolysis or for use in a battery is known as an *electrolyte*. (See Art. **2238.**) Other salts of metals in solution besides the above may be used as the electrolyte, with corresponding metals as electrodes.

2500. Battery and Connections.—A battery of two cells in series will be sufficient if small currents are desired. Cells should be used giving approximately a constant current. If constant-current batteries are not available, use three or four cells of some other type, and insert a resistance which may be varied to keep the current constant. The various forms of cells will be described later. Connections should be made by means of insulated wires, as shown in Fig. 995, where S = switch for making and breaking the circuit; B = battery; R = resistance for keeping the current constant; G = galvanometer; T = trough containing plates and liquid;

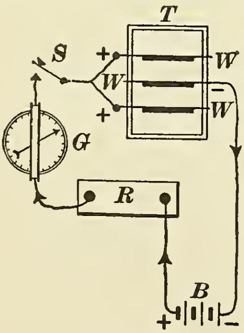


FIG. 995.

W, W, W = copper wires across top of trough from which plates are hung.

2501. After preparing the solution and setting up the apparatus, the positive plates should be hung in place, then the negative plate should be prepared and weighed; as soon as possible hang the negative plate in place; put in sufficient liquid to completely cover the plates; then, close the switch, noting the exact instant when the circuit is made. The deflection of the galvanometer needle should be noted from time to time, and any change in the deflection corrected by changing the resistance R. After sufficient time has elapsed, open the switch, again noting the exact time.

As soon as possible, take out the negative plate, wash and dry it carefully several times, and accurately weigh it. Then, find the amperes that have been flowing by the following formulas:

Let w_1 = the original weight of gain plate;
w_2 = the weight after the current has passed;
t = time in seconds during which the current flows;
C = strength of current in amperes.

Then, if the weights are in grams,

$$C = \frac{w_2 - w_1}{.0003286\, t}. \qquad \textbf{(457.)}$$

If the weights are in grains,

$$C = \frac{w_2 - w_1}{.005068\, t}. \qquad \textbf{(458.)}$$

Rule.—*In order to determine the strength of current by measurement of copper deposited, subtract the original weight of the gain plate, in grams, from the weight as found after the experiment; divide this result by the length of time the current was flowing in seconds multiplied by the number of grams of copper which can be deposited by 1 ampere in 1 second.*

After finding the current which has been passing, the galvanometer constant can be found from formulas **448** and **450.**

Example.—The negative plate is a sheet of copper about $2\frac{1}{2}$ in. square and about $\frac{1}{32}$ in. thick. After cleaning, it weighs 29.62 grams. The current being allowed to pass for 75 minutes, the plate weighs 31.33 grams. A tangent galvanometer in circuit gave a deflection of 42°. (*a*) How many amperes were passing, and (*b*) what was the galvanometer constant?

Solution.—(*a*) In this example, $w_1 = 29.62$ grams; $w_2 = 31.33$ grams; $t = 75 \times 60 = 4{,}500$ seconds. Then, by formula **457**, the current

$$C = \frac{31.33 - 29.62}{.0003286 \times 4{,}500} = 1.1564 \text{ amperes. Ans.}$$

(*b*) Use formula **450.**

$$C = K \tan m^\circ.$$

$$\text{Tan } 42^\circ = .9004.$$

Then,

$$\frac{C}{\tan m^\circ} = K;$$

or,

$$\frac{1.1564}{.9004} = K = 1.2843. \quad \text{Ans.}$$

Note.—The weight of copper deposited per ampere per second may be taken in grains (Troy) instead of grams, and the result worked out in the same way. 1 gram = 15.432 grains (Troy).

Example.—Change the weights in the above example to grains and work out the results.

MEASUREMENT OF POTENTIAL.

2502. If two points between which a difference of potential exists are connected together by a conductor, a current will flow from one to the other, its value depending on the resistance of the conductor and the difference of potential between the two points.

If this conductor be the coil of a galvanometer, it is obvious that the divisions on the scale may be marked to read volts instead of amperes.

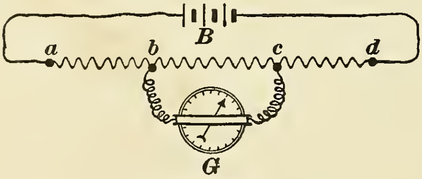


Fig. 996.

In Fig. 996 a current flows from the battery *B* through the resistance *a b c d*. There will, therefore, be a certain fall of potential along *a b c d*, and it may be desired to measure the difference of potential between *b* and *c*.

2503. If a galvanometer whose resistance is approximately that of the part of the circuit *b c* is connected to the points *b* and *c*, the current flowing from *a* to *b* will divide at *b*, and a part flow through the galvanometer *G*. The whole current will again flow from *c* to *d*. If the resistance of the galvanometer is known, the current flowing through it, as measured by the deflection of the needle, is also a measure of the difference of potential between *b* and *c*, but this difference of potential is not the same as it was before the galvanometer was connected.

The galvanometer being placed in parallel with a part of the circuit reduces the total resistance of the circuits, and as the distribution of resistance between *a* and *d* is changed, the distribution of the fall of potential will also be changed.

In order, therefore, to measure the difference of potential between *b* and *c*, the instrument used should be so constructed that it will not measurably alter the conditions of the circuit. If the galvanometer *G* in Fig. 996 have a very high resistance as compared with *b c*, so that the current passing through it will be a very small percentage of the total current in the circuit, the conditions will not be altered sufficiently to introduce any serious error.

2504. When a difference of potential exists between two points between which no current is flowing, as a battery with no external circuit made, it is usually the case that any considerable current flowing will reduce this difference of potential, owing to the internal resistance of the battery or other generator of the E. M. F.

To measure this difference of potential again requires a galvanometer of such resistance that a very small current will flow through it, in order that the conditions of the circuit shall not be sensibly changed; so that commercial measuring instruments that are constructed on the galvanometer principle are divided into two classes:

1. Instruments of low resistance, so arranged that a considerable current is required to give readable deflections, usually with the scales so marked as to give the deflection

of the needle the proper value in amperes of the current passing through the instrument. These are called **ampere-meters**, or more briefly **ammeters.**

2. Instruments of high resistance, so arranged that very small currents will give readable deflections, and with the scales usually so marked as to give the deflection of the needle the proper value in volts of the difference in potential between the points to which the instrument is connected. Such instruments are called **voltmeters.**

2505. Fig. 997 illustrates a method of measuring differences of potential, in which the principle of operation necessitates that *no current* be flowing through the galvanometer G. In the figure, $a\ b\ c\ d$ is a resistance through which a current is flowing, supplied by the battery B. It is desired to measure the difference of potential between b and c. S is a battery of standard cells, the E. M. F. of which is known, arranged so that one or more of the cells in series may be used.

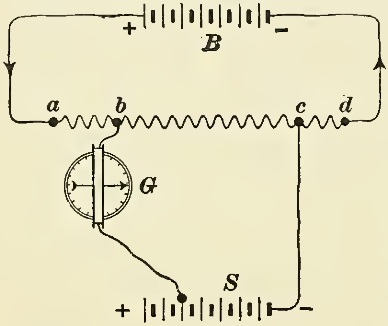


Fig. 997.

If the negative pole of the battery S be connected to the same end of $b\,c$ that the negative pole of B is connected, and the positive pole of S to the other end of $b\,c$, through the galvanometer G, it will be seen that, if the battery S had no E. M. F. of its own, the difference of potential between b and c would tend to drive a current through S from b to c, which would be indicated by the galvanometer. S has an E. M. F., however, which, from the way it is connected, opposes the passage of such a current, and if the E. M. F. of S exactly equals the drop in volts between b and c, no current will flow through S. To measure the difference of potential between b and c, it only remains to adjust the number of cells in S until the galvanometer G

indicates no deflection; then, multiplying the E. M. F. of each cell in S by the number used will give the drop in volts between b and c.

As no current flows through the galvanometer, it may be large or small, of high or low resistance, as long as it is sensitive to small currents. Consequently, various current strengths and differences of potential may with this method (called the **zero** method) be measured with a single galvanometer.

MEASUREMENT OF RESISTANCE.

2506. The resistance of a conducting body may be measured in a variety of ways. One of the most common is the **fall of potential** method, which consists of passing a current through the unknown resistance and measuring the amperes flowing and the drop in volts through the resistance. The resistance is calculated from Ohm's law. Fig. 998 shows the arrangement of the apparatus. $a\ b\ c\ d$ is a resistance of which it is desired to know the resistance $b\ c$. A current from the battery B flows through the ammeter $A\ M$ and the resistance. The drop in volts from b to c is measured by the voltmeter $V\ M$.

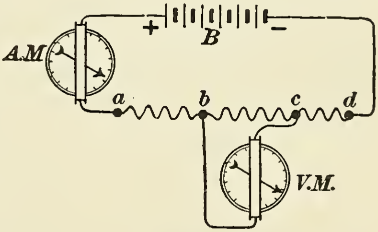


FIG. 998.

EXAMPLE.—1. If the current flowing from a to d be 2.2 amperes, and the drop from b to c be 6.25 volts, what is the resistance of the part of the circuit $b\ c$?

SOLUTION.—By formula **410,** $R = \frac{E}{C}$.

$E = 6.25$. $C = 2.2$. $R = \frac{6.25}{2.2} = 2.841$ ohms. Ans.

2. If the current be found to be 21.25 amperes, and the drop in potential 4.6 volts, what is the resistance? Ans. .2165 ohm.

2507. This method of measuring resistance is often not convenient, and many times impossible to use. Another method is to compare the unknown resistance with one or more known resistances in several ways. It may be done by connecting a known and the unknown resistance in series, and, on sending a current through the two, measuring the drop in volts across each. The resistances will be directly proportional to the fall of potential, and the current need not be measured.

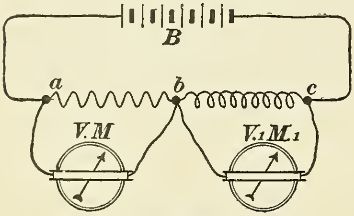


FIG. 999.

In Fig. 999, *B* is the battery, the current from which flows through the known resistance *a b* and the unknown resistance *b c*. Voltmeters $V M$ and $V_1 M_1$ measure the fall of potential across each. The same voltmeter might readily be used for both readings.

EXAMPLE.—1. If the resistance *a b* is known to be 2 ohms, and the drops as measured by $V M$ and $V_1 M_1$ are 4.25 volts in *a b* and 6.12 volts in *b c*, what is the resistance of *b c*?

SOLUTION.—As the resistances are directly proportional to the drops of potential,

$$4.25 : 6.12 :: 2 : x;$$

or,
$$\frac{2 \times 6.12}{4.25} = x = \frac{12.24}{4.25} = 2.88 \text{ ohms.} \quad \text{Ans.}$$

EXAMPLE.—2. If the drop through the known resistance is 6.28 volts and through the unknown 2.25, what is the unknown resistance if the known is 3.5 ohms? Ans. 1.254 ohms.

2508. Another way to attain the same result would be to connect the known and the unknown resistance in parallel, and measure the current in each. The currents would be in inverse proportion to the resistances. In Fig. 1000 the current from the battery *B* divides at *x*, a part flowing through the known resistance *a b* and the balance through the unknown

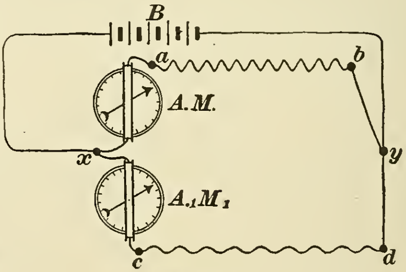


FIG. 1000.

resistance *c d*. Ammeter *A M* measures the current in *a b* and ammeter A_1 M_1 measures the current in *c d*.

It is to be noted that the ammeters and their connecting wires should be of such low resistance as not to add materially to the resistance of either branch of the circuit.

EXAMPLE.—If ammeter *A M* indicates 3.6 amperes and ammeter $A_1 M_1$ indicates 4.2 amperes, what is the resistance of *c d* if *a b* is 10.5 ohms?

SOLUTION.—As the currents are inversely proportional to the resistances,

$$4.2 : 3.6 :: 10.5 : x;$$

or

$$x = \frac{3.6 \times 10.5}{4.2} = \frac{37.80}{4.2} = 9 \text{ ohms. Ans.}$$

2509. In Fig. 1000 the drop along *c d* must be the same as that along *a b* (neglecting the ammeter resistances). If any point in *c d* be selected, a point in *a b* can be found that will have the same difference of potential between it and *x* that the point in *c d* has.

If a galvanometer is connected across from the point in *c d* to the point in *a b*, no current will flow through it, as there is no difference of potential between the points. Fig. 1001 represents this condition.

It is obvious that the resistance from *a* to *n* must be the same proportion of the total resistance *a b* that the resistance from *c* to *m* is of *c d*, in order that the drop in *a n* shall be the same as in *c m*. If the point *n* be moved to any point in *a b*, the point *m* must be correspondingly moved on *c d*, in order that there may be no current flowing though *G*, and that the proportion *a n* : *a b* :: *c m* : *c d* may still hold good.

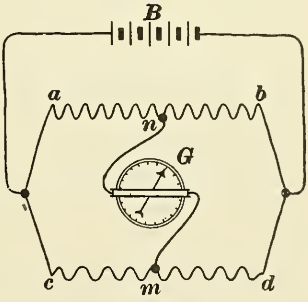


FIG. 1001.

It is also evident that the same proportion holds good for *n b* and *m d*; i. e., *n b* : *a b* :: *m d* : *c d*.

From the above proportions,

a n : *c m* :: *a b* : *c d*, and *n b* : *m d* :: *a b* : *c d*.

Therefore, *a n* : *c m* :: *n b* : *m d*.

That is, the resistance of *a n* is to the resistance of *c m* as the resistance of *n b* is to the resistance of *m d*.

From this proportion, it is evident that if *a n*, *c m*, and *m d* be known, the resistance of *n b* may be readily calculated.

This affords a ready means for measuring resistance, which, as will be shown, is very flexible and universally applicable.

2510. In Fig. 1002, *M*, *N*, and *P* are three known resistances, which may be varied by known amounts. An unknown resistance *X* is connected to *c* and *b*, completing the branch *a c b* of the circuit from *a* to *b*. Through this circuit a current flows from the battery *B*. Any one of the three resistances *M*, *N*, and *P* may be adjusted until the galvanometer *G* indicates that the points *c* and *d* are at the same potential; then, from the proportion given in Art. **2509,** $M : N :: X : P$.

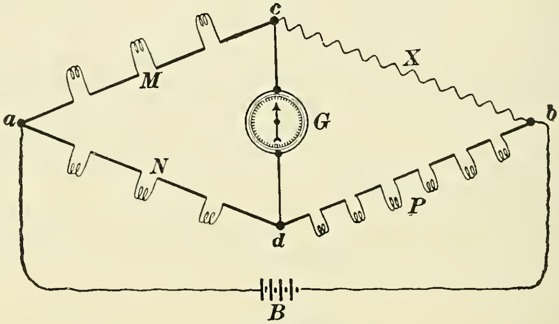


FIG. 1002.

It is obvious that if *M* be equal to *N*, *X* will be equal to *P*, while if *X* be a very high or very low resistance it may be measured equally well by changing the ratio of *M* to *N*. In any case,

$$X = \frac{M}{N} \times P. \qquad \textbf{(459.)}$$

This method of measuring resistance is known as the Wheatstone bridge method, and the instrument used is called a **Wheatstone bridge,** or, more commonly, a **bridge.**

In practice, the arms M, N, and P of the bridge are made up of a number of carefully prepared resistance coils, accurately adjusted to different resistances, fixed in a box, on the top of which are arranged blocks of brass, which form the terminals of the coils. The brass blocks are so situated that by inserting a metallic plug between any two of them the corresponding resistance coil is cut out, or *short-circuited;* that is, the current passes from block to block through the plug instead of going through the coil, as this path offers practically no resistance to the current. In this way the resistance of the arms of the bridge is changed.

Fig. 1003 shows a section of a box of coils showing the brass blocks and the method of cutting out the coils. *a*, *b*, *c*, *d*, *e*, *f*, and *g* are the brass blocks, to which are connected the coils *1*, *2*, *3*, *4*, *5*, and *6*. The brass plug *P* is made to fit tightly between the blocks.

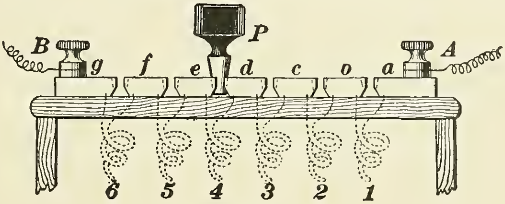


FIG. 1003.

The current from the battery is not allowed to flow continuously through the resistance coils, which might introduce errors owing to the heating effect of the current, but the battery circuit and galvanometer circuit are each provided with a key. On pressing the battery circuit key, the current passes through the bridge, and on then pressing the galvanometer key, it is seen, from the motion or lack of motion of the needle, if the proportion of resistance is correct. It is usual to make the arms M and N of comparatively few coils, with ratios of 10; for example, 1, 10, 100, and 1,000 ohms. By cutting out, for instance, all but the 1-ohm coil in one arm, and leaving all the coils in the other, the ratio of M to N is 1,111 to 1, or 1 to 1,111, as the case may be; so that an instrument thus arranged would measure resistances varying from 1,111 times the largest value of P to $\frac{1}{1111}$ of the smallest value of P.

For bridge measurements requiring a considerable degree of accuracy, it is best to use a sensitive reflecting galvanometer, which will respond to very slight differences in potential between c and d (in Fig. 1002).

2511. In Fig. 1004 is shown an arrangement of a bridge in which $H\ G$ corresponds to arm M (in Fig. 1002), $E\ F$ to arm N, $A\ B\ C\ D$ to arm P, and x to the unknown resistance. K, K' are the keys for closing the battery and galvanometer circuits, respectively. The number against each coil represents the resistance of that coil. It will be seen that with the resistances in $A\ B\ C\ D$, a great number of combinations can be made with suitable cutting in or cutting out of coils by means of the plugs, as at a, b, c, d, etc.

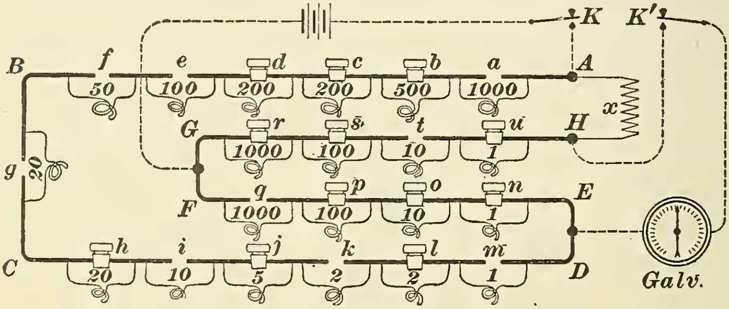


FIG. 1004.

EXAMPLE.—1. If in the figure, as shown, the galvanometer shows no deflection on pressing the keys K, K', what is the resistance of x?

SOLUTION.—In arm M ($H\ G$) the 10-ohm coil is in circuit, the others are short-circuited by the plugs r, s, and u. In arm N ($E\ F$) the 1,000-ohm coil is in circuit, the rest being short-circuited by the plugs n, o, and p. In arm P ($A\ B\ C\ D$) the 1,000, 100, 50, one 20, 10, one 2, and one 1 ohm coils are in circuit, the rest being short-circuited by the plugs b, c, d, h, j, and l. The resistances are, therefore,

$M = 10$ ohms;
$N = 1{,}000$ ohms;
$P = 1{,}000 + 100 + 50 + 20 + 10 + 2 + 1 = 1{,}183$ ohms;

and by formula **459**, $X = \frac{M}{N} \times P$, or $\frac{10}{1{,}000} \times 1{,}183 = 11.83$ ohms. Ans.

EXAMPLE.—2. What plugs would have to be *inserted* in P to measure a resistance of 21.7 ohms in x, if the 1-ohm coil only be used in M and the 10-ohm coil only be used in N?

Ans. *a*, *b*, *c*, *e*, *f*, *g*, *h*, *l*, *m*, or *a*, *b*, *d*, *e*, *f*, *g*, *h*, *k*, *m*.

2512. The resistance of the coils is usually stamped on the top of the box, that for each individual coil being marked beside the space between the brass blocks to which the coil is attached, so that, after having made the necessary adjustments, it is easy to read off the resistance in either arm of the bridge by adding the figures opposite the spaces unfilled by plugs.

The coils themselves are wound on spools of insulating material, and in reliable instruments are carefully standardized. In order that the current flowing through a resistance coil of a considerable number of turns should not create a magnetic field which might affect the galvanometer, the coils are wound *non-inductively;* that is, for each turn around the spool in one direction is wound a turn in the opposite direction, so that the magnetic effects are neutralized. In working with a sensitive galvanometer this precaution is very necessary. The usual method of winding the spool is to measure off the length of wire required and fold it in the middle; then, starting at this fold, the two parts of the wire are wound on as one wire. A current circulating in a spool so wound will pass through one half the wire in one direction and the other half in the reverse; so the magnetic effects, as well as the self-induction, are rendered practically zero.

In making resistance measurements with a Wheatstone bridge, it is not necessary to know either the current flowing or the E. M. F. of the source of current; so almost any source of a steady current of low E. M. F. is suitable for bridge work. It is customary to use two or three cells of battery, except for measuring high resistances, when more cells, up to 30 or 40, should be used.

2513. For measuring low resistances, a modification of the Wheatstone bridge, known as the **slide wire** or **meter** bridge, is used. A diagram of this bridge is shown in Fig. 1005. A wire *a b* of uniform cross-section is stretched between the heavy copper blocks *c* and *d*. *R* is a known and *X* an unknown resistance, both of which are connected at one end to the heavy copper block *e*, and at the other to the blocks *c* and *d*, respectively. The galvanometer is connected between the block *e* and a contact piece *n*, sliding on the wire *a b*. It will be seen that this is a form of the Wheatstone bridge where the arms *M* and *N* are replaced by *R* and *a n*, and the adjustable resistance by *n b*. From the consideration of the principles of the Wheatstone bridge,

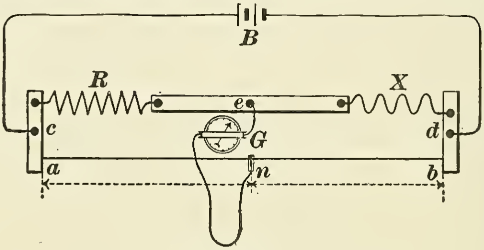


FIG. 1005.

$$R : X :: a\,n : n\,b;$$

or,
$$X = R \times \frac{n\,b}{a\,n}.$$

The copper blocks *c*, *e*, and *d* are made heavy, so that they will introduce no appreciable resistance into either arm of the bridge. As the wire *a b* is of uniform cross-section, its absolute resistance need not be known; as the resistance of the two parts *a n* and *n b* will be directly proportional to their lengths, the formula

$$X = R \times \frac{n\,b}{a\,n}$$

will hold good if *a n* and *n b* represent *length* instead of *resistance*. It is not even necessary to know the actual lengths of *a n* and *n b*; their ratio is sufficient. It is customary, however, to make the length of the wire *a b* one meter in this form of slide-wire bridge; whence the name *meter bridge*. The slider *n* is usually arranged so that one end slides along a scale the length of the wire, divided into any convenient number of divisions; in the case of a meter bridge, into millimeters; so that the lengths *a n* and *n b* may be read directly from the scale. The known resistance R is not usually made adjustable; instead *standard coils* are used, the usual sizes being 0.1, 1, and 10 ohms, the particular coil used being selected according to the resistance X. This makes the construction of the bridge much cheaper than the ordinary form, and as standard resistance coils of great accuracy may be purchased already prepared, the bridge may be cheaply and easily constructed.

2514. These standard resistance coils are usually of the form shown in Fig. 1006. The resistance coil itself is enclosed in a brass shell, and the whole filled with paraffin. The two projecting wires are of heavy copper, and serve as terminals. In order to insure good contact, when great accuracy of measurement is required, the terminals of the copper bars c, c, and d, where the resistances R and X are attached, are usually made in the form of mercury cups, instead of binding-posts, so that in connecting the standard resistance coil it is only necessary to hang the ends of the terminals in the mercury cups.

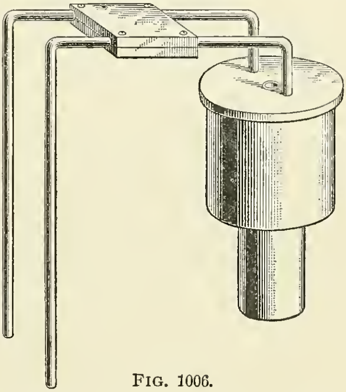


FIG. 1006.

It is not at all necessary that the wire of the slide-wire bridge be stretched out straight, as shown in Fig. 1005. This is a very convenient way to make such a bridge, but they are often built with the wire wrapped around an insulating cylinder, or stretched around the edge of a support, which may be circular or square, or of other shape; the main point being to support a length of wire so that the ratio of the distance between any point on the wire and one end to the whole length of the wire may be determined.

2515. The slide-wire bridge is more especially suited, as stated, to the measurement of low resistances, such as determining the *specific resistance* of metals, etc. The **specific resistance** of a conducting substance is the resistance of unit length of unit cross-section of that substance; that is, the resistance of a piece of metal 1 centimeter long, whose area of cross-section is 1 square centimeter, is its specific resistance. Expressed in ohms, the specific resistance of flint glass is 16,700,000,000,000,000,000, and that of annealed silver is .000001500, their ratio being about 1 : 11,000,000,000,000,000,000,000,000. To prevent the constant repetition of zeros in writing these and similar values, prefixes have been adopted to express multiples or submultiples of a unit, as per the following list:

MULTIPLES.

Prefix.	Amount of Multiplication.		
	Expressed in Words.	Expressed in Figures.	
deka	ten times	10	10
hecto	one hundred times	100	10^2
kilo	one thousand times	1,000	10^3
mega	one million times	1,000,000	10^6
bega	one billion times	1,000,000,000	10^9
trega	one trillion times	1,000,000,000,000	10^{12}
quega	one quadrillion times	1,000,000,000,000,000	10^{15}

SUBMULTIPLES.

Prefix.	Amount of Division.		
	Expressed in Words.	Expressed in Figures.	
deci	one-tenth	1 ÷ 10	10^{-1}
centi	one-hundredth	1 ÷ 100	10^{-2}
milli	one-thousandth	1 ÷ 1,000	10^{-3}
micro	one-millionth	1 ÷ 1,000,000	10^{-6}
bicro	one-billionth	1 ÷ 1,000,000,000	10^{-9}
tricro	one-trillionth	1 ÷ 1,000,000,000,000	10^{-12}

Using these prefixes, the specific resistance of flint glass would be said to be 16,700 quegohms (16,700 × 1,000,000,000,000,000 = 16,700,000,000,000,000,000 ohms) and that of annealed silver 1.500 microhms, since

$$\frac{1.500}{1,000,000} = .000001500 \text{ ohm.}$$

In Art. **2301** the resistance of various metals for 1 inch of length and 1 square inch in area has been given. These values may be reduced to *specific resistance* by performing the necessary calculations. The specific resistances of some of the substances commonly termed insulators are given in Table 85.

TABLE 85.

Substance.	Specific Resistance.
Mica	84 tregohms
Gutta-percha	449 tregohms
Hard rubber	28 quegohms
Paraffin	34 quegohms
Porcelain	540 quegohms
Flint glass	16,700 quegohms
Olive oil	1 tregohm
Lard oil	350 begohms

Table 86 gives the specific resistance of some of the more common solutions used as electrolytes.

TABLE 86.

Specific resistance of various electrolytes in ohms at 50° F

Liquid.	Specific Gravity.	Specific Resistance.
Copper sulphate Saturated solution	1.205	29.30
Zinc sulphate Saturated solution	1.440	33.60
Zinc sulphate	Solution giving least resistance.	28.22
Common salt		4.70
Sal ammoniac		2.50
Sulphate of soda		11.30
Sulphuric acid		1.38
Nitric acid		1.29
Hydrochloric acid		1.32

TEMPERATURE COEFFICIENT.

2516. The resistance of any conducting body changes with changes in the temperature. In the case of electrolytes, non-metallic substances (insulators and carbon), an increase in temperature decreases the resistance, while in the metals and their alloys an increase of temperature increases the resistance. The percentage of change of resistance with unit change of temperature is known as the **temperature coefficient.**

Thus, a piece of copper wire which is known to have a resistance of 10 ohms at a temperature of 32° F. is found to have a resistance of 11.11 ohms at 82° F. These changes in resistance, due to variations of temperature, become quite important in practical work, and allowance must generally be made, in all calculations involving conductors subject to changes of temperature, for the increase or decrease of their resistance.

2517. The formulas representing the effects of the *increase* of temperature upon the conductivity of a substance may be written as follows:

Assume r_1 = original resistance;
r_2 = resistance after rise of temperature;
a = temperature coefficient for each degree Centigrade;
b = temperature coefficient for each degree Fahrenheit;
$C°$ = degrees Centigrade rise of temperature (See Table 88.)
$F°$ = degrees Fahrenheit rise of temperature.

Then, $$r_2 = r_1\,(1 + a\,C°), \qquad \textbf{(460.)}$$

and $$r_2 = r_1\,(1 + b\,F°). \qquad \textbf{(461.)}$$

The values of a and b may be found in Table 87.

2518. The formulas for the decrease of resistance with *decrease* of temperature may likewise be stated.

Let r_1 = original resistance;
r_2 = resistance after lowering of temperature;

a = temperature coefficient for each degree Centigrade;
b = temperature coefficient for each degree Fahrenheit;
$C°$ = degrees Centigrade fall of temperature;
$F°$ = degrees Fahrenheit fall of temperature.

Then,

$$r_2 = \frac{r_1}{1 + a\,C°}. \qquad \textbf{(462.)}$$

$$r_2 = \frac{r_1}{1 + b\,F°}. \qquad \textbf{(463.)}$$

TABLE 87.

TEMPERATURE COEFFICIENTS FOR VARIOUS METALS.

Name of Metal.	For Centigrade. (*a*)	For Fahrenheit. (*b*)
Silver	.00377	.002094
Copper	.00388	.002156
Gold	.00365	.002028
Aluminum	.00390	.002167
Platinum	.00247	.001372
Iron	.00453	.002517
Tin	.00365	.002028
Lead	.00387	.002150
Antimony	.00389	.002161
Bismuth	.00354	.001967
Mercury	.00088	.000489
German silver	.00044	.000244

EXAMPLE.—A copper conductor has a resistance of 15 ohms at a temperature of 20° C. What will be its resistance (*a*) at 50° C.? (*b*) at 8° C.?

SOLUTION.—(*a*) The original resistance = 15 ohms = r_1. The change in temperature is $50° - 20° = 30°$ C. = C°. Then, since this is an increase, formula **460** will apply, for which, from Table 87, $a = .00388$ for copper, and we have $r_2 = 15\,[1 + (.00388 \times 30)] = 16.746$ ohms. Ans.

(*b*) In this case the change of temperature is a decrease, and $C° = 20 - 8 = 12$. Then, by formula **462,** the changed resistance $r_2 = \dfrac{15}{1 + (.00388 \times 12)} = 14.33$ ohms. Ans.

For metals having a different temperature coefficient from that of copper, the foregoing formulas should be changed by inserting the proper constant, taken from Table 87, in place of .00388 or .002156.

2519. The following table of Centigrade and Fahrenheit degrees is given to facilitate the rapid conversion from one scale to another.

TABLE 88.

TABLE OF CENTIGRADE AND FAHRENHEIT DEGREES.

Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.	Deg. C.	Deg. F.
0	32.0	26	78.8	51	123.8	76	168.8
1	33.8	27	80.6	52	125.6	77	170.6
2	35.6	28	82.4	53	127.4	78	172.4
3	37.4	29	84.2	54	129.2	79	174.2
4	39.2	30	86.0	55	131.0	80	176.0
5	41.0	31	87.8	56	132.8	81	177.8
6	42.8	32	89.6	57	134.6	82	179.6
7	44.6	33	91.4	58	136.4	83	181.4
8	46.4	34	93.2	59	138.2	84	183.2
9	48.2	35	95.0	60	140.0	85	185 0
10	50.0	36	96.8	61	141.8	86	186.8
11	51.8	37	98.6	62	143.6	87	188.6
12	53.6	38	100.4	63	145.4	88	190.4
13	55.4	39	102.2	64	147.2	89	192.2
14	57.2	40	104.0	65	149.0	90	194.0
15	59.0	41	105.8	66	150.8	91	195.8
16	60.8	42	107.6	67	152.6	92	197.6
17	62.6	43	109.4	68	154.4	93	199.4
18	64.4	44	111.2	69	156.2	94	201.2
19	66.2	45	113.0	70	158.0	95	203.0
20	68.0	46	114.8	71	159.8	96	204.8
21	69.8	47	116.6	72	161.6	97	206.6
22	71.6	48	118.4	73	163.4	98	208 4
23	73.4	49	120.2	74	165.2	99	210.2
24	75.2	50	122.0	75	167.0	100	212.0
25	77.0						

Relations of Thermometric Scales:

I.—*To convert Fahrenheit to Centigrade, subtract 32, multiply by 5, and divide by 9.*

For example, 50° Fahrenheit $= \frac{(50 - 32)\,5}{9} = 10°$ Centigrade.

II.—*To convert Centigrade to Fahrenheit, multiply by 9, divide by 5, and add 32.*

For example, 100° Centigrade $= \frac{9 \times 100}{5} + 32 = 212°$ Fahrenheit.

INSULATION.

2520. In order to transmit electricity from one point to another, that is, to make the electric current follow a definite path in a conductor, it is necessary that the conductor should be separated from all points between which and the conductor there is a difference of potential by substances whose resistance is so high that that difference of potential can establish no appreciable current.

If two conductors supplying current to a lamp, for example, were laid directly on the ground, the current would flow directly from one conductor to the other through the earth, the earth being a good conductor. If the wires be surrounded by glass tubes, the resistance offered to the passage of the current from wire to wire through the glass and the earth would be so great that the current would be infinitesimal, and the full strength of the current could be utilized in the lamp. Or, if the wires were suspended in the air upon glass knobs attached to poles, again the resistance between conductors, or from the conductors to the earth, would be comparatively enormous. This resistance is known as the **insulation resistance** of the circuit, and, it is obvious, should be as great as possible.

2521. In almost all electrical appliances, insulating materials are as necessary as conducting materials, and the measurement of the *insulation resistance* of such apparatus is often very important.

In telegraph and telephone line construction, bare iron or copper wires are used, and are supported on glass knobs. From the high specific resistance of glass, it would be reasonable to suppose that this would insulate the wires very thoroughly from the earth, which would be the case were it not for the fact that the surface of the glass insulators is always covered with a film of dust and moisture, which is of much less resistance than the glass. Glass insulators are, therefore, made so as to give a considerable length of surface between the point of attaching the wire and the point of support of the glass. Fig. 1007 shows such an insulator, which is supported by a wooden pin with a thread cut on the end, which screws into the thread moulded in the glass *B*. The wire being fastened in the groove *C*, any leakage of current from the wire must pass over the surface of the glass from *C* to the supporting pin. The length of this surface is materially increased by the groove *A*. This form of insulator is known as a *petticoat insulator*.

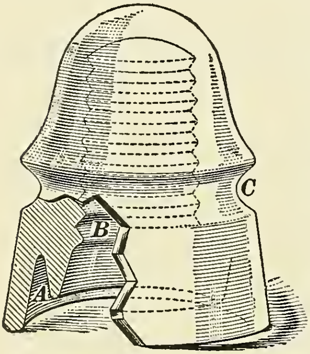


FIG. 1007.

The insulation resistance of one of these insulators would, of course, be very high, even if considerable moisture were present, but as in a long line strung on these insulators the insulation resistances are all in multiple, the total insulation resistance of the line may be low.

2522. Fig. 1008 shows an easy method of testing the approximate resistance of a line *L*, in which *G* is a galvanometer, *B* a battery, and *R* a known resistance, which should be high, say 10,000 ohms. *K* is a key or switch, which, when contact is made with terminal *b*, connects the resistance *R* through the galvanometer to the battery *B*; when contact is made to the terminal *a*, the battery is connected to the earth by the earth plate *E*, which may be a metal plate buried in moist earth, or the wire may be attached to a water or gas pipe, which, being buried in the earth, makes an excellent earth connection.

By connecting the battery to the resistance R by means of the switch K, the needle of the galvanometer will be deflected a certain amount, which should be noted. Then, on connecting the battery to the earth plate E, the circuit will be completed through the insulation resistance between the line L and the earth. The current flowing will again produce a deflection of the galvanometer needle. The currents which flow through the known resistance R and the insulation resistance of the line L will be inversely proportional to those resistances; so, knowing the galvanometer constant, the currents and, from their ratio, the insulation resistances of the line may be calculated.

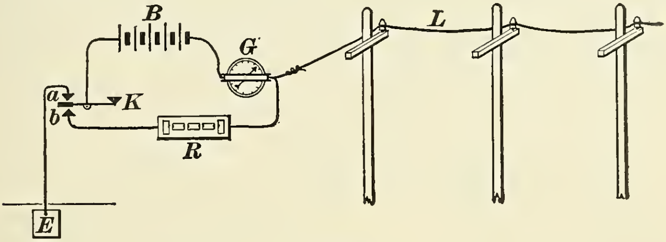


FIG. 1008.

If a tangent galvanometer be used, the resistances will be *inversely proportional to the tangents of the angles of deflection;* that is,

$$R : \tan d_1 = I : \tan d,$$

where $R =$ known resistance;
$I =$ insulation resistance;
$d =$ the angle of deflection when R is in circuit;
$d_1 =$ the angle of deflection when I is in circuit.

From the above proportion,

$$I = \frac{R \tan d}{\tan d_1}. \qquad \textbf{(464.)}$$

That is, *the insulation resistance of a line is equal to a* *given resistance multiplied by the quotient obtained by dividing the tangent of the angle of galvanometer deflection when that resistance is in circuit by the tangent of the angle of deflection when the circuit is through the line.*

Example.—The known resistance = 10,000 ohms; the deflection of the galvanometer when R was in circuit was 60°; the deflection of the galvanometer when I was in circuit was 33°; what was the insulation resistance of the line in ohms?

Solution.—$\tan d^\circ = 1.732.$
$\tan d^\circ_1 = .649.$

$R = 10,000$; therefore, by formula **464**,

$$I = \frac{10,000 \times 1.732}{.649} = 26,700 \text{ ohms, nearly.} \quad \text{Ans.}$$

2523. As the number of paths for the current through the insulation increases with the length of the line, the insulation resistance of the line decreases as the length of the line increases; so the total insulation resistance *multiplied* by the length of the line gives the insulation resistance per unit of length. The usual unit of length for overhead telegraph and telephone lines is one mile.

Example.—What is the insulation resistance per mile in the above example if the line be 7.5 miles long?

Solution.— $26,700 \times 7.5 = 200,250$ ohms, or .2 megohm, practically. Ans.

This is about the insulation resistance required for ordinary telegraph and telephone work.

The above method of testing requires a sensitive galvanometer of fairly low resistance, and gives approximately precise results for resistances not exceeding about 30,000 ohms.

If the resistance much exceeds this limit, a shunt may be used with the galvanometer of any convenient multiplying power; for example, 100. The deflection through the known resistance being noted, the shunt should be removed for the line resistance measurement. Thus, a current through the insulation resistance of $\frac{1}{n+1}$ of the current through the known resistance (in the above case $\frac{1}{100}$) will give the same deflection, and the line insulation resistance will be a corresponding multiple of the known resistance.

To obtain more accurate results, allowance must be made in each case for the resistance of the galvanometer and battery. Usually, these are not a sufficient per cent. of the total resistance of either circuit to affect the result much.

2524. Another good method of measuring insulation resistance is to make this resistance one arm of a Wheatstone bridge, as represented in Fig. 1009. By making the resistance of M great in proportion to N, resistances as high as 2,000,000 ohms may be measured with a bridge as ordinarily arranged. (See Fig. 1004.)

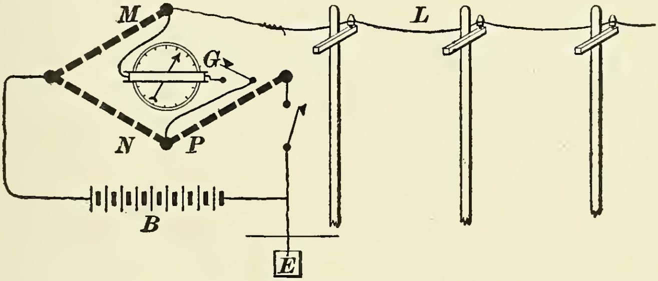


FIG. 1009.

2525. By **grounding** the distant end of the line L, the resistance of the conductor which makes up the line may be measured by the same methods. *Grounding* a circuit consists in connecting it electrically with the earth, usually by means of a metal plate buried in *moist* earth, or to the pipes of a water or gas system. Grounding is conventionally represented as at E, Figs. 1008 and 1009. The resistance of the earth is so slight that for small currents it may be usually neglected, if the grounding is well done.

2526. The insulation resistance of apparatus for electric light and power work must be considerably greater than that for telegraph and telephone use, and the wire used is, except in special cases, covered with insulation instead of being bare.

This insulation must not only have a high specific resistance, but it must be able to meet various other requirements. In wire for overhead construction, for example, the insulation must stand the abrasion of tree branches, etc., be reasonably fireproof, water-proof, able to withstand the action of the weather, and flexible enough to allow the wire to be reeled or strung in place without injury to the insulation.

It is obvious that many substances of high specific resistance, such as glass or porcelain, would not fill some of the above conditions. In fact, there is scarcely any *one* substance that would answer. The best grades of insulated wire are usually made with a layer of rubber, or some compound composed largely of rubber, surrounding the wire, and protected by an additional covering of braided cotton or similar device, soaked in some reasonably fireproof and weather-proof compound.

In order to thoroughly test the insulation resistance, contact should be made with the whole outer surface of the insulation. This is best done by immersing the wire in a tank of water, slightly salted to make it conducting, as shown in Fig. 1010. The tank is of metal, and the insulation resistance is measured between the water surrounding the wire and the wire itself, as shown at *a b*. Connection with the water is made by a binding-post attached to the metal tank, or if the tank be glass or china, a metal plate is used, dipping in the water.

FIG. 1010.

2527. A long piece of wire prepared for test in this way would have a large area of insulating material between two conducting bodies, i. e., the wire and the water. On connecting the wire and water to the poles of a battery, a charge of electricity will spread itself over the inner and outer surfaces of the insulation, which will cause a momentary rush of current from the battery. Another phenomenon which also appears is that known to telegraph engineers as

electrification. The exact nature of this phenomenon is not known, but it has been held by eminent authority to be a *polarization* of the insulation, and its effect is to cause a continuation of the first rush of current due to the static charge, that gradually grows smaller and smaller, until after the lapse of some few minutes the current becomes steady. On disconnecting the battery and replacing it with a piece of wire, a back current will flow through the wire from surface to surface of the insulation until it is *depolarized.*

In testing the insulation resistance of long pieces of wire in water, these effects may interfere materially with readings, especially if the Wheatstone bridge method be used. With the following methods of testing, however, it is usually sufficient to wait, after closing the current, until the current has become steady before taking readings.

In testing long cables for submarine use, or long pieces of wire, it is sometimes customary to take the resistance readings after the electrification has continued for a certain definite time, usually 1 minute. This is always so stated in the results of the test, thus: "Insulation resistance per mile after 1 minute's electrification, 400 megohms."

Where the surface area of the insulation is small, the electrification is hardly perceptible, and ordinarily will have no effect on the readings, even if a bridge be used.

2528. The following method is like that described in Art. **2522,** Fig. 1008, except that as the insulation resistance of a short length of well-insulated wire would be quite high, it would probably be necessary to use a shunt with the galvanometer.

Fig. 1011 is a diagram of connections for such a measurement. The shunt S is connected in parallel with the galvanometer G by inserting a metal plug between the metal blocks at a. The switch K connects either the metal tank T or the known resistance R with the battery B.

The best method of procedure is to first read the deflection through the insulation resistance of the wire W with the shunt disconnected. Then connect in the shunt and read the deflection through the known resistance R, changing the shunt S until the deflection is approximately the

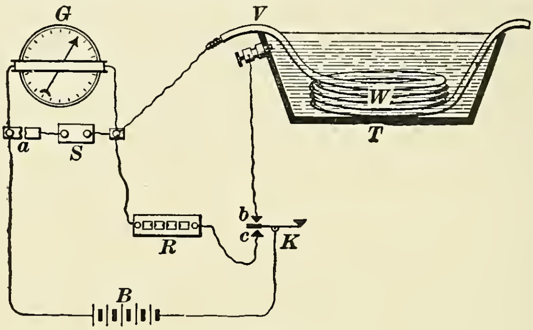


FIG. 1011.

same as before. Then, knowing the multiplying power of the shunt used, the insulation resistance of the wire may be calculated as in Art. **2522.**

EXAMPLE.—In a tangent galvanometer the resistance is 600 ohms; deflection through insulation $= 38°$; with a shunt of 8 ohms, deflection through resistance $= 42°$; resistance $R = 10{,}000$ ohms. What is the insulation resistance?

SOLUTION.—Multiplying power of shunt $= \frac{600}{8} + 1 = 76$.

The deflection through the resistance $= d° = 42$, and the deflection through the insulation $= d°_1 = 38$. Then, $\tan d° = .9004$; $\tan d°_1 = .7813$.

To get the true value of the current in resistance R, $\tan d°$ must be multiplied by the multiplying power of the shunt.

$.9004 \times 76 = 68.43 =$ tangent of the angle to which needle would be deflected if shunt were not used. (68.43 is the tangent of about 89° 10′, at which point a tangent galvanometer could not possibly be read with a precision of under 50%.)

By formula **464,**

$$I = \frac{10{,}000 \times 68.43}{.7813} = \frac{684{,}300}{.7813} = 876{,}000 \text{ ohms, nearly. Ans.}$$

2529. It is evident that the theory of the above method of testing insulation resistance is that, if the voltage of the battery be constant, the current sent through the two resistances will be inversely proportional to those resistances.

The current from the battery is very small in either case, although when passing through the resistance R it is about

70 times as much as when the insulation resistance is in circuit. This difference would scarcely affect an ordinary battery, and as the actual insulation resistance of a specimen of wire is seldom wanted with an accuracy greater than 95–90% (that is, within 5 or 10%), this method gives good results.

2530. Another method of measuring insulation resistance that does not require the use of a shunt with the galvanometer is shown in Fig. 1012. To make the test, a battery of 100 cells is required. The cells should be uniform in kind, size, and condition.

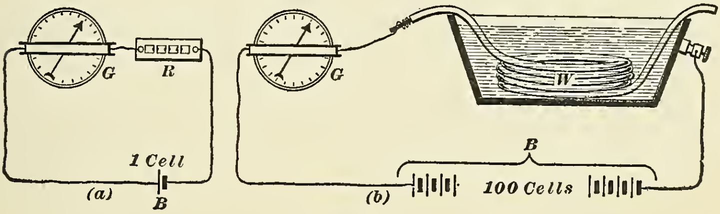


FIG. 1012.

The galvanometer should be connected in series with the insulation resistance and the cells, as shown in Fig. 1012 (*b*), and the deflection noted.

The galvanometer should then be connected in series with one cell of the battery and a known adjustable resistance, which should be high (say 10,000 ohms, as before), and the deflection under these conditions [Fig. 1012 (*a*)] noted. The resistance should be adjusted so as to give about the same deflection as the first reading.

It is evident that the 100-cell battery would give 100 times the current through the resistance and galvanometer that the single cell does, and assuming that the angle of deflection of the galvanometer is directly proportional to the current flowing, 100 times the deflection, if that were possible. So, calling d the deflection through the resistance, with 1 cell, then $100\,d$ would be the deflection with 100 cells; but the deflection through the insulation resistance with 100 cells is known; call it d_1.

Then, as before, $I : R :: 100\,d : d_1$, or if $x =$ the number of cells and $R =$ the extra resistance,

$$I = \frac{R\,x\,d}{d_1}. \qquad \textbf{(465.)}$$

That is, *the insulation resistance of a cable is equal to a given resistance multiplied by the quotient obtained by dividing the deflection through the given resistance with one cell by the deflection through the cable with a number of cells, this product being multiplied by that number.*

Example.—If the deflection with 100 cells through the insulation resistance was 37, and through 9,000 ohms with 1 cell was 42, what was the insulation resistance?

Solution.—In this example the given resistance $= 9{,}000$ ohms $= R$; the deflection through this resistance with 1 cell $= 42° = d$; the number of cells used for the insulation test $= 100 = x$; and the deflection through the insulation with these cells $= 37° = d_1$. Then, by formula **465,**

$$I = \frac{9{,}000 \times 100 \times 42}{37} = 1{,}020{,}000 \text{ ohms, nearly, or about 1 megohm. Ans.}$$

This method is based upon the supposition that the available E. M. F. of the battery will be directly proportional to the number of cells, which, with a little care in the selection of cells, will be the case, and that the current taken from the cells will not be great enough with either measurement to affect the E. M. F., which will again be the case, and the precision of this form of test will be amply good for most insulation measurements.

2531. As many insulations deteriorate after having been under water some time, readings should be taken of the insulation resistance at intervals during a considerable time to observe this deterioration, if there be any. For example, readings taken after the wire had been immersed 15 minutes, 1 hour, 3 hours, 10 hours, 24 hours, would show any effect that wetting would have that would be serious.

If a break occur in the insulation under the water, the water will come in contact with the metal wire, and the ensuing electrolysis will liberate bubbles of gas, which will alternately collect and pass off at the break. This will so vary the resistance that the current will not be steady enough to allow its value to be read. The galvanometer needle will irregularly swing back and forth, and it will be useless to attempt to measure the insulation resistance, especially as the action would indicate defective insulation.

2532. It is not altogether necessary to immerse the wire in water, although this is very convenient, as the water makes contact with the entire outer surface of the insulation, as the wire does with the entire inner surface, besides testing the water-proof qualities of the insulation.

For some tests the wire may be closely wrapped around a smooth bright metal bar—a section of shafting for example—and the resistance between this bar and the wire measured. Or two pieces of the wire may be twisted together, and the resistance between the two wires measured.

It is often desirable to test the insulating qualities of sheets of paper, mica fiber, or similar substances. A convenient way to prepare them for such a test is to make two smooth brass plates smaller than the pieces of insulation to be tested, which should be placed between them. The insulation resistance may then be measured between the two brass plates, and from the area and length (thickness) of the piece of insulation between the plates its specific resistance may be calculated. Another way is to wrap the insulation around a smooth metal bar as before—a piece of shafting for example—and bind the outside closely with fine bare copper wire. The insulation resistance may then be measured between the wire and the bar. Many other methods of preparing insulation for tests will suggest themselves as occasion requires.

ELECTRICAL APPARATUS.

2533. The following is a description of the free electrical apparatus with which the student is furnished in connection with this subject. Directions are given for performing certain experiments calculated to help the student to a better understanding of the subject of Electrical Measurements. Unless the student has had previous instruction of a like nature, he is earnestly requested (though not required) to make all the experiments mentioned, and to keep a record of his results by answering the questions under the heading Experiments with Electrical Apparatus, which follows the description of the instruments. He may forward his record to the School, if he so desires, for correction and approval when he sends his answers to the questions on Electrical Measurements, but he will be marked and his work computed in connection with his work on the questions above referred to.

DESCRIPTION OF APPARATUS.

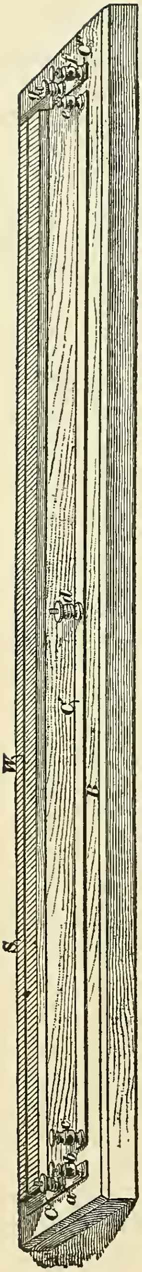


FIG. 1013.

2534. The Slide-Wire Bridge.—This is a Wheatstone bridge of the slide-wire form (Art. **2513**), and is illustrated in Fig. 1013. The slide wire *W*, made of German silver wire, is stretched across a scale *S*, of the same length as the wire, divided into 1,000 equal divisions, every 5th division being marked, and every 50th division numbered, in both directions, starting from each end. Three strips of copper *C*, *C'*, *C''* are fastened to the board, as shown, and to these are screwed the terminals *a* and *b*; *c*, *d*, and *e*; *f* and *g*, respectively.

From an inspection of the figure, it will be seen that the strip *C* corresponds to the piece *d* in Fig. 1005; *C'* corresponds to the middle section *e*, and *C''* to the piece *c* in the same figure.

The **resistance coils** furnished for use with this bridge are of the form illustrated in Fig. 1014. The resistance coil is made up of a coil of German silver wire *R*, insulated with silk, wound on a wooden bobbin, or spool, *S*. Through the head of the spool two pieces of heavy copper wire *t*, *t* are driven, the ends of the resistance coil being soldered to these wires; they form the terminals of the coil. The two wires (*t*, *t*) are the same distance apart as the holes in the terminals *b*, *c* or *e*, *f*, Fig. 1013, so that a coil may readily be connected in its proper position by slipping the two terminal wires through those holes, and clamping them in position by means of the milled-head screws.

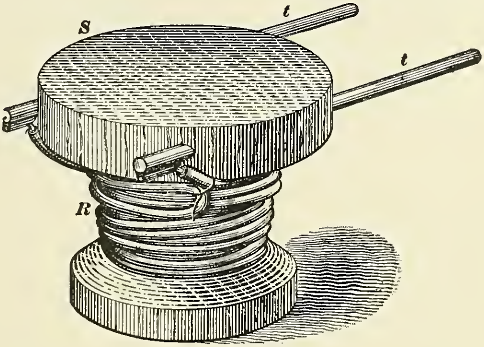


FIG. 1014.

With each bridge are furnished two resistance coils, one of 1 ohm, the other of about 10.5 ohms, resistance. These may be readily distinguished, as the 1-ohm coil is covered with a strip of leather, while the wire of the other is exposed. Also, the 10.5-ohm coil is made of much finer wire than that used for the 1-ohm coil.

2535. The **galvanometer** is a combination tangent and detector galvanometer, available for use in either capacity. The detector galvanometer consists of a coil of insulated wire wound on a wooden form, and divided in the center to allow for the support of a magnetic needle which swings in the center of the coil. A perspective section of the coil and its support is shown in Fig. 1015. Here *B* represents the wooden form on which the coil is wound in two parts, *C C* and *C′ C′*. An

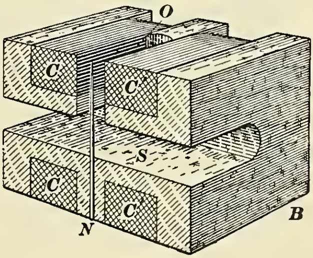


FIG. 1015.

opening is made in the form, passing completely through the coils, thus forming the space *S* in which the magnetic needle is free to swing. Another opening *O* is made in the top of the partition between the two parts of the coil, extending through to the space *S*. This provides a means for placing the magnetic needle in position. In the center of the wooden form is placed an ordinary steel needle *N*, extending nearly to the top of the coils in the center of the opening *O*; this needle forms the pivot on which the magnetic needle swings.

The magnetic needle and the pointer that is attached to it are illustrated in Fig. 1016. The construction is as follows: The magnetic needle *M* is made from a piece of flat steel about $\frac{1}{2}$ inch long and $\frac{3}{16}$ inch wide, pointed at the ends. Before being hardened, a hole is punched in the center of this piece, and when the piece has been hardened and magnetized, a short glass tube *G*, closed at the top, is fastened in the hole. This forms the bearing which rests on the pivot in the center of the coils; the smooth glass surface resting on the steel point makes a bearing that is remarkably free from friction, so that the galvanometer is very sensitive. The pointer which indicates the deflection of the needle is a fine glass rod *P*, blackened at the ends, and attached in the middle to a light tin tube *T*, which slides over the glass bearing *G*, and is held in place by its friction with the bearing.

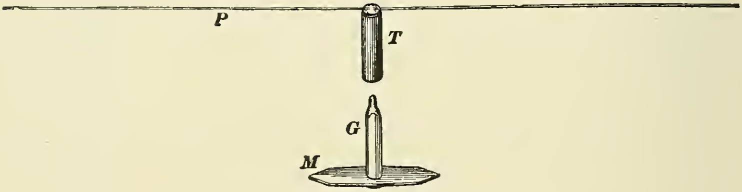


FIG. 1016.

The coils and pivot are mounted in the center of a circular wooden case that is turned from a solid block; a paper scale is mounted over the coils, and is divided on one side into degrees (numbered to 90 by tens each way from a zero mark), and on the other side the scale is marked to represent tangents. The ends of the pointer on the needle move over the scale divisions and indicate the angle of deflection of the needle. The whole is protected by a circular glass cover, held in place by a ring of brass wire sprung into a groove under the top edge of the wooden case, as will be seen by examining the instrument. Connection to the coil is made by means of two binding-posts on the case, shown at *a*, *b* in Fig. 1017.

When the instrument is used as a tangent galvanometer, the case containing the magnetized needle and coil is placed within the circle *c* on a cross support having a dowel which fits into a hole in the center of the case. The galvanometer may, therefore, be turned through any angle, while still keeping the magnetized needle in the center of the large circle. On this circle are wound four coils of two turns each of No. 18 insulated wire, and the ends of these coils are brought out to terminals on the base. Thus, the terminals *d*, *d'* form the ends of one coil, *e*, *e'* are the terminals for the second coil, and so on. The galvanometer should be so placed within the coil that the scales are one on each side of the circle. When the whole instrument is then turned so that the glass pointer is at right angles to the coil and over the zero marks, the axis of the magnetized needle will be in the plane of the coil.

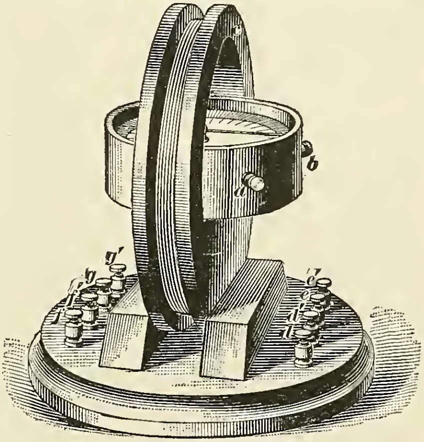


FIG. 1017.

When the instruments are received, they should be carefully unpacked and prepared for use.

In setting up the galvanometer, the glass cover should first be removed by springing out the brass wire ring and introducing a thin knife-blade or a toothpick under the edge of the glass. The cotton which is placed on the glass pointer to protect it while in transit must then be carefully removed, after which the glass should be replaced and the ring sprung in. The apparatus is then ready for use.

2536. A diagrammatic representation of the connections of the bridge for ordinary resistance measurements is shown in Fig. 1018. From this it will be seen that the battery *B* (which is the Leclanche cell already furnished the student) is connected to the copper contact strips to which the ends of the slide wire are also fastened. The known resistance *R* and the unknown resistance *X* are connected between the outer and middle strips.

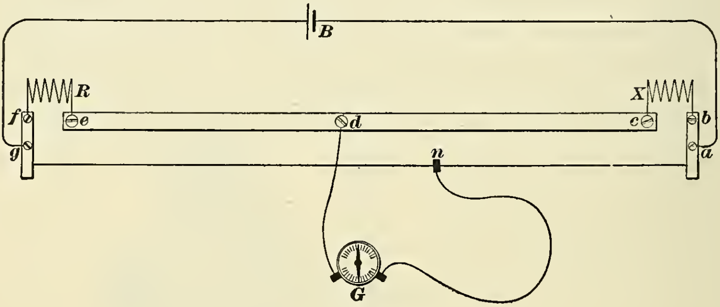


FIG. 1018.

One terminal of the galvanometer is connected to the middle strip at *d* by a wire clamped under the screw, the other terminal being connected to the proper point on the slide wire by a piece of insulated wire having a contact piece at the end. This end is pressed on the slide wire and moved along until a point is reached where alternately removing it from and touching it to the slide wire produces no deflection of the galvanometer. With care in observing that the galvanometer needle is actually *not* deflected even a fraction of a scale division, this point may be very accurately located. By reading the number of the division on the scale at this point, the relation between the lengths of the slide wires $g\,n$ and $n\,a$ may be readily found, and the value of the unknown resistance X found by applying the formula given in Art. **2513**, but substituting the point a for b and g for a, as these letters in Fig. 1018 correspond to the ends of the wire in Fig. 1004, $X = R\frac{n\,a}{n\,g}$.

For example, suppose the 1-ohm coil to be used as R, and that the position of the point n, where an application of the galvanometer lead to the slide wire produces no deflection of the galvanometer, to be opposite division 415 on the scale starting at g; this will be the third division from the line marked 400, in the space between 400 and 450. The corresponding number read on the scale which starts from a will be 585. The position of this point n will hereafter be called the *bridge reading;* in this case the bridge reading is 415 or 585. We may now find the value of the unknown resistance X by the formula above,

$$X = 1 \times \frac{585}{415} = 1.41 \text{ ohms.}$$

The accuracy of this bridge does not require that the results be carried out to more than three significant figures.

If the relative position of X and R, Fig. 1018, be reversed, the formula will obviously read $X = R\frac{n\,g}{n\,a}$. For uniformity in all problems given in this section, the known resistance will be considered to be in the position shown in Fig. 1018.

In using the detector galvanometer, see that it rests on a level surface, so that the needle and pointer will swing clear. This may be tested by causing the needle to deflect (e.g., by momentarily connecting the galvanometer to the battery terminals), first in one direction and then in the other; the needle should, after two or three oscillations, return to the same position from either side.

When using the tangent galvanometer, the instrument may be leveled by means of the three leveling screws in the base; in fact, the galvanometer may, for most purposes, be kept in position on this base.

See that no magnets or large pieces of iron are near the galvanometer, or, if this is unavoidable, see that the relative position of the galvanometer and the magnets or pieces of iron is not changed while making a test.

Set the galvanometer up in such a position that the ends of the pointer coincide with the zero divisions of the scale.

If either the pointer or pivot is not exactly central, it may be that each end will not coincide with the zero marks at the same time. If this is the case, get one end into the proper position and take all readings from that end of the pointer.

Do not rub the glass cover with a piece of cloth or paper before taking a reading, as the static charge thus induced on the glass will attract the glass pointer, thereby causing it to indicate incorrectly.

In making all experiments, note on a piece of paper or in a note-book the apparatus used, and how; if necessary, draw a diagram of connections, etc. Write down each result as soon as each part of the experiment is completed; *do not trust to memory for results.* Make all experiments twice, if possible, thus checking the first results. By taking the above precautions and exercising care in taking the readings, reliable and instructive results may be obtained with this simple apparatus by performing the experiments mentioned in the succeeding pages.

EXPERIMENTS WITH ELECTRICAL APPARATUS.

Experiment 1.—As stated above, one of the resistance coils has 1 ohm resistance, the other about 10.5 ohms. It is desirable that this second coil have 10 ohms resistance, to facilitate calculation, etc. The process of winding these coils is as follows: The given length of wire is cut off and folded in the middle; the ends are then soldered to the copper terminals and the doubled wire wound on the spool, the middle of the length of the wire thus coming on the outside of the spool, where it is held by a piece of thread. By cutting this thread the middle of the wire is released, and a length may be unwound, the insulation scraped off, and the two bare wires firmly twisted together, which will have the effect of lessening the resistance of the coil. For this experiment, then,

(*a*) Find the resistance of the fine wire resistance coil, using the other coil as the standard, and calling its resistance 1 ohm.

(*b*) Figure out the bridge reading which would result if the fine wire resistance coil were of 10 ohms resistance.

(*c*) Lessen the resistance of the fine wire coil, a little at a time, in the manner described, until it is of 10 ohms resistance. State the various steps of the operation fully.

(*d*) From the results of experiment (*c*), what is the total length of wire in the fine wire coil as furnished?

Experiment 2.—Measure off about 40 feet of the insulated wire furnished with the first set of apparatus. Connect this in the place of the unknown resistance X, Fig. 1018. Use the 1-ohm coil for the known resistance and measure the resistance of the wire.

(*a*) What is the resistance per foot of the wire?

(*b*) Assuming it to be pure annealed copper, what is the diameter of the wire, calculated from the above measurement?

Experiment 3.—Connect up the bridge, battery, and galvanometer as shown in Fig. 1019, using the 10-ohm coil as the known resistance R. It will be seen that in this case the galvanometer takes the place of the unknown resistance, and a plain wire of the galvanometer circuit, in the ordinary method of connection. (Fig. 1018.)

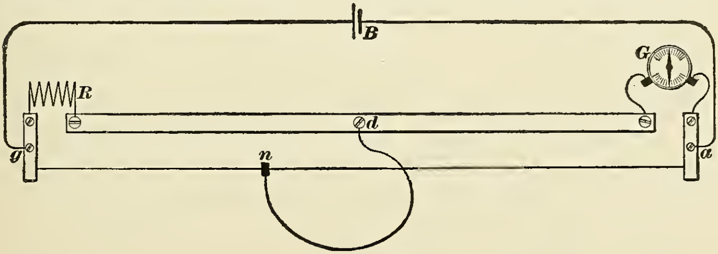


FIG. 1019.

When the connections are made, a current will flow through the galvanometer, giving a certain deflection. If the wire that takes the place of the galvanometer circuit in the ordinary method is pressed down on the slide wire at *any other point* than where no current passes through it, the amount of current in the circuit will be changed and the deflection of the galvanometer will correspondingly change. Consequently, if a point on the slide wire is found where connecting the wire produces *no change* in the galvanometer deflection, that point will give a bridge reading from which the galvanometer resistance may be calculated by the formula already given.

(*a*) Perform the foregoing experiment and calculate the resistance of the galvanometer.

(*b*) Explain the theory of this measurement.

Experiment 4.—Scrape the insulation off one end of each of two short pieces of wire for about 6 inches. Wrap the bared ends firmly, one around one of the terminals of the 10-ohm coil and the other around the other terminal, using about 3 inches of the bared ends. This will leave 3 inches or so projecting from each terminal (besides the insulated part), which should be connected to the terminals of the galvanometer, thus connecting the 10-ohm coil and the galvanometer in parallel, or, in other words, shunting the galvanometer with the 10-ohm coil.

(*a*) Calculate the combined resistance of the galvanometer and its shunt.

(*b*) Measure this combined resistance on the bridge by the method described in Experiment 3, using the 1-ohm coil for the known resistance. How do the two results compare?

(*c*) What is the multiplying power of this shunt?

Experiment 5.—Check up the results of the measurements given in Experiments 1 and 3 by reversing the positions of the known resistance and the unknown resistance, using the modified form of the formula as given in Art. **2536.** How do the results compare?

Experiment 6.—In measuring the resistance of apparatus located at some distance from the bridge, it is necessary to take into account the resistance of the wires leading to the resistance that is to be measured. This can best be done by measuring the resistance of these leads separately, their circuit being completed by twisting or pressing the distant ends of the leads together, either before or after the regular resistance measurement is made, and deducting this resistance from the total resistance of the circuit measured.

Perform this experiment, measuring the resistance of the 1-ohm coil located 20 or 30 feet away from the bridge. Use the 10-ohm coil as the standard, and some of the wire supplied with the first lot of apparatus for the leads. Give the total resistance of the circuit, the resistance of the leads, and the resistance of the coil.

Experiment 7.—Secure a piece of the carbon from an arc lamp, 3 or 4 inches long. If coated with copper, scrape the copper off or eat it off with acid.

(*a*) Twist the bared ends of two short pieces of wire around the carbon at points as near the ends as possible, and measure the resistance of the carbon between these two points.

(*b*) Measure carefully the diameter of the carbon and the length between the points where the lead wires are attached, and from these figures calculate the resistance of the carbon for an inch cube; i. e., what would be the resistance of a piece of the carbon experimented upon if of 1 square inch sectional area and 1 inch long?

SUGGESTIONS FOR EXPERIMENTS.

2537. If the student has access to an electric-light station he can measure the resistance of the various circuits of the dynamos or other appliances, or of the lines, at times when the station is not running, or can measure such other resistances, of between .05 and 200 ohms, as the opportunity may allow.

Other standard resistances may be prepared on the same lines as those furnished with a bridge; if made of higher or lower resistance than those furnished, they will increase the range of resistances that may be measured, although accurate measurement of high resistances (500 ohms or over) should not be expected.

The best method of calibrating the tangent galvanometer is to compare it with a standard direct reading instrument—a Weston ammeter, for example. If this can not readily be done, it may be calibrated by the copper sulphate electrolysis method described in Art. **2496** and following. The weight measurements should be carefully made on a delicate pair of scales. Any good apothecary has such a pair of scales, and would probably perform the weighings for a small sum.

In this method of calibration, copper wires may be substituted for the copper plates described in Art. **2498,** they being easier to clean and handle. Two wires should be used, coiled into open spirals, one about twice the diameter of the other; the smaller should be placed inside the larger, and connected to the negative pole of the battery, thereby becoming the electrode to be weighed.

Many other useful applications of this apparatus which might be mentioned will occur to the student as he advances in the Course.

PRACTICAL MEASUREMENTS.

INSTRUMENTS.

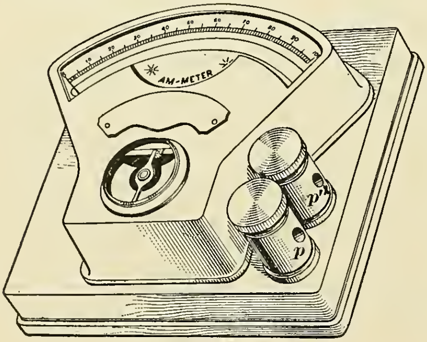


FIG. 1020.

2538. Most of the apparatus and tests so far described are more for laboratory use than for the shop or station, and now such accurate and reliable portable measuring instruments are made that many measurements before referred to may be made with as great a degree of precision and much greater facility than with the various galvanometers and other apparatus already described. Some of the best forms of portable instruments made are those known as the Weston instruments. Their general form is shown in Fig. 1020. These instruments are made on the principle of the D'Arsonval galvanometer (Art. **2476**), as shown in the sectional view, Fig. 1021.

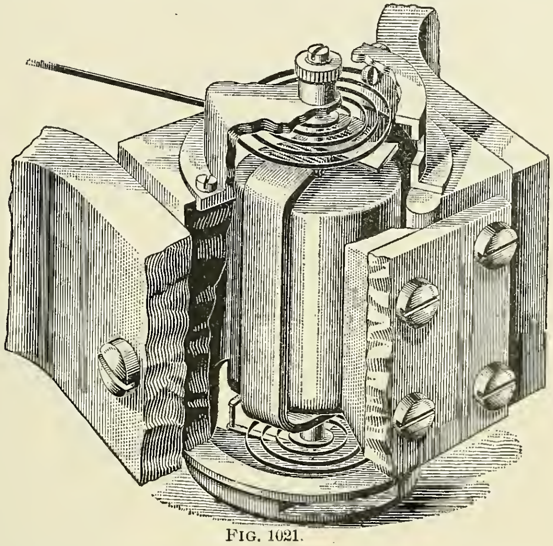


FIG. 1021.

Fig. 1022 shows the magnetic circuit of this form of instrument. The permanent magnet *A A* has soft iron pole-pieces *P*, *P* fastened to it by the screws *S*, *S*, and bored

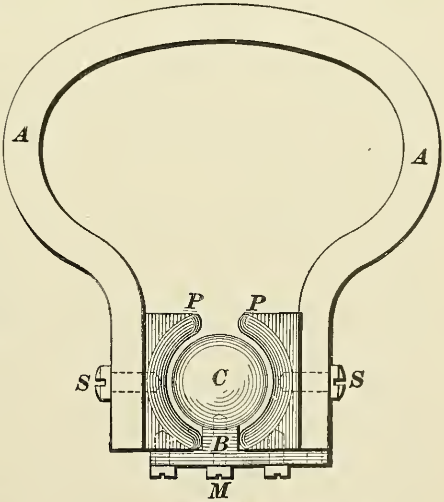


FIG. 1022

out to make a cylindrical opening. In the center of this opening is a stationary soft iron cylinder *C*, supported in place by a screw *M* passing through a lug on the brass plate *B*. This cylinder being of less diameter than the opening through the pole-pieces, there is left a narrow gap between the pole-pieces and the iron core, as shown. The lines of force from the permanent magnet pass across this space, making a strong and uniform magnetic field.

The movable part of the instrument is shown in Fig. 1023. It consists of a rectangular coil *C* of fine wire wound on an aluminum or thin copper bobbin, which is suspended vertically between two delicate jeweled bearings. Two flat horizontal spiral springs *S*, *S* oppose the tendency of the coil to rotate, and at the same time conduct the current to the suspended coil.

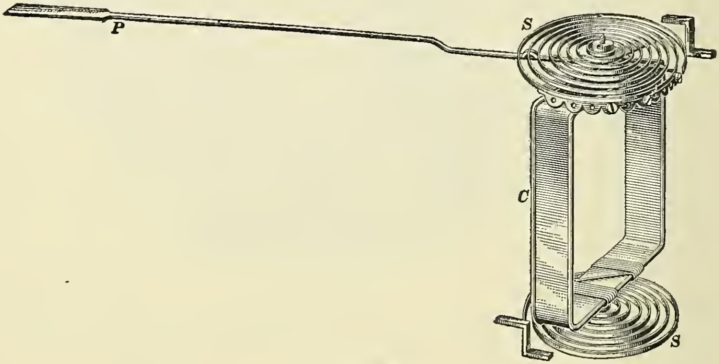


FIG. 1023.

A thin aluminum pointer *P*, attached at right angles to the coil, moves over a scale and indicates the deflection of the coil from its normal position, which is as shown in Fig. 1021. On a current being sent through the coil by means of the springs *S*, *S*, there is a tendency for the coil to move through the magnetic field (Art. **2438**), which it will do until the torsion of the spiral springs equals the force with which the coil tends to move, when the coil will come to rest, and the pointer will indicate the angle of deflection of the coil.

The magnetic field being practically uniform, the angle of deflection is closely proportional to the current in the coil, so the scale divisions are very uniform, as is shown by Fig. 1024, which is a scale about three-fourths size.

The copper or aluminum bobbin on which the coil is wound, in moving through the magnetic field, has an electromotive force set up in it which causes a current to circulate around the bobbin as long as the bobbin moves. This current circulates in the opposite direction to the current in the coil; hence, it tends to oppose the motion of the coil. As this tendency exists only when the bobbin is moving, it has the effect of preventing the needle from swinging too far over the scale, thus bringing it quickly to rest at the proper point.

FIG. 1024.

This *damping effect* is due almost entirely to the currents in the bobbin. The friction is so slight that it has practically no effect on the position the needle will take. This is shown by the fact that the needle having been deflected by a current will respond to very minute variations in that current; that is, the instruments are very *sensitive*.

An instrument whose moving system possesses this feature of coming to rest quickly at the proper point is known as a **dead-beat** instrument; this is a very important feature, for it assists the rapidity of taking measurements very materially. The moving system is practically the same for all direct-current Weston ammeters and voltmeters. If the instrument is designed for a voltmeter, a high resistance, located in the back of the case, is connected in series with the movable coil; if for an ammeter, the coil is connected in parallel with a short, thick piece of copper or some alloy, so that only a small part of the current passes through the movable coil, and the resistance of the instrument is extremely low. By reason of this extremely low resistance of the ammeters and the high resistance of the voltmeters, they consume very little energy indeed, and may be left continuously in circuit without undue heating.

For example, a 15-ampere Weston ammeter has an internal resistance of .0022 ohm; when measuring a 10-ampere current, the drop (C R) is .022 volt, and the watts expended (C E) = .22, or about one thirty-four-hundredth $\left(\frac{1}{3,400}\right)$ of a horsepower.

The resistance of a 150-volt voltmeter is about 19,000 ohms. Measuring 110 volts, the instrument would take $\frac{110}{19,000} = .0058$ ampere, nearly, with a consumption of energy of .638 watt, nearly, or about $\frac{1}{1,200}$ of a horsepower.

The conducting parts of the instrument are made of an alloy having a very low temperature coefficient, so that moderate changes in the temperature of the instrument do not affect its readings appreciably. Beneath the needle just inside the scale is a mirror. On looking down on the needle, by getting the needle directly over its reflection in the mirror, errors due to not getting the needle in line with the scale (known as **parallax**) are eliminated. These several good features make these instruments very reliable and convenient for making all sorts of electrical measurements, and as they may be obtained in a great variety of ranges, their use is very general.

There are many other forms of portable instruments made, most of them being constructed on the same general principle as the Weston, and they may often be used in making various measurements to good advantage.

SIEMENS DYNAMOMETER.

2539. Another instrument which is largely used for measuring currents is the Siemens dynamometer. This instrument is constructed on the same fundamental principle that the Weston and many other electromagnetic instruments are, namely, that a conductor carrying a current will tend to move, if in a magnetic field, with the direction of the lines of force at an angle to the direction of the current.

The working parts of this instrument, one form of which is shown in Fig. 1025, consists of two rectangular coils of wire, one, *F*, fixed, the other, *M*, movable. The normal position of the movable coil is with its plane at right angles to the plane of the fixed coil, and it is suspended in this position by a fiber. To the top of the coil is also attached a light helical spring *S*, the other end of which is attached to a milled nut *T*. On turning this nut the spring will be tightened, thus acting to move the coil.

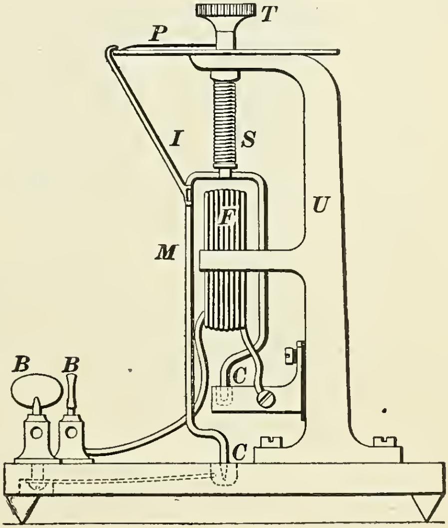


FIG. 1025.

The two coils are connected in series, connection being made to the movable coil by means of mercury cups *C*, *C*, into which the ends of the coil dip.

On sending a current through the two coils in series, the mutual action of the two coils tends to move them into parallel planes. The effect is to rotate the movable coil about its vertical axis; by turning the milled nut, a tension may be put on the spring which will return the coil to its original position. The force exerted by the spring on the coil is proportional to the angle through which the milled head attached to the spring is turned; so, by a pointer *P*, attached to the milled head, the force required to pull the coil back to its central position may be indicated.

A pointer *I*, attached to the movable coil, is opposite a zero mark on the scale when the movable coil is at right angles to the fixed coil.

As the two coils are in series, doubling the current in one coil doubles it in the other, so the mutual force of both coils is doubled, and the force acting on the movable coil is quadrupled; that is, the force on the movable coil, hence the torsion in the spring necessary to bring the pointer on the coil back to zero, is *proportional to the square of the current.*

These instruments are seldom made direct reading, but are furnished with a table which gives the deflections corresponding to various current strengths. Intermediate values may be interpolated or calculated.

The fixed coil is usually wound in two parts of unequal number of turns and size of wire; either coil may be used, thus varying the range of the instrument. This form of instrument is especially useful, as it may be used equally well for alternating current as for direct, since there is no iron or other magnetic material used in its construction.

EDISON CHEMICAL METER.

2540. As stated, most of the measuring instruments in commercial use depend on the electromagnetic effect of a current for their action; perhaps the only electrochemical current meter that is in commercial use is the Edison chemical meter, which is extensively used by the Edison Illuminating Companies. In this instrument a fixed proportion of the current passing through the meter is shunted through an electrolytic bath consisting of two zinc plates dipping in a solution of sulphate of zinc. The plates, solution, and connectors are all mounted in little glass jars, and two jars are set up in each meter, one to act as a check on the other. At the end of a certain fixed time (usually thirty days) the jars and their contents are replaced by others, and the ampere-hours of current that have been used by the customer calculated from the gain in weight of the negative plate. By various ingenious devices in the several parts of the meter, the effects of various sources of error are almost entirely removed. Great care, however, must be exercised in removing the jars and caring for their contents. Fig. 1026 gives a view of the latest type of Edison chemical meter.

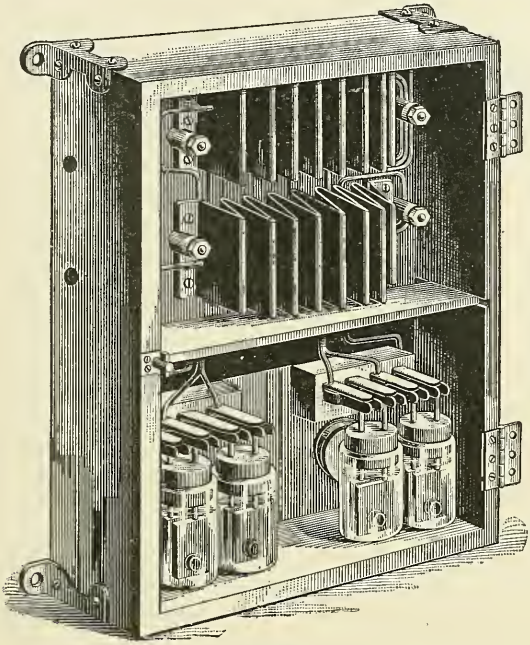


FIG. 1026.

CARDEW VOLTMETER.

2541. The representative instrument of the class that measures the heating effect of the current is the **Cardew voltmeter,** illustrated in Fig. 1027.

In this instrument a long wire *w*, usually of some platinum alloy, is stretched from end to end of the long tube *a*; each end is fastened to the dial end of the tube; the wire then passes over pulleys at the end of the tube and back to the dial end, where a spring attached to the middle of the wire keeps it stretched taut. On a current being sent through the wire, the heat caused by the passage of the current expands the wire; the addition to its length is taken up by the spring, and the motion of the middle of the wire which is attached to the spring is transmitted to a needle *b* by suitable multiplying gear, so that the motion of the needle over the dial is a measure of the amount of expansion of the wire.

The wire is usually of small diameter and considerable specific resistance, so that it in itself has resistance enough to allow the instrument to be used as a voltmeter for potentials less than about 100 volts without external resistance. This voltmeter may be used either for alternating or direct currents, is remarkably dead-beat, and simple in construction. Its internal resistance is low for a voltmeter, and, in consequence, it takes considerable current, enough in many instances to seriously affect some conditions of an experiment. This instrument has no particular law of deflections by which the scale is divided; the principal divisions are determined by comparing the instrument with a standard, and the intermediate divisions interpolated.

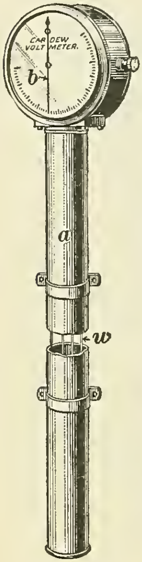


Fig. 1027.

There are several other instruments made on this principle, commonly known as *hot-wire instruments;* the Cardew is the best known.

WATTMETERS.

2542. The energy expended in a circuit being the product of the current and the electromotive force, these factors may be measured separately, and multiplied together to obtain the number of watts expended. Instruments have been designed, however, which automatically perform this multiplication, thus measuring watts directly, one of the best known being the **Siemens** wattmeter.

This instrument is of the same general form as the current-meter, Fig. 1025, the difference between the instruments being that in the wattmeter the two coils are not in series. This instrument measures at all times the product of the current in any circuit and the difference of potential between the ends of that circuit. The stationary coil is in series with the outside circuit; consequently the total current flows through it. The movable coil is in series with a resistance great enough to prevent the full difference of potential between the mains sending more than a small amount of current through the movable coil. This coil and the resistance are then connected in parallel with the rest of the circuit, as shown in Fig. 1028, where *F* is the fixed coil of the instrument; *M*, the movable coil; *R*, the resistance that is connected in series with *M*; *D*, the source of electricity, and *C*, the external circuit, the energy expended in which it is desired to measure.

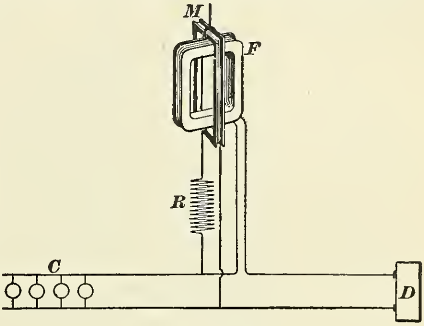


FIG. 1028.

It is evident that if the drop in volts through the circuit *C* be constant, the current through *M* will also be constant. The force acting on the movable coil will then vary directly as the current in the coil *F*; the potential being constant, the watts expended in the circuit will also vary directly as the current. If the current in the coil *F* is constant, variations in the E. M. F. will vary the current in the coil *M* in the same proportion, and the force on the coil *M* will then vary directly as the E. M. F.; the current in the circuit *C* being constant, the watts will also vary directly as the E. M. F. So, in either case, the force acting on the movable coil (consequently the force required to bring it to zero position) varies directly as the watts. When variations occur in both current and potential simultaneously, the same holds true, and the force required to bring the coil to the zero position is proportional to the watts.

The general appearance of the wattmeter is almost precisely the same as that of the current dynamometer. The resistance used with the movable coil is usually made a separate piece of apparatus; when made so that it has no *self-induction*, the wattmeter may be used for measuring the energy expended in alternating-current circuits.

THE THOMSON RECORDING WATTMETER.

2543. The Siemens wattmeter gives the instantaneous value of the watts expended in the circuit. The Thomson wattmeter is, as its name indicates, a recording meter, and its readings give the product of the watts and time, i. e., the watt-hours. The construction is simple; the principle is, broadly, that of the Siemens dynamometer. The movable coil is not held to zero position, but revolves, and does not surround the fixed coil, but revolves between two fixed coils. The movable coil is really a small *drum-wound armature*, provided with a small commutator made of silver to prevent oxidation. The effect of using the commutator is to make the effective plane of the coil (armature) take a position at right angles to the plane of the fixed coils.

The connections of this instrument are made on the same principle as those of the Siemens wattmeter. The fixed coils are in series with the circuit, and the movable coil and a resistance in series with it are in parallel with the circuit.

The amount of energy expended in the circuit is measured by the rotation of the movable coil, a worm on the shaft on which the movable coil is mounted engaging with a set of gears which operate a dial similar to a gas-meter dial, so that the energy expended in a certain given time in the circuit may be read directly from the dial in watt-hours.

The friction of the apparatus being exceedingly small, the retarding force on the coil that opposes its tendency to rotate is imparted by a thin copper disk attached to the shaft on which the movable coil is mounted. This disk is rotated between the poles of strong permanent magnets; the lines of force from the magnets cutting the disk set up electromotive forces between adjacent points on the disk;

the disk being of copper, the resistance between those points is very low, so that a considerable current may flow. This current tends to retard the rotation of the copper disk, and this tendency increases directly as the speed. As in the Siemens wattmeter, the force acting to rotate the movable coil increases directly as the watts; therefore, the number of revolutions of the moving system of the meter will be directly proportional to the watts expended in the circuit. This meter may be used for either alternating or direct currents, and gives very accurate results. Fig. 1029 represents the Thomson meter with the cover removed.

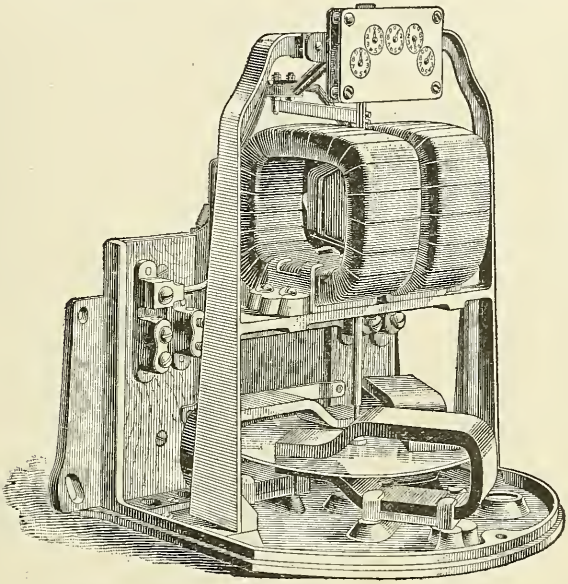


FIG. 1029.

SHALLENBERGER METER.

2544. This meter is constructed on a similar principle to the Thomson meter, but is designed to be used only on alternating-current circuits, as is also the **Duncan** meter. Other recording meters are in use, but are usually complicated in construction and rather unreliable in operation.

In addition to the Siemens wattmeter, there have recently been introduced general forms of portable direct-reading wattmeters, which are giving good satisfaction, and are more convenient to use than the Siemens form.

SWITCHBOARD INSTRUMENTS.

2545. In lighting and power stations and similar places, it is often desirable to know the approximate number of amperes, volts, or watts delivered by a machine or machines, and for this purpose instruments have come into use which, while not sufficiently accurate for making reliable measurements to a great degree of precision, are very useful in indicating approximately the output of a machine or the load on a circuit. The scales of such instruments are usually large and open, so they may be read from a distance. Many forms of such instruments are made by different manufacturers; their principle of operation is usually the electromagnetic effect of the current, but their details of construction will not be described here.

MEASUREMENTS WITH COMMERCIAL INSTRUMENTS.

2546. Most of the measurements previously described as being made with some form of galvanometer can be made with good commercial instruments—the Weston, for example.

In the Weston instruments, the terminals of the ammeter are both on the same (right) side of the instrument (see Fig. 1020), and are made large and heavy, while in the voltmeters the terminals are on opposite sides, are made small, and are usually covered with rubber, in order that they may be handled without danger from shocks. Some of the voltmeters are made with the resistance coils in two sections, of such relative value that, when only one section is in circuit, the scale readings are some convenient submultiple of the readings when both sections are used. In this case the instrument is provided with an extra binding-post on the left side, and the scale divisions have two values; for instance, the voltmeter with a range of 150 volts may have the resistance so divided that by using the third binding-post the range will be 15 volts and the scale divisions will have $\frac{1}{10}$ their former value.

Measurements of current strength or difference of potential are very simple. To measure the number of amperes flowing in a circuit, it is only necessary to connect an ammeter of proper capacity in series with the rest of the circuit, as shown in Fig. 1030, and read the amperes directly from the position of the pointer on the scale. The resistance of the ammeter, as pointed out, is so low that it will not affect the total resistance of the circuit appreciably. The difference of potential between two points in a circuit, or the E. M. F. of a battery, or other source of E. M. F., may be readily measured by connecting the terminals of a voltmeter to the proper points of the circuit and reading the voltage direct, as shown in Fig. 1031.

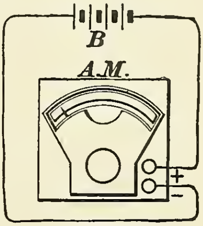


FIG. 1030.

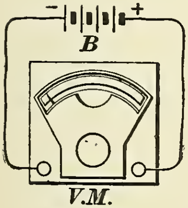


FIG. 1031.

By using instruments of the proper range, very low or very high resistances may be measured. Fig. 1032 shows a way of measuring a very low resistance—in this case a section of copper rod. Here a current from the battery *B*, measured by the ammeter *A*, flows through the section of copper rod *R*, and the drop between the points *C* and *D* is

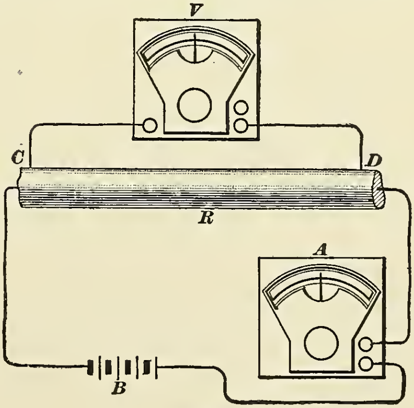


FIG. 1032.

measured by the voltmeter V. As the drop through a short section of copper rod would be very slight, except with an enormous current, a voltmeter capable of measuring very small differences of potential must be used. They may be had to measure from 0 to .05 volt, such an instrument being known as a *millivoltmeter*.

EXAMPLE.—If, in the above figure, the reading of the ammeter be 34.5 amperes, and that of the millivoltmeter be .00875 volt, what is the resistance of the copper rod between C and B?

SOLUTION.— $R = \frac{E}{C} = \frac{.00875}{35.4} = .000247$ ohm. Ans.

High resistances may be measured in a similar manner by using a low-reading ammeter (mil-ammeter) and a high-reading voltmeter. The high-reading voltmeters may also be used to measure very high resistances, such as insulation resistances; the method of connecting up for such a test is shown in Fig. 1033. Here R is the insulation to be measured, $B\,C$ a battery or other source of E. M. F., which should be as high as convenient, as long as it is within the range of the instrument, $V\,M$ the voltmeter, and K a switch for shunting the resistance R. As the resistance of the switch K is practically nothing, it is evident that when it is closed the voltmeter is connected directly to the terminals of the battery and will measure its E. M. F., and when the switch K is open the resistance R is in series with the voltmeter. The formula for finding the value of R in ohms is

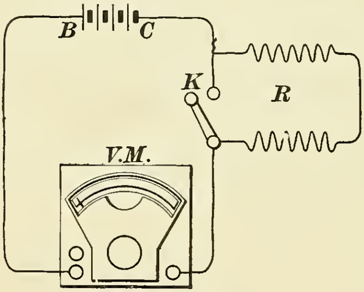


FIG. 1033.

$$R = r\left(\frac{d}{d_1} - 1\right), \qquad \textbf{(466.)}$$

where $d =$ deflection of voltmeter with the resistance R *not* in circuit, $d_1 =$ deflection of voltmeter with resistance R in circuit, and $r =$ resistance of voltmeter.

This formula is obtained as follows: The E. M. F. of the battery $B\,C$ being constant, the drop through the voltmeter only or the voltmeter and resistance in series will be the same, that is, $C\,r = C_1 R + C_1 r$.

As the deflection of the voltmeter needle is proportional to the current, this may be written

$$d\,r = d_1 R + d_1 r;$$

or,

$$d\,r - d_1 r = d_1 R;$$

or,

$$\frac{d\,r}{d_1} - \frac{d_1 r}{d_1} = R;$$

or,

$$\frac{d\,r}{d_1} - r = R;$$

hence,

$$r\left(\frac{d}{d_1} - 1\right) = R,$$

which is the formula given.

In the simple case where the resistance to be measured is just equal to the voltmeter resistance, it is evident that the deflection of the voltmeter with the resistance in series with it would be half that when the voltmeter alone is in circuit, which satisfies the equation as follows:

Given, $r \times (\frac{2}{1} - 1) = R.$

Then, $r \times 1 = R,$

and $r = R.$

Example.—If the E. M. F. of the battery, as measured by the voltmeter, is 100 volts, and the deflection, when the resistance to be measured is in circuit, is 40 volts, what is the value of that resistance in ohms if the resistance of the voltmeter is 18,000 ohms?

Solution.—In this case $d = 100$, $d_1 = 40$, $r = 18{,}000$. Then, by formula **466**,

$R = 18{,}000\,(\frac{100}{40} - 1) = 18{,}000 \times 1.5 = 27{,}000$ ohms. Ans.

2547. It will be seen that this method of testing insulation is on exactly the same principle as that shown in Fig. 1008 (Art. **2522**), the known resistance in this case being that of the instrument itself. Slight variations in the E. M. F. of the source of supply do not affect the result very materially, and when an approximately constant known E. M. F. is available, such as an electric-lighting circuit, insulation tests may be made with great facility by merely connecting the voltmeter in series with the E. M. F. and the insulation resistance. On the assumption that the E. M. F. has a constant (known) value, a table may be prepared showing the insulation resistance corresponding to various deflections of the voltmeter.

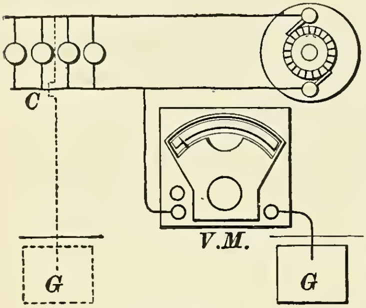


FIG. 1034.

This affords a ready means of testing the insulation resistance of lighting circuits where the E. M. F. of the dynamos is constant, if the voltmeter is of high resistance; by connecting the voltmeter between one side of the circuit and the ground, the deflection of the needle will give the insulation resistance of the *other side* of the line, or will show the presence of a "ground," as represented in Fig. 1034. Both sides may be tested in this manner, and it is usual to provide a small switch or other convenient apparatus for readily making the desired connections.

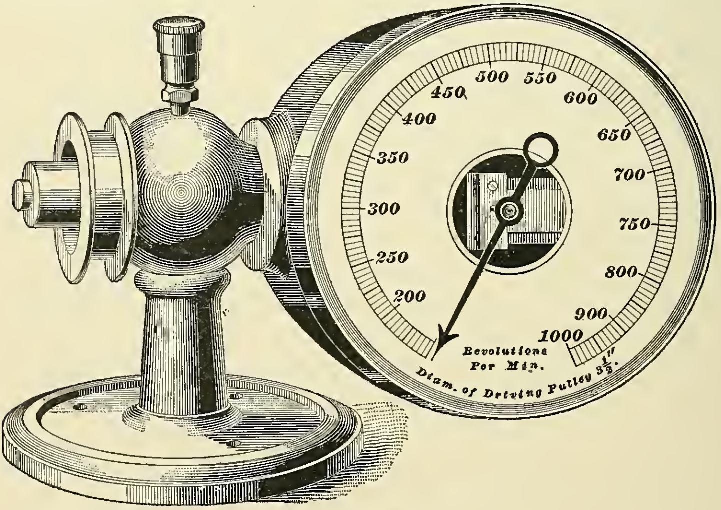


FIG. 1035.

2548. For many electrical measurements, it is necessary to know the *rate of revolution* of certain moving parts

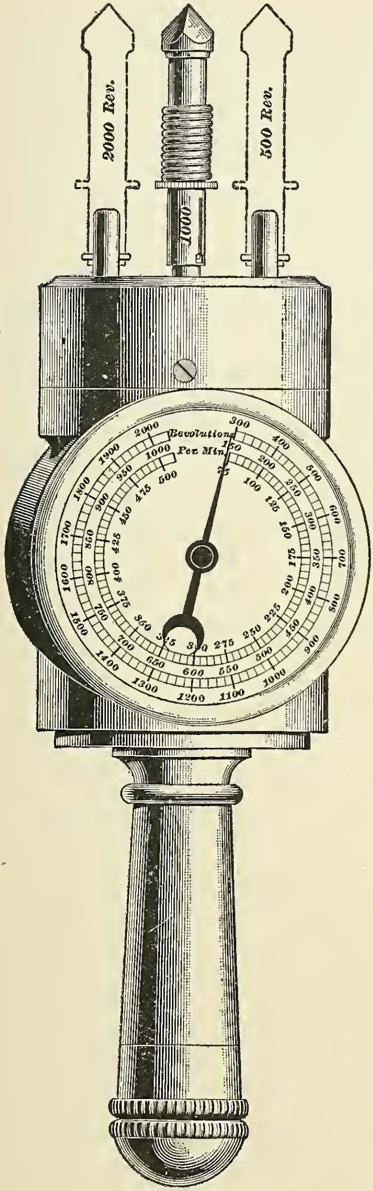


FIG. 1036.

of machinery. The number of revolutions made by the machinery in one minute or other length of time does not necessarily give its rate of revolution, so that for accurate work the ordinary *revolution counter* is scarcely suitable. Instruments called **tachometers** are made which indicate by the position of a needle on a dial the *rate of revolution* of the apparatus to which they are connected. The principle of these instruments is similar to that of a centrifugal engine governor; two weights are thrown out from their center of rotation by centrifugal force, and their tendency to move is overcome by a spring. By suitable gearing the motion of the weights is made to actuate a pointer which moves over a suitably divided dial, thus indicating the rate of rotation of the weights.

Fig. 1035 shows a form of tachometer which, being belted to a pulley of suitable diameter by a light belt, will give the rate of revolution of that pulley.

The form shown in Fig. 1036 is intended to hold in the hands. A three-sided point on one of the spindles of the instrument is intended to be thrust into the center mark of a revolving shaft, when the rate of revolution of that shaft is indicated on the dial. It is usual to make three little ridges in the sloping sides of the center mark of the shaft with a three-sided punch (supplied with the instrument), to insure that the point on the tachometer shaft will not slip when the instrument is applied.

2549. Electrical measurements may be broadly stated to be *measurement of current.* The principal methods of measuring current and their general applications have been described, but for particular cases these methods often require much modification in detail, and to arrive at certain results many combinations of such methods may be made, according to the requirements of the case in hand. Some of these combinations and modifications for special cases will be described where necessary in succeeding sections.

BATTERIES.

DEFINITIONS.

2550. An **electric battery** is a combination of a number of separate electric sources. Thus, a voltaic or galvanic battery (see Arts. **2238** and **2239**) is a combination of a number of separate voltaic cells properly joined together.

The term battery is also applied to a combination of Leyden jars (see Art. **2232**) properly joined together so as to form a so-called **Leyden battery.** A battery of this kind, however, has very little practical value, in the present state of the art, compared to the value of the two great classes of batteries treated of in this discussion, namely:

I. **Primary batteries.**

II. **Secondary** or **storage batteries,** sometimes called **accumulators.**

2551. A **primary battery** is a combination of a number of **primary cells** so as to form a single source.

2552. A **secondary** or **storage battery** is a combination of a number of **secondary cells** so as to form a single electric source.

2553. Primary batteries, as well as secondary batteries, depend for their operation upon the chemical action which takes place between certain different substances when brought into contact with each other. The whole theory and operation of cells and batteries being thus dependent on chemical action, it is necessary to give here some principles of **chemistry,** which is that science which treats of the composition of matter, of the changes produced therein by the action of heat and other natural forces, and of the action and reaction of different kinds of matter upon each other.

PRINCIPLES OF CHEMISTRY.

2554. Chemical action is that which produces a change in the chemical condition of matter, and may be action of **decomposition,** i. e., splitting a substance up into other distinct substances; or action of **recomposition,** i. e., uniting two or more different substances into a new one.

2555. Decomposition can not go on indefinitely; if a substance be split up by decomposition, and the resulting substances (if possible) be again split up, and so on, a point will be reached where substances are found which by no known process can be further decomposed. Such substances are called **elements.**

2556. A **chemical compound** is the union of two or more elements to form a new substance. Compounds may be formed by the union of two or more compounds; but this is merely a new union of the elements which originally formed the uniting compounds.

2557. There have so far been discovered about seventy-two substances which seem to be elements. About half of these are very rare; the balance, the more important, are given in Table 89. To prevent constant repetitions of their names, and to aid in expressing the composition of substances, there has been assigned to each element a *symbol*, consisting of the initial letter, or the initial letter and another letter of its Latin name, which is often different from its common name. These symbols are given in column 2, Table 89.

2558. The exact nature of chemical action is not known, any more than is the exact nature of electricity or heat; but it is similar to other physical phenomena in that chemical action is a manifestation of energy. This energy is apparently stored up in the atoms of the elements as *potential energy*, and causes such atoms to have an *affinity for*, or tendency to combine with, other atoms, this affinity being greater or less according to the relative amount of potential energy stored in the combining atoms. Under the proper conditions, these affinities cause the atoms to combine, which allows their potential energy to appear as kinetic energy, usually in the form of heat. Thus, chemical combination develops kinetic energy, while to perform chemical decomposition, kinetic energy must be supplied.

2559. The heat given out by the formation of a compound is known as the **heat of formation** of that substance. The amount of this heat has been measured in the case of some substances, and tables giving these values are published in most works on chemistry.

NOTE.—The heat of formation is usually expressed in calories (per gram of the substance), the calorie used being the *lesser*, or *gram-degree*, calorie which is the heat required to raise 1 gram of water 1° C. It is obviously .001 of the calorie defined in Art. **1130.**

2560. An element is the ultimate *substance* to which any compound can be *chemically* subdivided. As has been explained in Art. **1089,** all matter (every substance) is (mechanically) composed of *molecules*, they being the smallest *particles* into which the substance can be *mechanically* subdivided without being resolved into its elements. The molecules are each made up of a number of *atoms* of the elements of which the substance is composed, and in any given substance each molecule is always composed of the *same total number of atoms* of its elements combined in the same proportions; if the proportionate number of atoms of any element is changed, a new substance is formed. When an element exists uncombined, its atoms do not exist alone, but group together into molecules, just as the atoms of the different elements group together to form the molecules of the compound.

2561. By very careful analysis and measurement, it has been determined that *elements always combine in certain fixed proportions or multiples of those proportions;* from this fact, it is possible to assign to each element a number, which number, or a multiple of it, will represent the proportion, by weight, of that element which enters into any compound. To this number is assigned the name **atomic weight,** and these numbers represent the *relative weights* *of the atoms* of the elements. The *actual* weight of an atom has been calculated, but is unimportant here.

Hydrogen being the lightest of the elements, the weight of its atom is taken as the unit, and the atomic weights of all other elements calculated therefrom. The atomic weights of the more common elements will be found in column 3 of Table 89.

TABLE 89.

THE PRINCIPAL ELEMENTS.

1. Name of Element.	2. Symbol.	3. Atomic Weight.	4. Valency.	5. Chemical Equivalent.	6. Electro-chemical Equivalent. Grams per Coulomb.
Aluminum	*Al*	27.00	III	9.00	.00009324
Antimony	*Sb*	120.00	V	24.00	.00024860
Arsenic	*As*	75.00	V	15.00	.00015540
Barium	*Ba*	137.00	II	68.50	.00070960
Bismuth	*Bi*	208.90	V	41.78	.00043280
Boron	*B*	11.00	III	3.66	.00003792
Bromine	*Br*	79.95	I	79.95	.00082100
			VII	11.42	.00011840
Cadmium	*Cd*	112.00	II	56.00	.00058020
Calcium	*Ca*	40.00	II	20.00	.00020720
Carbon	*C*	12.00	IV	3.00	.00003098
Chlorine	*Cl*	35.45	I	35.45	.00036730
			VII	5.07	.00005252
Chromium	*Cr*	52.10	II	26.05	.00026990
			VI	7.44	.00007708
Cobalt	*Co*	59.00	II	29.50	.00030560
			VIII	7.38	.00007646
Copper	*Cu*	63.40	I	63.40	.00065680
			II	31.70	.00032840
Fluorine	*Fl*	19.00	I	19.00	.00019680
			VII	2.57	.00002663
Gold	*Au*	197.30	I	197.30	.00204400
			III	65.77	.00068140
Hydrogen	*H*	1.00	I	1.00	.00001036

TABLE 89—*Continued.*

1. Name of Element.	2. Symbol.	3. Atomic Weight.	4. Valency.	5. Chemical Equivalent.	6. Electro-chemical Equivalent. Grams per Coulomb.
Iodine	*I*	125.85	I	125.85	.00130300
			VII	17.98	.00018630
Iron	*Fe*	56.00	II	28.00	.00029010
			IV	14.00	.00014500
Lead	*Pb*	206.95	II	103.48	.00107200
			IV	51.74	.00053600
Magnesium	*Mg*	24.30	II	12.15	.00012590
Manganese	*Mn*	55.00	II	27.50	.00028490
			VII	7.86	.00008143
Mercury	*Hg*	200.00	I	200.00	.00207200
			II	100.00	.00103600
Nickel	*Ni*	58.00	II	29.00	.00030040
			VIII	7.25	.00007510
Nitrogen	*N*	14.03	V	2.81	.00002911
Oxygen	*O*	16.00	II	8.00	.00008288
			VI	2.67	.00002766
Phosphorus	*P*	31.00	V	6.20	.00006423
Platinum	*Pt*	195.00	II	97.50	.00101000
			IV	48.75	.00050500
Potassium	*K*	39.11	I	39.11	.00040520
Selenium	*Se*	79.00	II	39.50	.00040920
			VI	13.17	.00013640
Silicon	*Si*	28.40	IV	7.10	.00007355
Silver	*Ag*	107.90	I	107.90	.00111800
Sodium	*Na*	23.05	I	23.05	.00023880
Strontium	*Sr*	87.60	II	43.80	.00045370
Sulphur	*S*	32.06	II	16.03	.00016610
			VI	5.34	.00005532
Tellurium	*Te*	125.00	II	62.50	.00064750
Tin	*Sn*	119.00	II	59.50	.00061640
			IV	29.75	.00030820
Zinc	*Zn*	65.30	II	32.65	.00033820

The names of the non-metallic elements are printed in *italics.*

2562. In indicating the elements which make up any substance, the symbols of those elements are commonly used; also, to indicate the number of atoms (if more than one) in the molecule, a small number is suffixed to the symbol. The expression of the chemical constitution of a substance by means of the symbols, with the relative number of atoms of each element suffixed, is called the **chemical formula** of that substance.

Thus, a substance (water) whose formula is H_2O is composed of hydrogen and oxygen, and each molecule of water is composed of two atoms of hydrogen and one of oxygen. The atomic weights of H and O (from Table 89) are 1 and 16, respectively. Therefore, any weight of water is composed of $2 \times 1 = 2$ parts by weight of hydrogen and $16 \times 1 = 16$ parts by weight of oxygen. It follows, then, that the weight of 1 molecule of water will be $2 + 16 = 18$; if 1 gram of water were decomposed there would result $\frac{2}{18} = .1111+$ gram of hydrogen and $\frac{16}{18} = .8889-$ gram of oxygen. (Compare this with Art. **2493.**)

2563. An apparent exception to the above statements is the metal mercury, which seems to unite with most of the other metals in all proportions, forming **amalgams** of the metals. These amalgams, however, are generally considered to be merely mechanical mixtures, and not true chemical compounds. Some metals, such as zinc, gold, silver, lead, and others, form amalgams at ordinary temperatures, it being merely necessary to clean the surface of the metal thoroughly before placing it in contact with the mercury.

An amalgamated metal in a chemical formula is indicated by placing the symbol Hg after and above the symbol of the metal amalgamated; thus, Zn^{Hg}.

2564. As the same elements occur in many different substances, it is evident that they must be capable of replacing each other; that is, in a molecule of a given substance the atoms of one element present can be replaced by a certain number of atoms of another, thus forming a new compound. This number is not necessarily the same as the number of atoms of the element replaced. For example, *one* atom of carbon (C) can replace the *four* atoms of hydrogen (H) contained in *two* molecules of water ($2H_2O$), forming CO_2. From this it follows that *the weight of one element which will replace unit weight of another element in a compound is not always the same as the ratio of the atomic weights of the two elements.* The weight of the replacing element may be said to be the (chemical) equivalent of unit weight of the replaced element. Taking for the standard replaced element the lightest, hydrogen, and calling its unit weight 1, the least weight of any other element which will replace 1 part by weight of hydrogen in a compound, or will unite with 1 part by weight of hydrogen, is its actual **chemical equivalent,** or *combining weight.* (See column 5, Table 89.) The chemical equivalent of hydrogen is evidently the same as its atomic weight, 1. Now, in *binary* compounds (compounds consisting of only two elements) containing hydrogen, the proportion of the hydrogen in the compound is never *less* than 1 atom of H to 1 atom of the other element, though often more; consequently, from the definition, the chemical equivalent is never greater than the atomic weight, and may be less.

2565. The ratio of the atomic weight to the chemical equivalent of an element is thus equal to or greater than 1. In fact, as it is considered that there are comparatively few atoms of each element in any molecule, this ratio is always small, and always a whole number. This ratio is also *the number of atoms of hydrogen which would be required to replace one atom of the given element*, and is called the **valency,** or **atomicity,** of the element. The valency of the more common elements is given in column 4, Table 89. It can also be shown from the above statements that the number of atoms of one element required to replace a given number of atoms of another element which are in combination with a given number of atoms of a third element is *inversely proportional to the respective valencies of the replaced and replacing elements.*

For example, each molecule of water (H_2O) contains two atoms of H and one of O. If the O, whose valency is II, is replaced by Cl, whose valency is I, *each* of the two atoms of H will combine with one atom of Cl, forming, not H_2Cl_2, but two molecules of HCl (written $2HCl$). Thus, two atoms of Cl of valency I are required to replace one atom of oxygen of valency II.

2566. Elements sometimes combine in other proportions than the above statements would allow; but such substances are seldom *stable*, readily uniting with additional atoms of the proper elements until the proportions are as indicated by the valencies. However, some elements seem to have two different valencies, as will be noticed in column 4, Table 89; the lower valencies give the more stable compounds. It should be remembered that the above principles of atomic weight, valency, etc., are not the expression of any chemical theory, but the result of long and careful observation and measurement; this, however, is not infallible, and apparent violations of the foregoing statements will be encountered, though they may be generally used with consistent results.

2567. At ordinary temperatures and pressures, five of the elements, hydrogen, oxygen, nitrogen, fluorine, and chlorine, are gases; mercury and bromine are liquids; while all the rest, including all metals excepting mercury, are solids. All the solids except carbon have been liquefied at various temperatures.

Few elements, except the more common metals and oxygen and nitrogen, are extensively used or found in an uncombined state; they usually occur in various combinations, which are divided into three classes: *acids*, *bases*, and *salts*.

2568. An **acid** may be defined as a compound containing hydrogen, which hydrogen may be replaced by a metal when presented to it in the form of an oxide or hydrate. The combination of oxygen with any other single element is called an **oxide.** A **hydrate** is the substance formed by the union of an element, or, more often, a metallic oxide, with the elements of water. This should not be confounded with a **solution,** which is merely a mechanical mixture of some solid with water, or similar liquid.

Acids are usually sour to the taste, and will readily cause chemical action. Most acids contain oxygen, being formed from the union of an oxide of a non-metal and water; but in some few acids oxygen is absent. Table 90 gives a list of the more common acids, with their chemical formulas and other data.

TABLE 90.

COMMON ACIDS.

1. Name of Acid.	2. Chemical Formula.	Specific Gravity.	
		3. Pure Acid.	4. Commercial Acid. (Average.)
Hydrochloric........	HCl	1.227	1.14 to 1.16
Hydrobromic........	HBr	1.515	....
Nitric...............	HNO_3	1.530	1.33 to 1.41
Sulphuric............	H_2SO_4	1.846	1.70 to 1.83
Chromic	H_2CrO_4	.....	....

NOTE.—HCl and HBr are in reality gases, which dissolve readily in water, forming the liquid known by the above names. The specific gravity given is that of the solution (maximum).

2569. A **base** is a compound, usually an oxide or hydrate, of a metal, which metal is capable of replacing the hydrogen of an acid when the two are in contact. Some particularly active bases are known as *alkalies*, which are soluble in water. The principal alkalies are sodium hydrate, $NaOH$, potassium hydrate, KOH, and ammonium hydrate (aqua ammonia), NH_4OH.

2570. A **salt** is the substance resulting from the replacement of the hydrogen of an acid by the metal of a base. The action of the stronger acids and bases on each other is very violent. By some chemists the acids are considered to be *salts of hydrogen.*

Some combinations of non-metallic elements act in many ways similar to the metals, and can form oxides and hydrates and bases, and replace the hydrogen in acids, and tnese groups of elements should be included in the above definitions. Such a group is NH_4, which is sometimes called **ammonium.** There are several other similar groups. They act and may be handled as elements; their valency is the difference between the valency of the separate elements. Thus, in the above case, NH_4, the valency of N is V, while that of H_4 is $4 \times \mathrm{I} = \mathrm{IV}$; hence, the valency of $NH_4 = \mathrm{V} - \mathrm{IV} = \mathrm{I}$.

2571. In chemistry, substances are given names in accordance with their composition, although many of the more common substances have popular names. For example, the crystals of copper sulphate are popularly known as blue vitriol. Ordinary compounds of a metal and non-metal are named from both components. The Latin name of the metal is given first, and for its last syllable, *um*, is substituted *ic*; to this is added the name of the non-metal, with its last syllable changed to *ide.* Thus, a compound of iron and sulphur is named *ferric sulphide.*

In the case where an element has two valencies, distinction is made between the two compounds which may be formed from the same elements by substituting the termination *ous* for *ic* in the name of the metal, when referring to the compound having the *lower proportion* of the non-metallic element, which is usually oxygen. For example, copper forms two oxides: *cupric oxide*, CuO, and *cuprous oxide*, Cu_2O. The prefixes *per* and *proto* are sometimes used instead of the terminations *ic* and *ous*, respectively. The names of acids are derived from their principal constituent (aside from the hydrogen) by changing the last syllables of its name to *ic.* Thus, the principal acid formed from sulphur is called *sulphuric acid* (H_2SO_4). The acids which do not contain oxygen are distinguished by the prefix *hydro*, as *hydrobromic acid* ($HBr.$). (See Table 90.)

2572. The names of the salts resulting from the action between bases and acids are derived from the name of the acid by taking the first syllable of the principal constituent of the acid and adding *ate.* Thus, salts formed from bases and sulphuric acid are named *sulphates*, and from nitric acid, *nitrates.*

In the case where oxides are formed with non-metals of more than one valency, the acids formed from such oxides by their union with water take the *ic* and *ous* terminations, just as the metallic compounds do. For example, there are two oxides of sulphur, SO_2 and SO_3. The acid formed from the first is known as *sulphurous acid*, and from the second *sulphuric acid.* The salts of an acid ending in *ic* have the termination *ate*, as stated above, while the salts of the *ous* acids end in *ite.* It is evident, from the use of the terminations, that the *ate* salts and the *ic* oxides contain greater proportions of oxygen; *ite* salts and *ous* oxides are usually unstable, readily combining with oxygen to form the higher salts or oxides.

ELECTROCHEMISTRY.

2573. A current of electricity passing through a conducting liquid will decompose the liquid, the amount of the various elements set free being proportional to the quantity of electricity passing through the liquid. (Art. **2493.**)

The amount (weight) of any element which will be liberated by a given quantity of electricity is *proportional to the chemical equivalent of that element;* hence, from the amount of hydrogen (or other element whose valency is always I) set free by unit quantity of electricity, the amount of any other element that will be set free by the same quantity of electricity may be calculated. The chemical equivalent of hydrogen being 1, the amount of hydrogen liberated by one coulomb of electricity becomes a multiplier, and by multiplying the chemical equivalent of each element by this multiplier the **electrochemical equivalents,** or the weight of each element liberated per coulomb, results. (See column 6, Table 89.)

2574. There is reason to believe that *all chemical action generates* E. M. F.; but in order that this E. M. F. may be utilized, the chemical action must take place in and between conducting bodies.

In order that the E. M. F. may be continuously maintained—that is, that the chemical action be continuous—at least one of the bodies acted upon must be a liquid. This liquid is called the **electrolyte** (see Art. **2238**). An electrolyte does not necessarily contain water; it may even be made by melting one of the elements of the cell.

2575. The simplest form of a cell consists of at least two bodies, of which one at least must be a liquid, in and between which two bodies the chemical action goes on which generates the E. M. F. In order that this E. M. F. may be utilized to cause a current to flow, provision must be made for connecting an external circuit with the two bodies between which the action takes place.

Such a cell is usually composed of an electrolyte (often called the *exciting liquid*), into which are placed two conducting bodies; at least one of the bodies is metallic, and it is between this body, called the **anode,** and the electrolyte that the chemical action takes place.

Strictly speaking, the surface of contact between the liquid and the metal is the place of action, and would more properly be called the anode.

The other body, called the **cathode,** serves mainly as a means of connecting the external circuit to the electrolyte, the other end of the circuit being connected to the anode. Connection is actually made to the parts of the anode and cathode which project from the cell, these being then called the **electrodes** of the cell. This is the usual construction, although in some cells the chemical action takes place between two different liquids, in which case whatever solid conducting bodies are used act merely as connectors or terminals.

Examples of such cells will be pointed out in the description of the various types.

2576. The chemical action which takes place is as follows: When the two elements of the cell are placed in the electrolyte, the fact of there being a *chemical affinity* between the various substances in the cell sets up a difference of potential, which appears as between the terminals of the cell; this affinity may or may not set up a chemical action, but so long as the external circuit is open, whatever action may occur is only local, and its energy appears as heat. The reason that no chemical action occurs is that the atoms, having combined, have given up most of their potential energy, and so remain in the combinations they have assumed; as soon, however, as the external circuit is closed, the difference of potential which exists equalizes itself, and causes a momentary current to flow through the external circuit *from* the cathode *to* the anode, the cathode being at the higher potential.

This current decomposes the electrolyte, breaking up the compounds therein and restoring to the various atoms their potential energy. Some of these atoms then unite with the material of the anode, and the E. M. F. is maintained, causing the flow of current to be continuous.

ELECTROCHEMICAL CALCULATIONS.

2577. In the following we shall show the exact relation existing between the current and the chemical work; that is to say, we shall show how to calculate the amount of chemical work which a given current can perform, and, conversely, the quantity of current evolved when a definite amount of chemical work is done.

2578. Whenever an electrolyte is decomposed by a current, the resolved elements have a tendency to reunite. This tendency is termed **chemical affinity.** Thus, when an electric current has been sent through a solution of zinc sulphate ($ZnSO_4$), and the solution has thereby been split up into zinc, oxygen, and sulphur, then, as soon as the current ceases to flow, the zinc exhibits a tendency to recombine chemically with the disintegrated solution. This tendency represents the strong chemical affinity of zinc for oxygen and sulphur. Also, when acidulated water has been decomposed electrically, the separated oxygen and hydrogen tend to reunite.

2579. This tendency to reunite is strikingly shown by an electromotive force which is set up in the solution after the decomposing current ceases. This E. M. F. can be shown to exist by connecting a galvanometer in circuit with the decomposed solution. The deflection of this instrument will show that this E. M. F. due to chemical affinity acts in the *opposite* direction to the E. M. F. of the decomposing current. In other words, it is an *opposing electromotive force.*

2580. Careful measurement has shown that when hydrogen and oxygen combine with each other, an electromotive force of 1.47 volts is set up. From this it follows that no water can be decomposed unless an electromotive force of at least 1.47 volts is utilized; for it requires this much alone to offset the opposing E. M. F. of recombination.

2581. With every electrolyte there is a similar minimum E. M. F. necessary to produce continuous decomposition. This E. M. F. can be calculated for any electrolyte if the heat of formation and the electro-chemical equivalent of its constituents are known. The heat of formation is the **thermochemical equivalent** of the substance. By the thermochemical equivalent is meant the amount of heat liberated by the chemical combination of the molecular weight of one substance with another. This energy is usually expressed in gram-calories; that is, the amount of heat necessary to raise the temperature of one gram of water 1° Centigrade. This thermochemical equivalent is a value found by careful experiment. Thus, one gram of zinc, for instance, converted into zinc sulphate ($ZnSO_4$), is known by experiment to give off about 4,000 heat-units as it combines. In Table 91 the heat of formation of various substances with oxygen is given.

TABLE 91.

HEAT OF COMBINATION WITH OXYGEN.

1 Gram of	Compound Formed.	Calories or Gram Degrees of Heat Produced.
Hydrogen	H_2O	34,000
Carbon	CO_2	8,000
Sulphur	SO_2	2,300
Phosphorus	P_2O_5	5,747
Zinc	ZnO	1,301
Iron	Fe_3O_4	1,576
Tin	SnO_2	1,233
Copper	CuO	602

2582. Electrochemical Equivalent.—Experiment has shown that when 1 C. G. S. unit of current (Art. **2268**) passes through water, it liberates .0001036 gram of hydrogen. Now, since 1 C. G. S. unit of current equals 10 practical units or coulombs (Art. **2278**), it is evident that 1 coulomb will liberate only $\frac{1}{10}$ of this weight of hydrogen, or 1 coulomb liberates .00001036 gram of hydrogen. This quantity of hydrogen is always liberated by 1 coulomb of current, and similarly 1 coulomb of current will liberate a certain definite weight of any other electrolytic substance. The weight thus liberated by 1 coulomb, or by 1 ampere flowing for 1 second, is termed the **electrochemical equivalent** of the substance, and may be found tabulated for the most important elements in column 6 of Table 89.

2583. Total Weight Liberated by Chemical Action.—Experiment has furthermore shown that the total weight of any substance liberated by chemical action is directly proportional to the quantity of current flowing, so that if

z = electrochemical equivalent of any substance;
Q = number of coulombs;
W = weight in grams of liberated substance;

then, $W = Q \times z.$ **(467.)**

EXAMPLE.—A current of .5 ampere was passed through an acidulated solution of water for 10 minutes. What weight of hydrogen was evolved?

SOLUTION.— $t = 10$ minutes $= 600$ seconds. $C = .5$ ampere. Then, by formula **405,** $Q = Ct = .5 \times 600 = 300$ coulombs. Referring to column 6, Table 89, we find for hydrogen $z = .00001036$; hence, by formula **467,** $W = 300 \times .00001036 = .003108$ gram of hydrogen. Ans.

2584. Heat Formation by Chemical Action.—In Art. **2581** the formation of heat during chemical action was explained. Experiment has shown that the total heat generated during chemical action is proportional to the weight of the substance liberated; so that if

h = calories evolved per gram of substance;
W = weight in grams of substance liberated;
H = total calories evolved;

then, $$H = W \times h;$$

but, inserting the value of W as given by formula **467,** we have as the total heat evolved in calories

$$H = Q \times z \times h. \qquad \textbf{(468.)}$$

2585. Relation of Heat and Work.—It can be shown by a simple calculation, knowing the mechanical equivalent of heat, that 1 calorie is equivalent to 4.2 joules of work. This being the case, formula **468** may be written to express the total joules J of energy required to liberate a given weight of any substance; for, if J = total joules of energy, then, by utilizing the notation of Art. **2584,**

$$J = 4.2\, Q \times z \times h. \qquad \textbf{(469.)}$$

2586. It is possible to express this work in another manner. By referring to Art. **2332,** it will be seen that the work done in joules in any electrical circuit can be expressed by the product of volts and coulombs of that circuit; or, if

E = volts;
Q = coulombs;
J = joules;

then, $$J = E \times Q. \qquad \textbf{(470.)}$$

2587. Calculation of E. M. F. Produced by Chemical Action.—Comparing formulas **469** and **470**, we see that they are different expressions for the same quantity J, namely, the electrical work in joules done in the circuit; or,

$$J = 4.2\ Q \times z \times h, \text{ and}$$
$$J = E \times Q.$$

But, when two quantities are each equal to a third, they are equal to each other; hence,

$$E \times Q = 4.2\ Q \times z \times h.$$

Dividing both sides of this equation by Q gives us an expression for the electromotive force in volts, namely,

$$E = 4.2\ h \times z. \qquad \textbf{(471.)}$$

This important result may be expressed in words as follows: *The electromotive force in volts of any chemical reaction is equal to the product of the electrochemical equivalent of the separated substance, the heat of combination of the substance per gram degree, and the constant 4.2.*

Example.—Calculate the opposing E. M. F. set up by the hydrogen tending to unite with the oxygen during the decomposition of water.

Solution.—Refer to Table 89, column 6, where for hydrogen $z =$ .00001036, and to Table 91, column 3, where for hydrogen $h = 34{,}000$; then, by formula **471**, $E = 4.2 \times 34{,}000 \times .00001036 = 1.479$ volts. Ans.

Note.—In the same way we may calculate the opposing E. M. F. set up and effecting any particular electrolysis.

2588. In the above, the E. M. F. is calculated on the assumption that all the chemical energy developed is converted into electricity, and that none of the energy appears as heat. Practically, however, some heat is generated in almost every case during the electric activity. This is only a secondary consequence of the electric resistance of the cell. If the cell offers a negligible resistance, then the amount of heat electrically developed by the current would also be negligible, and all the chemical energy developed by chemical changes in the cell would be liberated outside the cell, that is, in the external circuit.

2589. It should be remarked, however, that owing to the incompleteness of our knowledge of thermochemical equivalents, and of the exact nature of the electrochemical actions in the cell, the E. M. F. of a cell can only in a few instances be practically predetermined.

ELECTROCHEMICAL THEORIES.

2590. If more than one set of actions can take place in a cell, the effect of the various actions on the E. M. F. of the cell may be determined by properly applying formula **471** to each action; the E. M. F. of each action may be then added or subtracted to get the resulting E. M. F., according to the nature of the action.

If the substance forming the anode has an affinity for one or more elements of the electrolyte, and the substance forming the cathode has an affinity for the other element or elements of the electrolyte, it is evident that the tendencies of these elements to combine with the anode and cathode, respectively, will assist each other, and the E. M. F. of each action will add together in giving the resulting E. M. F. of the cell.

If the substances forming the anode and cathode, respectively, have each an affinity for *the same* element or elements of the electrolyte, it is again evident that the tendency of these elements to combine with the anode will be partly balanced by their tendency to combine with the cathode; hence, the E. M. F. which would result from either action alone must be *subtracted* from the other to obtain the resulting E. M. F.

This explains the case spoken of in Art. **2243,** where it is stated that a cell with zinc and iron as elements will give a less E. M. F. than a cell using zinc and graphite (carbon). In the zinc-iron cell a part of the electrolyte has an affinity for both the zinc and the iron, while in the zinc-graphite cell the electrolyte has no affinity for the graphite, and the full E. M. F. of the action on the zinc appears. The meaning of the Electromotive Series (Art. **2241**) should now be clear.

2591. The action which takes place in a cell if the electrolyte be an acid is the decomposition of the acid, the liberation of the hydrogen, and the formation of a salt of the metal of the anode by its union with the balance of the acid. If the electrolyte be a solution of some salt, the action is more complicated, and is about as follows: The water of the solution is decomposed, its oxygen uniting with the anode, forming an oxide, and the hydrogen is liberated, as before. The oxide which is formed then unites with the salt in solution, forming salts of the metal oxidized; these formations increase the E. M. F. which the cell would have were water alone the exciting liquid and the oxidation of the anode the only action.

2592. Thus it is seen that in almost all the chemical actions which occur in batteries, the attacking of the anode results from the decomposition of the electrolyte, hydrogen being liberated. This decomposition of the electrolyte takes place throughout the space between the anode and cathode; the hydrogen, however, does not appear throughout the electrolyte, but only at the surface of the cathode; for as soon as the electrolyte is decomposed into its elements, the union of the metal of the anode with the elements of the electrolyte with which it can combine can only take place with the atoms of the particular molecules which were at the surface of the anode; the free hydrogen atoms of these molecules then unite with the free elements of what was the next layer of molecules, reforming the original electrolyte; the displaced hydrogen atoms of these newly formed molecules unite with the free elements of the next layer of molecules, and so on all across the liquid, until at the point where the current leaves the liquid (the cathode) there are left the free hydrogen atoms of the last layer of molecules. These then appear at the surface of the cathode, providing the cathode is not a substance with which these free atoms may unite.

2593. The accompanying diagrams represent this action in the case of the zinc (Zn), sulphuric acid (H_2SO_4), and copper (Cu) cell, given in Art. **2240.** One single line of the molecules of the electrolyte between the zinc and the copper plate is represented at *a*, Fig. 1037. Each molecule of the acid is made up of one atom of S and four of O, united with a molecule consisting of two atoms of H.

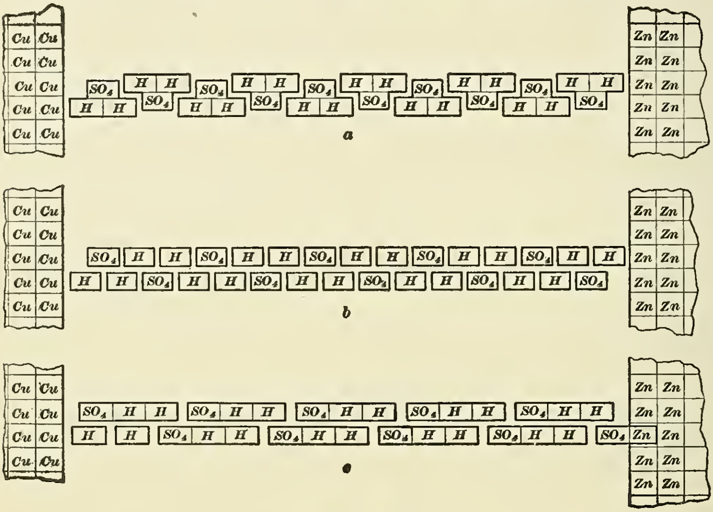


FIG. 1037.

No current is supposed to be flowing through the liquid. For convenience, it is assumed that the molecule of SO_4 is not decomposed by the current; actually, it probably is, but since in its action with the zinc it unites with it as a whole, the assumption is allowable.

In *b*, Fig. 1037, the molecules are represented as decomposed by a current, and each individual molecule of SO_4 and atom of H is separate. Now, the molecule of SO_4 at the right has a greater affinity for, or tendency to combine with, the zinc than it has to combine with the free atoms of H, and does so, as represented at *c*, Fig. 1037, and the remaining molecules of SO_4, not being in contact with the zinc, unite with the free atoms of H all the way across the liquid, until at the copper plate there are no free molecules of SO_4 left to combine with the last atoms of H in the line; consequently, the free atoms of H appear in the form of gas at the copper plate. Of course the newly formed molecules of H_2SO_4 are immediately decomposed and reunited, which process continues with inconceivable rapidity, the result being a practically continuous formation of $ZnSO_4$ at the anode and the liberation of H at the cathode as long as any current flows through the electrolyte. Now, it is evident that all the energy required to decompose *all but one molecule* in any line of molecules across the electrolyte is immediately given up by the reunion of the free atoms. For this reason *the distance between the anode and the cathode, or their size, does not affect the amount of electrolyte decomposed by a given force, nor the energy required to perform the decomposition.*

POLARIZATION AND DEPOLARIZATION.

2594. If at or near the cathode there is some substance, such that the free hydrogen left by the decomposition of the electrolyte may unite with it, the energy liberated by such formation will add to the E. M. F. of the cell. If this free hydrogen can not unite with some substance at the cathode, it collects on the surface in bubbles as a gas. In addition to the reduction of the E. M. F. of the cell due to decomposition of the electrolyte, the formation of hydrogen also acts disadvantageously, as it forms in a layer on the surface of the cathode, which enormously increases the internal resistance of the cell, thus diminishing the current which the E. M. F. of the cell can send through any given external resistance. The formation of hydrogen on the surface of the cathode is known as **polarization,** and its removal, by any means, mechanical or chemical, is called **depolarization,** the agent used being called the **depolarizer.**

2595. If merely mechanical means of depolarization be used, the result is to prevent the increase of internal resistance of the cell; by causing the liberated hydrogen to *recombine* at the cathode, by chemical means, not only is the internal resistance not increased, but the actual E. M. F. of the cell is increased.

2596. Various mechanical devices for depolarizing cells have been used; the cathode has been arranged to be agitated in the liquid, or to be entirely removed from the liquid at intervals; or the cathode, and in some instances both electrodes, have been made in the form of disks, dipped for about half their diameter into the electrolyte. On rotating the disks, the hydrogen is prevented from forming on the cathode by its motion.

The power for performing these various movements has usually been derived from clockwork, and in some instances from the current given out by the battery. It is evident that such devices are commercially of little value, especially as chemical depolarizers may be easily used.

2597. The depolarization by chemical means may be accomplished by surrounding the negative element (cathode) with a solid or liquid substance, with which the free hydrogen may combine. This combination usually merely disposes of this element, and prevents the bad effects of a deposit on the cathode. Under these circumstances the compound formed at the cathode is usually water, the depolarizer being a substance rich in oxygen, with which the hydrogen combines. This water has the effect of diluting the electrolyte, already weakened by the combination with the anode; but, by properly selecting the depolarizer with reference to the electrolyte, the chemical combination at the cathode may be such that it will, either directly or by further combination, replace the part of the electrolyte which has combined with the anode, thus keeping the electrolyte of the same composition and strength throughout the life of the anode or of the depolarizer. Instances of both these classes of chemical depolarization will be noted in the description of the various cells.

2598. The *rate* at which any depolarizer will depolarize depends on many conditions; and no depolarizer will keep the E. M. F. of a cell constant for all currents, for, after a certain limiting current has been reached, the limit depending on the sizes of the various parts of the cell, the formation of the free element of the electrolyte is more rapid than its absorption by, or recombination with, the depolarizer, and the surplus will collect on the cathode.

In the case of depolarizers which, by the formation of water, dilute the electrolyte, the E. M. F. will become less with continued use of the cell, even if the current output be small. These facts should be remembered in dealing with the various depolarizers.

2599. From the preceding remarks, it appears that in order to give a high E. M. F., the metal chosen for the anode must be one whose salts have a comparatively high value for their heat of formation. Such metals are potassium, sodium, strontium, calcium, and magnesium; potassium salts having the highest heat of formation, the others, in the order given, having lower.

2600. Having a high heat of formation means, however, that the metal has a great affinity for the elements necessary to form its salts or oxides; this being the case, they are liable to combine with such elements whenever the opportunity presents itself, taking them from the air, from water, or from salts of other metals which have a lesser affinity for the salt-forming elements. Consequently, the metals in the list given could not be used in the presence of acids or solution of salts, or even of water, without decomposing the liquid and rapidly forming salts or oxides, the whole of the energy of the action appearing as heat. In order, then, to have a practicable cell, the metal should not be attacked by the exciting liquid except as the exciting liquid is decomposed by the passage of the current. This is the reason for the extensive adoption of zinc, that being a metal whose heat of formation is comparatively high, at the same time not high enough to cause its salts and oxides to be formed with any degree of rapidity unless the necessary elements are presented to it in a free state, as they are in a voltaic cell by the decomposition of the electrolyte.

Besides, zinc is actually the cheapest of the metals, excepting iron, and, in proportion to the amount of kinetic chemical energy possessed, is cheaper than any other metal which could be used.

2601. Batteries, as sources of electrical energy, are used mainly in cases where a current is required very intermittently, such as in ringing bells, lighting gas, etc., or where a small but steady current is required for long periods of time, as in telegraphy and telephony, or for laboratory and testing purposes. Their general use on a large scale, as sources of electrical energy for lighting or power purposes, is prohibited, at least at present, by the comparatively great cost of the material consumed, and the expense of installation and maintenance.

For example, the bichromate battery is about the cheapest in point of cost of materials consumed, and in this the materials alone would cost about 28 cents per horsepower per hour on a large scale, while the cost of electrical energy, using dynamos, is about 5 or 6 cents per horsepower per hour, ordinarily, and in many cases is much less. The cost of material in the silver chloride battery is about $135 per horsepower per hour.

This high cost of the power does not, however, prevent batteries from being largely used for the purposes outlined above, and their practical application is an important part of electrical engineering.

CELLS.

CLASSIFICATION.

2602. The various classes of voltaic cells may be divided as follows:

Cells in Which There Is No Depolarizer.—This is the simplest form of cell, and, on account of polarization, cells of this class, commonly called *open-circuit cells*, are not used for other than intermittent work.

2603. Cells With a Depolarizing Electrolyte.—In this class of cells the electrolyte is of such a nature that either no hydrogen is formed or the liquid contains a substance with which the hydrogen unites. As this action takes place mainly at the cathode, there is little distinction, as far as action goes, between this latter type and cells with a liquid depolarizer.

2604. Cells With a Liquid Depolarizer.—In this class of cells, however, the cathode is surrounded by a depolarizing liquid, which is prevented, by mechanical means, from mixing with the electrolyte.

The means usually employed are either to separate the two liquids by a porous partition, which allows of their electrical contact without mechanical mixture of the two, if their respective specific gravities be nearly the same; or, if these differ considerably, gravity will keep the two liquids apart, one over the other in the containing vessel.

2605. Cells With a Solid Depolarizer.—This class is identical in action with the class preceding, the depolarizer, however, being a solid instead of a liquid.

If the solid depolarizer is granular, or in the form of powder, it is often necessary to employ a porous partition between the cathode surrounded by the depolarizer and the electrolyte. This is merely to keep the depolarizer in place, and is dispensed with if the depolarizer is formed into a paste or solid body upon the cathode. In fact, the depolarizer may itself form the cathode, if it be a solid conducting material, the office of the cathode being primarily to establish a connection between the electrolyte and the external circuit.

2606. Cells in Which an Elementary Substance Is Applied to the Cathode, Acting as a Depolarizer.—This substance may be applied mechanically or chemically. In the former case, the body, in the form of a gas or liquid, is made to appear at the cathode by pumping or forcing it to that place from some external source of supply. In the chemical method, the cathode is surrounded by solid or liquid substances, which by their action on each other evolve some elementary body which combines with the free element of the decomposed electrolyte.

This is distinct from the action of cells with a liquid or a solid depolarizer, as the production of the elementary depolarizing body substance is independent of the electrical action of the cell, *going on all the time, whether the cell is in use or not*, variations in the current output of the cell having no influence on its production.

Voltaic cells are ordinarily classed as "single-fluid" and "two-fluid" cells; but as such a classification has little reference to this principle of operation, it will not be used in this discussion. The number of different kinds of cells that have been made is very large indeed, but they can all be subdivided into one of these general classes. Only a few typical cells of each class will be described.

CELLS WITH A NON-DEPOLARIZING ELECTROLYTE.

2607. This class of cells includes the cells of the **Volta** type, which consists generally of an electrolyte of acid or saline solution, into which are placed two or more plates of metal, one of which (usually of zinc) is acted on by the electrolyte.

2608. A simple form of this cell is illustrated in Art. **2238.** Its materials are zinc, dilute sulphuric acid, and copper, which give an E. M. F. of about .9 volt.

Many modifications of the form of this type of cell have been suggested and used, such as making the elements in strips and rolling them around each other in a helical form, with insulating material between, etc.; but all are open to the objection of rapid polarization.

2609. In place of copper as a cathode many other elements have been used, notably the **Smee** cell, using platinum or platinized silver, and cells of various makes in which the cathode is of iron.

In cells of this and other types, impurities in the zinc set up local actions, which diminish the E. M. F. of the cell and cause a wasting of the zinc. These local actions are almost wholly prevented by amalgamating the zinc, which is usually done.

If a good quality of drawn or rolled zinc is used, this precaution is hardly necessary.

2610. Not long after the first use of the zinc, sulphuric acid, and copper battery, it was found that the copper or other metallic cathode could be advantageously replaced with porous carbon, and many cells were so constructed. The E. M. F. of such a cell is about 1.35 volts ordinarily. To prevent the electrolyte from becoming exhausted too quickly, there is sometimes placed in the cell a porous earthenware pot or cup, filled with strong sulphuric acid. As the dilute acid outside the porous cup becomes weaker, the stronger acid oozes through the sides of the porous cup and keeps up its strength. In some instances the carbon cathode has itself formed the porous cup. An objection to the use of porous cups in this type of cell is that its pores are liable to become clogged by deposits of zinc sulphate from the solution.

2611. Other acid electrolytes have been used in this type of cell. With either nitric or hydrochloric acids (diluted) the E. M. F. is not sensibly different from that with sulphuric acid as the electrolyte.

2612. Of the saline electrolytes, the best exciting liquid is considered to be a solution of ammonium chloride (sal ammoniac). The E. M. F. of a zinc, ammonium chloride, and carbon cell is about 1.15 volts.

2613. There are a great number of cells of this type in use for ringing bells, gas lighting, and doing other intermittent work. They are all alike in principle, but their mechanical construction differs somewhat. In the **Law** open-circuit cell, the carbon electrode is in the form of a hollow cylinder, enclosing a smaller hollow cylinder, each with a wide slit in one side. These cylinders hang vertically in the electrolyte, and the zinc hangs in the space formed by the slits in the side, being suspended from the cover of the cell. This is an excellent form of cell, being well worked out in its mechanical details.

2614. In the **Little Giant** cell, the hollow carbon cylinder is continuous, except for a hole in the side for the circulation of the electrolyte, and the zinc, in the form of a rod, is suspended in the center of the carbon cylinder. The top of the cylinder is extended to form the cover of the cell, and the zinc is insulated from it by a porcelain bushing.

2615. The **Hercules** cell employs a corrugated solid carbon cylinder, the zinc element being made of sheet zinc bent into a cylinder surrounding the carbon.

2616. Many other forms of carbon or zinc elements may be and are used. The particular shape of the carbon has comparatively little to do with the satisfactory working of the cell, care and good design in the construction being more important. The element should be of such shape as not to be easily broken in transit, and, being usually molded into shape under pressure, should be of such proportions that it is cheap to make.

2617. In all the cells of this type the carbon is made as porous as possible, and of such shape that the surface exposed to the liquid is very large compared with the surface of the zinc. Thus, the average area of the internal circuit of the cell is made large, and at the same time advantage is taken of the slight depolarization, occurring with a porous carbon of large surface, due to the oxygen which porous carbon absorbs from the air, with which some of the evolved hydrogen combines. The E. M. F. of this type of cell is, therefore, slightly higher than those which employ a non-oxidizable metallic cathode, such as platinum. This depolarizing action takes place slowly, and, therefore, hydrogen will form on the cathode if a considerable current be taken from the cell, thus increasing the internal resistance. In intermittent work this is not objectionable, as the hydrogen is soon absorbed when the external circuit is opened.

2618. Another salt which has been much used in solution as an electrolyte is sodium chloride (common salt). The heat of formation of sodium chloride being greater than that of ammonium chloride, the energy required to decompose the electrolyte is greater; therefore, the E. M. F. of cells using this electrolyte is slightly lower, that of a zinc, sodium chloride, and carbon cell being about 1.08 volts.

However, this electrolyte being very cheap and of common occurrence, many makers of batteries have employed it. It has also been proposed to use sea-water as an electrolyte, by placing in the ocean immense plates of zinc and copper or carbon. This has never been commercially accomplished, for the consumption of zinc makes the cost of the electrical energy too great for this method to compete with others now in use.

2619. Electrical buoys have been constructed, in which plates of carbon or copper and zinc enter or leave the water as the buoy is rocked by the waves, thus causing a light to flash or a bell to ring intermittently.

2620. Various other salts in solution have been used as electrolytes, such as ammonium nitrate, alum, potassium sulphate, zinc sulphate, zinc chloride, potassium hydrate (caustic potash), etc.

The E. M. F. of cells using solutions of these various salts as electrolytes may be found from the values given in Table 93.

2621. The effect of substituting various metals for the zinc in this type of cell may be found from Table 92, which gives the E. M. F. of the action of dilute sulphuric acid on various metals. The values given in this table may be taken for the E. M. F. of cells using a platinum or carbon cathode; if other metals which appear in the table be used as the cathode, the E. M. F. of the action of the acid upon them must be subtracted to get the E. M. F. of the cell. (See Art. **2590.**) With other electrolytes, the substitution of other metals for zinc reduces the E. M. F. in about the same proportion as in the table.

TABLE 92.

E. M. F. OF THE FORMATION OF VARIOUS SULPHATES.

Metal.	Formula of Sulphate.	E. M. F. Volts.
Potassium	K_2SO_4	2.35
Zinc	$ZnSO_4$	1.35
Cadmium	$CdSO_4$	1.05
Lead	$PbSO_4$	.90
Tin	$SnSO_4$	.88
Iron	$FeSO_4$	.83
Aluminum	$Al_2(SO_4)_3$	.70
Copper	$CuSO_4$	.47
Silver	Ag_2SO_4	.30
Platinum	None	.00

TABLE 93.

E. M. F. OF ZINC (PURE) WITH VARIOUS ELECTROLYTES.

Electrolyte. Acids.	E. M. F. Volts.	Electrolyte. Saline Solutions.	E. M. F. Volts.
H_2SO_4	1.35	$NaCl$	1.08
HCl	1.40	$ZnSO_4$	1.32
HNO_3	1.43	NH_4Cl	1.15
		$NaOH$	1.35
		KOH	1.38
		Ordinary H_2O	0.90

CELLS WITH A DEPOLARIZING ELECTROLYTE.

2622. The best known cells of this type are the so-called **bichromate** cells. These consist, broadly, of a zinc-carbon couple, with an electrolyte composed of a solution of some acid or other exciting liquid, mixed with a proportion of the *bichromate salts* of some metal. The bichromate salts are a peculiar series of salts formed by the oxide of chromium having the formula Cr_2O_7, which is an unstable oxide, appearing only in combination with some other metal, such as potassium or sodium, forming the *bichromate salts* of those metals. The mixture usually employed as an electrolyte is sulphuric acid and potassium bichromate, $K_2Cr_2O_7$, although sodium bichromate, $Na_2Cr_2O_7$, is somewhat superior for the purpose, which is to act as a depolarizer. This office the bichromate salts perform perfectly, as they have a large proportion of oxygen, as is seen from their formulas; consequently, the hydrogen liberated by decomposition of the electrolyte is consumed as fast as generated, forming water and a salt known as *chrome alum*, which forms in crystals of a purplish color. This results in a high E. M. F. (usually about 2 volts).

2623. The chemical actions in this class of cells are complicated; one result is the formation of chromic acid by the action of the acid in the electrolyte on the $K_2Cr_2O_7$, which will slowly attack the zinc whenever in contact with it, whether there be any current flowing or not. This leads to the device—which is almost universally adopted—of lifting the anode, or both elements, from the liquid when the cell is not in use. Cells which are in continuous use are liable to have their internal resistance increased by a deposit of the crystals of chrome alum on the cathode, these crystals being poor conductors. In certain forms of cells the construction is such that this is not liable to occur.

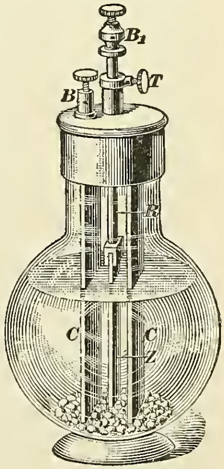


FIG. 1038.

2624. A familiar type of bichromate cell is the **Grenet** cell, shown in Fig. 1038, which consists of a bottle-shaped glass jar with a hard rubber or porcelain cover. From this cover two flat carbon plates C, C are suspended, parallel to and a short distance from each other, as shown; between them hangs a zinc plate Z supported by a sliding rod R, which may be drawn up until the zinc is entirely out of the liquid; it is held in any position by the thumb-screw *T*. On the top of the brass rod is a binding-post B_1, the other terminal of the cell being the binding-post *B*, which is connected to the two carbon plates.

The electrolyte is composed of 3 parts of potassium bichromate, dissolved in 18 parts of water, to which is added 4 parts of sulphuric acid.

The E. M. F. of such a cell is 1.92 to 2 volts.

At ordinary temperatures, variations in the proportion of bichromate in the solution, within moderate limits, do not vary the E. M. F. or the internal resistance very much. Variations in temperature vary the internal resistance, but not the E. M. F., the internal resistance decreasing as the temperature increases. With the above proportion of sulphuric acid and bichromate in the solution, the sulphuric acid is first exhausted. Theoretically, for an equal life of both substances in the electrolyte, the correct proportions should be

$$\left.\begin{aligned} H_2SO_4 &= 7.0 \\ K_2Cr_2O_7 &= 3.0 \end{aligned}\right\} \text{parts by weight,}$$

which proportion is often used. In fact, however, it is more necessary to keep up the strength of the depolarizer, that is, the bichromate, so the first given proportion will really give better results.

2625. A great variety of batteries of this type has been made, especially abroad, where they are called **Poggendorf's** cell; they do not differ in principle or material from the Grenet cell, but in mechanical details are more suited to general work. They are usually built with several cells, the various elements being connected in series to give an E. M. F. of 6 to 10 or more volts. All the elements are simultaneously raised out of or lowered into the liquid by a lever or windlass arrangement, as shown in Fig. 1039, which represents a battery of five cells all alike. The elements are of zinc and carbon, there being three plates of zinc, *Z*, and four of carbon, *C*, in each cell. The plates are all suspended from a wooden cross-bar, so that they may be simultaneously raised or lowered by winding or unwinding

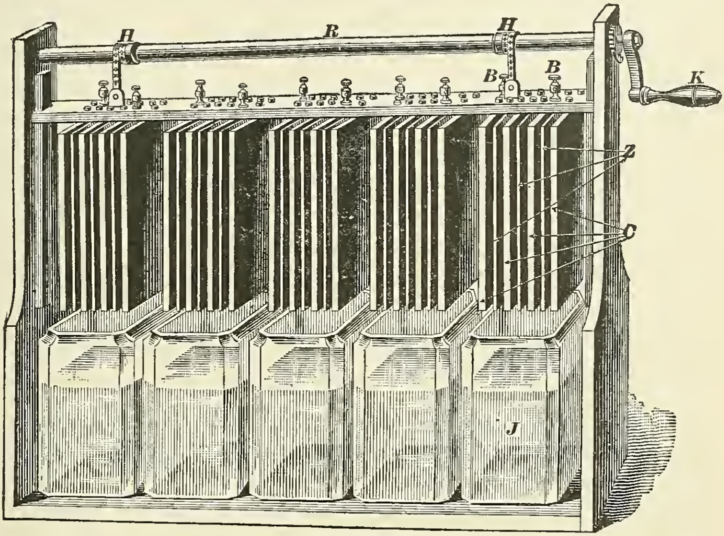


FIG. 1039.

the chains *H*, *H* upon the rod *R*, which is turned by means of the crank *K*.

The elements may thus be raised from the liquid contained in the jars *J* when the cells are not in use. The elements of each cell are provided with two binding-posts *B*, *B*, one of which is connected to the carbon and the other to the zinc plates. The various cells may then be used separately, or connected together in parallel or in series, as desired.

2626. An ingenious arrangement of bichromate cells for cautery work is that due to Chardin. In his battery the elements are normally held out of the liquid by a spring; by pressing a foot lever they may be gradually lowered into the liquid.

When just the ends of the elements are in the liquid, the internal resistance of the battery is considerable; but as the elements are lowered, this resistance decreases largely.

By varying the distance which the pressure on the foot lever causes the elements to dip into the liquid, a sensitive and easily managed method of control of the output of the battery is secured.

2627. Another type of bichromate cell consists of a closed vessel divided into two parts by a horizontal perforated partition. In one part the zinc and carbon elements are located. Enough liquid to fill one of the parts of the vessel is introduced; when the vessel is standing on one end, all the liquid is below the partition and the elements above, and in order to render the cell active, it is only necessary to turn the vessel completely over, when the liquid flows through the perforated partition and comes in contact with the zinc and carbon.

2628. Among the cells which may be said to belong to this class is a type of cell in which no free hydrogen or other gas is evolved in the decomposition of the electrolyte. Such electrolytes, which might more properly be called *non-polarizing*, are the solutions of some of the salts of metals having more than one valency. The salt containing the greater amount of the non-metallic element (the *ic* salt) is used as the electrolyte; on being decomposed, a salt of the metal of the anode is formed with a part of its non-metallic element, and the remainder is recombined to form the salt having the lesser proportion of the non-metallic element (the *ous* salt).

2629. An example of this type of cell is the **Pabst** cell, in which wrought iron and carbon are used as elements, and a solution of ferric chloride as the electrolyte. The ferric chloride is decomposed into ferrous chloride and free chlorine; the latter unites with the iron anode, resulting in an E. M. F. of .78 volt.

2630. Similar cells are also made, using a solution of ferrous sulphate as an electrolyte, the action being similar. There are other salts, with solutions of which zinc will combine without hydrogen being released, such as sulphite of potassium or of sodium, and non-polarizing cells are constructed, employing solutions of these salts as electrolytes.

2631. Most single fluid cells in which the electrolyte is depolarizing are open to the objection that the zinc is attacked by the electrolyte at all times, whether the external circuit be closed or not; besides this, with the exception of the bichromate cells, the materials of the electrolyte are usually expensive, and not readily obtainable, and the commercial use of such cells is limited.

CELLS WITH A LIQUID DEPOLARIZER.

2632. **Nitric acid,** being rich in oxygen, is largely used as a depolarizing liquid in this class of cells.

Its use is objectionable from the fact that when deprived of a part of its oxygen, it gives off a gas, nitric oxide, which, on combining with the oxygen of air, becomes nitrogen peroxide, NO_2, a disagreeable and even dangerous corrosive gas; consequently, the best of ventilation is essential where cells with this depolarizer are used.

2633. The principal cells using this depolarizer are the **Grove** and **Bunsen** cells, and some of their derivatives. In the Grove cell the positive element is zinc; the negative, platinum. The platinum element is placed inside a porous cup and surrounded with nitric acid; outside the porous cup is the exciting liquid, sulphuric acid diluted with water. The E. M. F. of the Grove cell is 1.9 volts at ordinary temperatures.

2634. The Grove cell is a very old type, and has been made in many forms, but the expense of using the platinum element has led to the adoption of the Bunsen cell, which substitutes a carbon element for the platinum. With commercial nitric acid, specific gravity about 1.33, the E. M. F. of the Bunsen cell is 1.89 volts ordinarily; if pure (fuming) nitric acid, specific gravity 1.53, be used, the E. M. F. is increased to about 1.96 volts. About .35 volt is due to the action of the depolarizer.

2635. Variations in the density of the nitric acid thus affect the E. M. F. of the cell only slightly, until the specific gravity of the solution falls to about 1.23; but at a density below this the acid has little or no effect as a depolarizer, although the liquid still contains about 30% of nitric acid.

As the commercial acid is most frequently used in the cell, only a small proportion of water is required to dilute it to a point where it can not be used. In fact, where commercial acid is used, only about 13% of the actual amount of the pure acid in the solution can be utilized, if nitric acid alone be the depolarizer.

The water formed at the cathode by the process of depolarization, therefore, is disadvantageous on account of its dilution of the depolarizer.

Several investigators have mixed sulphuric acid with the nitric, in various proportions, with good results. Sulphuric acid has a strong affinity for water, and will combine with it in considerable quantity; consequently, the water formed at the cathode is absorbed by the sulphuric acid, leaving the nitric acid at its full strength.

2636. Variations in the density of the exciting liquid also affect the E. M. F. of the cells to some extent, but not so much so as variation in the density of the depolarizer. The density ordinarily used is about 1.09 sp. gr. (13% by weight of acid). At this point the E. M. F. of the action of the exciting liquid on the zinc is about 1.53 volts.

As the action of pure water alone on zinc will give an E. M. F. of about .9 volt, variations of the density of the exciting liquid from 13% (by weight) of acid down to pure water will reduce the E. M. F. about .6 volt. Increasing the density of the liquid to about 1.23 gives a maximum E. M. F. (of the action of the acid on the zinc only) of about 1.6 volts; any further increase in the density does not increase the E. M. F. appreciably. To obtain the E. M. F. of the cell, to the above figures should be added the E. M. F.

due to the action of the depolarizer, about .35 volt, as stated above.

It is somewhat difficult to maintain sulphuric acid which has free access to the air at a density much above about 1.10, on account of the absorption of water from the air by the acid, and acid of about this density is ordinarily used.

2637. The proportions of the two acids in the cells are about 3 of exciting liquid to 1 of depolarizer, the depolarizer being of a specific gravity of about 1.33; with these proportions the cell will maintain its E. M. F. (within about 10%) for several days on a closed circuit.

The average internal resistance (as ordinarily constructed) is about 2 ohms.

2638. Many modifications of the Grove and Bunsen cells have been made, some consisting merely in changes in the mechanical arrangement of the parts, others substituting various depolarizers, exciting liquids, or elements.

For example, a carbon cup fitted with a tight cover has been used as cathode. On this being filled with nitric acid, the gas given off by the acid produces a pressure inside the cup, which forces the acid out through the pores of the carbon to the surface, where its depolarizing action takes place. This suppresses a part of the disagreeable fumes of the acid. To accomplish this same result, it has been proposed to cover the cell with an inverted vessel containing scrap tin, which will absorb the fumes. A layer of turpentine floating on the acid will prevent a large part of the fumes from being given off, as they combine with the turpentine.

2639. When iron or steel is placed in strong nitric acid it is not attacked, although this acid is a powerful oxidizing agent; but when the acid is diluted to about 1.20 sp. gr., or lower, the iron is strongly attacked.

Consequently, with a strong solution of nitric acid as a depolarizer, iron (usually cast iron) may replace the carbon element of the Bunsen cell, with good results, the E. M. F. being about 1.7 volts. Care must be taken, however, that the density of the depolarizer does not fall too low, or the negative element will be consumed. In fact, a cell of this class may be constructed with only iron and nitric acid as elements, in the following order: Iron (anode), dilute nitric acid, porous cup, strong nitric acid, and iron (cathode).

2640. A cell similar to the foregoing, except that the negative element is carbon instead of iron, known as the **Maêche** cell, gives an E. M. F. of 1.5 volts, and has the advantage of giving off a much less quantity of nitrous vapor than the Bunsen. By substituting ordinary water for the dilute acid in the Maêche cell, the E. M. F. is reduced to about 1.2 volts; but owing to the difference in specific gravity of the two liquids (nitric acid and water), they soon mix somewhat through the walls of the porous cup.

2641. The E. M. F. of this type of cell is really generated in two parts: one at the surface of the anode, due to the action of the electrolyte on the anode, and the other at or near the cathode, due to the action of the depolarizing liquid on the hydrogen evolved. (See Art. **2590.**) Varying the material of the anode or the electrolyte will then affect that part of the E. M. F. just as in a cell of the class given in Art. **2602,** and the amount by which the E. M. F. is reduced or increased may be found from the values given in Tables 92 and 93, making due allowance for the E. M. F. due to the depolarizing action. The effect on the E. M. F. of varying the depolarizer may likewise be calculated from the values given in Table 94.

TABLE 94.

DEPOLARIZING EFFECT OF VARIOUS SUBSTANCES.

Substance. Solids.	E. M. F. Volts.	Substance. Liquid.	E. M. F. Volts.
MnO_2(ordinary).....	.33	HNO_3 (concentrated)..........	.35
Pb_2O_3..............	.81	H_2CrO_4...........	.47
		Cl gas dissolved in water............	.64

2642. The foregoing values for the E. M. F. in Tables 92, 93, and 94 are about the average of the somewhat varying results of different experimenters.

The values also vary somewhat with different temperatures and degrees of concentration of the liquids; they will be seen to be approximately correct if compared with existing cells.

It will be seen from this table that either chromic acid or chlorine water (chlorine gas dissolved in water) used as a depolarizer would give a higher E. M. F. than nitric acid; but as these liquids decompose in the presence of air, they can not be commercially used, just as in the case of sodium or potassium as anodes. (See Art. **2599.**)

2643. Another important type of cell of this class is the **bichromate** cell, which differs from that described in Art. **2603,** in that the bichromate solution is not mixed with the electrolyte, but is separated from it by a porous partition, with the effect that the zinc is not seriously attacked on open circuit. As to the E. M. F., chemical action, etc., this type is not sensibly different from the bichromate cells described in Art. **2603.**

The bichromate solution is usually, with the cathode, placed in the outer vessel, the zinc and exciting liquid being inside the porous cup; the exciting liquid being usually sulphuric acid diluted with water to about 1.10 sp. gr., although solutions of sodium chloride or ammonium chloride are used.

2644. The depolarizing liquid is usually of the composition given in Art. **2624,** under the name **electropoion fluid.** A bichromate mixture is prepared by dealers in battery material, as follows (all parts by weight): Sulphuric acid, 2 parts, is mixed with water, 4 parts; in another vessel, 1 part of potassium bichromate is dissolved in 3 parts of boiling water, and while hot is mixed with the liquid first prepared. This liquid, when cold and more or less diluted, is suitable for use in most bichromate cells.

2645. The **Fuller** bichromate cell, one form of which is represented in Fig. 1040, is a very excellent cell of this type, being economical in operation. It consists of a glass jar containing the depolarizer (electropoion fluid diluted about one-half), into which is hung the carbon cathode *C*. In the center of the jar is placed the porous cup *P*, into which is poured a little mercury, and the zinc, which is in the form of a rod or wire *W*, with a conical lump *Z* cast on the end, placed in position. The mercury serves to keep the zinc well amalgamated.

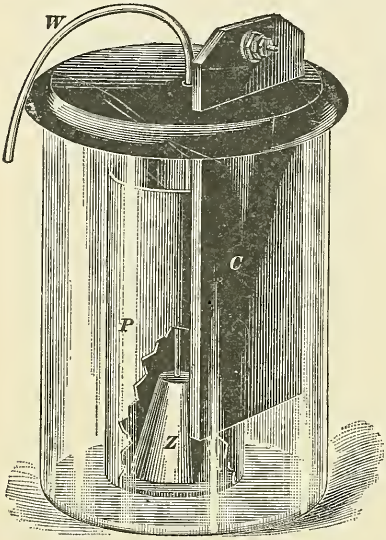


FIG. 1040.

The exciting liquid is either very dilute sulphuric acid, or, more commonly, pure water. The E. M. F. is 2.14 volts, and the internal resistance (of the type shown in Fig. 1039) usually about 1 ohm, depending, however, on the thickness and character of the porous cup. This type of cell is largely used for telegraphic purposes in England.

2646. Bichromate cells are often constructed in which the liquids employed have such a difference in their specific gravities that they may be placed one over the other in the cell, no porous partition being required to keep them from mixing.

2647. The **Partz** cell, one form of which is illustrated in Fig. 1041, is an example. This cell is a bichromate cell (see Art. **2643**), which uses a solution of sodium chloride, or of magnesium sulphate, as an electrolyte, surrounding the zinc *Z*, and a bichromate solution as a depolarizer, surrounding the carbon cathode *C*. The depolarizer, having a higher specific gravity than the electrolyte, remains at the bottom of the jar, and the two liquids are kept separate. As the depolarizer is weakened by use, it is from time to time strengthened by the introduction of crystals in the glass tube *T*, which is suspended in the cell, having a small opening below the normal level of the bichromate solution. The crystals used are what the manufacturers call sulphochromic salt, which is formed by the action of sulphuric acid on some bichromate solution, and when dissolved in water gives the same results as the electropoion fluid (Art. **2644**).

With the cell shown, which employs a 6-in. × 8-in. jar, the internal resistance is about 1 ohm with a solution of magnesium sulphate, and about .5 ohm with a solution of sodium chloride, the E. M. F. being the same, 1.9 to 2 volts, in either case. This cell is good for either open or closed circuit work, as the depolarization is very complete; at the same time, the local action on open circuit is almost imperceptible.

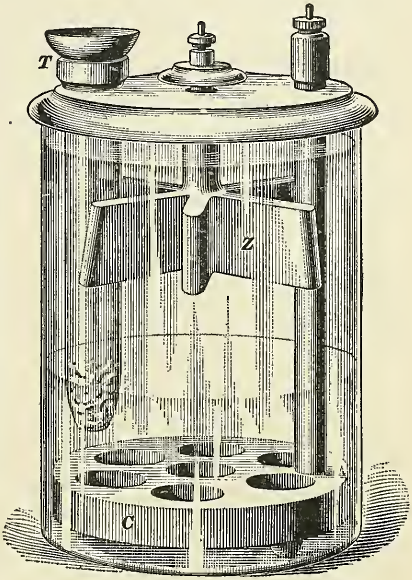


FIG. 1041.

The chrome alum solution which forms, being heavier than the bichromate solution, descends to the lower part of the cell, so that the crystals form beneath the carbon plate, which is slightly raised from the bottom of the jar; consequently, the formation of these crystals does not appreciably increase the internal resistance of the cell.

2648. Another form of gravity bichromate, known as the **Kousmine** cell, has its liquids arranged in the reverse order to the above. The electrolyte is sulphuric acid diluted to about 1.15 sp. gr. or less, and surrounds the zinc anode at the bottom of the jar. The depolarizer is a very weak solution of *potassium bichromate*, which floats on the sulphuric acid, being much lighter, and surrounds the carbon cathodes at the top of the jar. The heavy solution of chrome alum falls to the bottom, as in the Partz cell. The E. M. F. and actions of this cell are the same as in other bichromate cells, but its life is not long, the bichromate solution being soon exhausted by use.

2649. It can be readily seen that in cells of this class, consisting of the anode, exciting liquid, porous partition, depolarizing liquid, and cathode, a great number of different styles of cells may be constructed, by varying any of the four principal constituents, and a great many such variations have been made or suggested.

As pointed out in Arts. **2599** and **2600**, zinc is really the best and cheapest material for the anode; consequently, substituting other metals has not usually benefited the cell, except in special cases. The effect on the E. M. F. of such substitution may be readily found from Table 92, as before.

Great varieties of solutions have been used as electrolytes or depolarizing liquids; some with good results, and others without apparent reason, except to make a new cell.

2650. **M. D'Arsonval,** a French physicist, has made a series of cells, in which, by the action of the two liquids upon each other at their junction in the porous cup, an insoluble but conducting body is deposited in the pores of the porous cup, which prevents the gradual mixing of the liquids that usually takes place. For example, one of these cells is made up as follows: Zinc, solution of sodium hydrate (caustic soda), porous cup, ferric chloride, and carbon. The E. M. F. of this cell is about 2.7 volts; the action of the hydrogen on the ferric chloride reduces it to ferrous chloride and hydrochloric acid; at the same time ferric hydrate (which is insoluble, but a conductor) is formed in the pores of the porous cup.

2651. Various chloride salts have been used as depolarizers in cells of this class, the action being usually the reduction of the chloride to one containing a greater proportion of the metallic element. or else the entire reduction of the chloride, depositing the metallic element on the cathode; in either case the action of the hydrogen on the free chlorine forms hydrochloric acid.

2652. Many of the nitrate and sulphate salts have also been used as depolarizing liquids, and with a variety of electrolytes, generally acid; but the principal type of this class of cell, other than the Bunsen and the bichromate, is the type which employs as an electrolyte a salt of the metal of the anode, and as a depolarizer a salt of the metal of the cathode. The depolarizer is usually a salt formed by the same acid that formed the electrolyte salt; that is, if the electrolyte be a sulphate, the depolarizer is also a sulphate, etc. In this case the action is as follows: The passage of the current decomposes both liquids, and the hydrogen from the decomposed water unites with the non-metallic elements of the decomposed liquids, forming the acid from which the salt was formed, the metallic element of the depolarizer being deposited on the cathode; this acid attacks the anode, reforming the salt of which the electrolyte is composed. The electrolyte, therefore, is continually added to, while the depolarizer is continually reduced.

2653. Neglecting the intermediate reactions, which generally do not affect the E. M. F., it is evident that the E. M. F. of this type of cell is due to the energy given up by the formation of the salt of which the electrolyte is composed, less the energy required to decompose the salt of which the depolarizer is composed. Now, whatever may be the actual energy of the formation of the various salts, the *difference between the energies of formation of the same salts of any two metals is the same*, whatever the particular salt may be; for example, the *difference between* the heat of formation of zinc sulphate and that of copper sulphate is the same as the difference between the heats of formation of zinc nitrate and copper nitrate.

2654. It naturally follows, that with given metals for the anode and cathode, the E. M. F. should be the same, whatever salt of the two metals be used as electrolyte and depolarizer, respectively. This is borne out in practice, as experiments have shown the E. M. F. under these circumstances to be practically the same.

In order, then, to obtain a high E. M. F., it is necessary to use as an anode a metal whose salts have a high heat of formation, and as a cathode a metal whose salts have a low heat of formation, just as in the other classes of cells.

For commercial use, the same considerations apply as to the other classes; that is, the materials used in the cell must be easily and cheaply obtained, even if they do not result in the highest possible E. M. F. The cells which best realize this condition are the **Daniell** cell and its derivatives.

2655. The Daniell cell uses for the anode, zinc; for the electrolyte, a solution of (usually) zinc sulphate, $ZnSO_4$; for the cathode, copper; and for the depolarizer, a solution of copper sulphate, $CuSO_4$. Sometimes, in setting up the cell, dilute sulphuric acid is used instead of the zinc sulphate, but this soon forms a solution of zinc sulphate; hence, the result is the same as if the zinc sulphate were used originally. The E. M. F. of the Daniell cell is given several values by different investigators, ranging from 1.059 to 1.079 volts. The London Post Office uses this cell as a standard, and calls its E. M. F. 1.07 volts.

The original form of the Daniell cell consisted of a glass jar, into which the zinc, in the form of a cylinder, was placed. Inside the zinc was a porous cup containing the cathode, a strip of sheet copper. The porous cup was filled with the $CuSO_4$ solution and the outer jar with the $ZnSO_4$ solution.

2656. To prevent the gradual weakening of the depolarizer, it is usual to put a considerable amount of copper sulphate crystals (commonly known as *blue vitriol*) into the porous cup. As the liquid weakens, the crystals are gradually dissolved. Several modifications of the form of the original Daniell cell are in use, many of them designed to keep up the supply of copper sulphate as it is weakened.

2657. One such design, known as the **globe** or **balloon** cell, is shown in Fig. 1042, where Z is the zinc anode, P the porous cup, in which is the copper cathode C. To keep up the strength of the depolarizer, a glass globe G is filled with crystals of copper sulphate S and a little water, in which the copper sulphate gradually dissolves ; the solution, being heavier than the water, falls to the bottom of the neck of the globe and replenishes the solution in the porous cup. The neck of the globe extends down into the porous cup below the level of the liquid, so that the water may be retained in the globe. The globe rests on a ring of some soft material R, making a comparatively tight joint between the globe and jar, which prevents evaporation to a considerable extent. As ordinarily constructed, the globe holds about two pounds of copper sulphate crystals, which will usually last about six months. A cell similar to the above is used extensively for telegraph purposes in Russia.

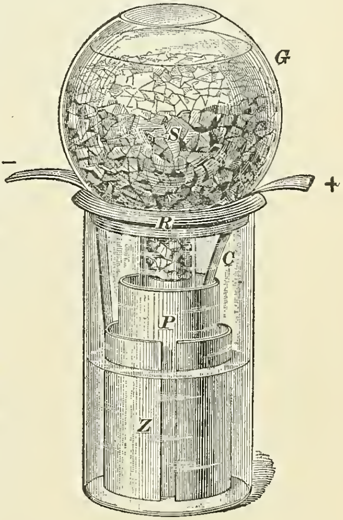


FIG. 1042.

2658. The specific gravity, at ordinary temperature, of a saturated solution of $ZnSO_4$ is about 1.44, while that of a saturated solution of $CuSO_4$ is about 1.20; hence, if saturated solutions of these salts are used, the zinc sulphate solution will be considerably heavier than the other; it has been found, however, that the best results are obtained from a saturated solution of copper sulphate, used with a solution of zinc sulphate diluted to a specific gravity of about 1.10. The considerable difference in weight between the two solutions has led to their arrangement, one over the other, in the cell, the heavier copper sulphate being at the bottom.

2659. In the **Hussey** and the **Gethin** cells a porous partition is used to separate the two liquids, in the form of a porous cup, located in the upper part of the jar. This cup holds the zinc and the electrolyte; beneath it is the copper, made in the form of a cross of sheet copper, which is surrounded by crystals of copper sulphate.

2660. Since the proportion of the two liquids in the jar varies from time to time, the porous partition does not always mark the point of separation of the two liquids, and it increases the internal resistance of the cell; consequently, the batteries of this type that are more generally used are those which do not use any porous partition at all, depending on the difference in the specific gravities of the two liquids to keep them apart.

Such cells are called **gravity cells,** or *gravity Daniell cells*, and are very extensively used for telegraph and fire-alarm work in this country.

2661. As long as a current is flowing through the cell, the chemical action keeps the boundary-line of the two liquids sharply defined; but when the current ceases to flow the solutions gradually intermix, and the *copper sulphate*, coming in contact with the zinc anode, sets up local actions, which cause a deposit of copper on the zinc, and a consumption of the zinc itself. To prevent this action, these cells should be used only on a circuit which is closed practically all the time, which is the case with telegraph and fire-alarm lines.

2662. Practically the first cell of this type to be used was the **Callaud** cell, illustrated in Fig. 1043. In this cell the zinc *Z* is in the form of a cylinder, suspended by hooks from the edge of the jar. The copper *C* is a flat strip bent into a circle, which rests on the bottom of the jar. Connection is made between it and the external circuit by means of a wire *W*, which is insulated with some rubber compound where it passes through the liquids. The position of the two liquids is shown in the illustration, the zinc sulphate ($ZnSO_4$) being at the top, as stated. This form of cell has been modified quite largely, it being now the practice to use large cast zincs instead of the cylinder of sheet zinc used in the Callaud form, which allows of a longer life for each cell.

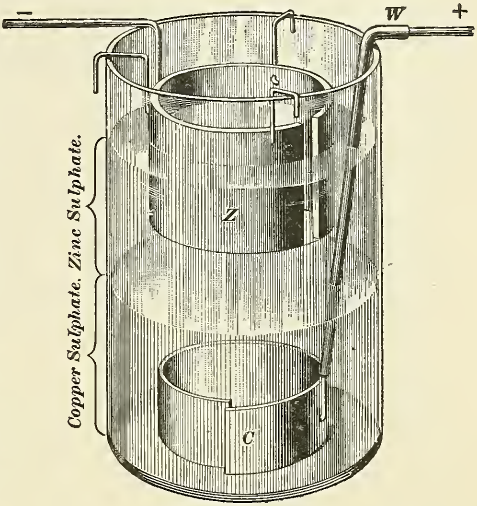


FIG. 1043.

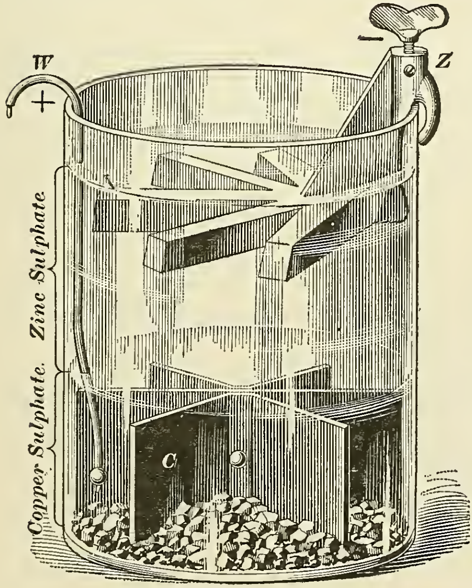


FIG. 1044.

2663. The form of gravity Daniell cell most used in this country is the familiar **crowfoot** cell, illustrated in Fig. 1044, where *Z* is the zinc, from the shape of which the cell gets its name; *C* is the copper, which is connected to the external circuit by the wire *W*, which is insulated where it passes through the liquid. When the cell is set up the copper cathode is surrounded with copper sulphate crystals. The standard form of this cell is of the following dimensions:

Jar, 6 inches diameter, 8 inches high. Copper, made from three pieces of thin sheet copper 2 inches wide and 6 inches long riveted together in the middle; the outside pieces are then spread out, making the copper of a six-pointed star shape. To the middle strip is riveted a piece of No. 16 insulated copper wire. (See Fig. 1044.)

The zinc is of the shape shown in the illustration, and weighs 3 lb. About 2 pounds of sulphate of copper crystals are required to charge the cell.

2664. The usual practice in charging is to set up the elements in the cell, put in the copper sulphate, and fill up with clean water until the zinc is covered; the cell is then allowed to stand for about 24 hours. By the action of the zinc on the copper sulphate solution, zinc sulphate is soon formed around the zinc, and the cell is ready for use.

If desired for immediate use, a solution of zinc sulphate may be prepared and poured into the jar with the copper sulphate solution; in this case the zinc should not be placed in position until the two liquids have separated, which will be indicated by the upper part of the liquid becoming nearly colorless, while the lower part is of a deep blue color.

2665. The average internal resistance of a crowfoot cell of this size is about 3 ohms, and its E. M. F. is the same as the other forms of Daniell cell, 1.07 volts.

2666. The maintenance of this type of cell is simple, it only being necessary to renew the supply of copper sulphate crystals when the solution becomes weak, which is indicated by the fall of the blue-colored liquid below the top of the copper cathode; besides this, the density of the zinc sulphate solution should be occasionally measured with a hydrometer, and if too dense (above about 1.15 sp. gr.) a part should be removed and replaced by water.

2667. With the crowfoot form of zinc there is considerable waste, due to the size of the "stub" which is left when the zinc has been consumed so that it can not be used. Several forms of zincs have been designed to prevent this waste as far as possible.

2668. One form, used by the Baltimore (Md.) Fire Department, is cast into a ring with upwardly projecting lugs, which have shoulders upon them, by which the zinc is supported by the edge of the battery jar. This form of zinc is illustrated in Fig. 1045. The ring itself being entirely below the level of the liquid in the cell, it can be almost entirely consumed.

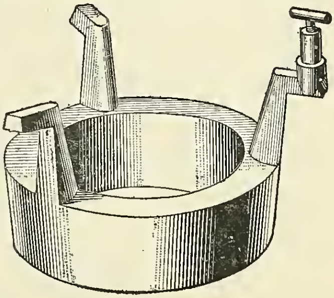


FIG. 1045.

2669. Fig. 1046 illustrates another similar form, known as the *pinnacle* zinc, from the fact that it is supported on a vertical rod of insulating material, which is fastened at the lower end to the copper. This rod projects up through the liquids and enters the cavity in the center of the brass supporting piece B, which is fastened to the zinc Z by the screws S, S. The complete cell is shown in Fig. 1047,

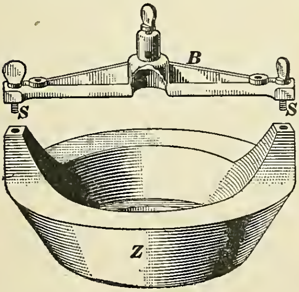


FIG. 1046.

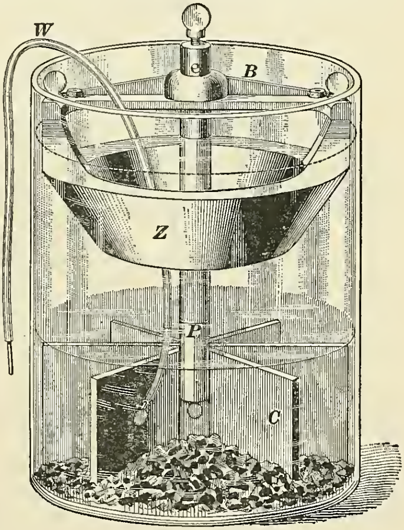


FIG. 1047.

Z being the zinc, *C* the copper, and *P* the rod of insulating material which supports the zinc by means of the supporting piece *B*.

2670. Another form of zinc in which there is no waste whatever is the D'Infreville *wasteless* zinc, illustrated in Fig. 1048. This zinc is cast with a conical lug *C* on the top, and a corresponding cavity in the under side of the zinc (see Fig. 1049). When the zinc is nearly consumed, it is removed from the support *B*, and the lug *C* inserted in the cavity of a new zinc, which is then put in place in the support *B*. The old zinc is then underneath, and is entirely consumed. Fig. 1049 shows a cross-section of this form of zinc, showing a new zinc *A*, a partly consumed zinc *B*, and the stub of a third *C*.

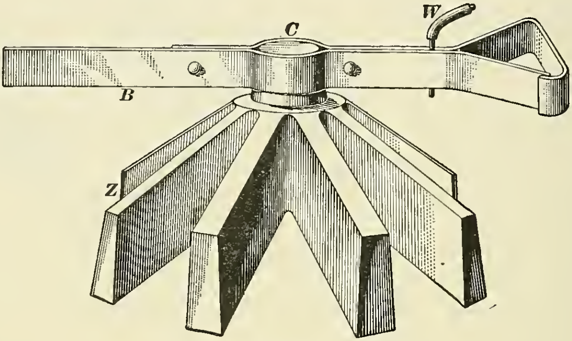


FIG. 1048.

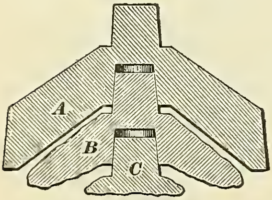


FIG. 1049.

The support *B* (Fig. 1048) also serves as a connector, the end of the connecting wire being sprung in between the two brass strips of which the connector is made, as shown at *W*.

2671. The Daniell cell, in various forms, has been used as a standard cell in laboratory work and for testing purposes. It is well adapted to such work, if too great a degree of accuracy is not required, as the E. M. F. is practically unaffected by moderate changes in temperature or in the density of either solution used, or by the length of time the cell is in operation. For ordinary work the E. M. F. of such a standard cell may be taken at the value given; that is, 1.07 volts. (See Art. **2655.**)

CELLS WITH A SOLID DEPOLARIZER.

2672. The depolarizers which are used in this class of cell are generally substances containing a large proportion of oxygen, with which the free hydrogen unites, forming water; the balance of the depolarizer is sometimes dissolved in this water, but more often remains at the cathode in a solid form, the water merely serving to dilute the electrolyte. In the first case the solution formed usually acts to keep up the strength of the electrolyte. (See Art. **2597.**)

2673. Some few of the non-metallic elements which exist in the solid state will unite directly with hydrogen, and might be used as depolarizing cathodes; such a substance is the metalloid *tellurium*. Such elements are rare and are not used in commercial forms of cells.

2674. Among the most widely used depolarizers are the oxides of manganese, copper, and lead, and the chlorides of some of the metals.

The several sulphates of mercury also have a large proportion of oxygen, and are used for this purpose.

2675. The **Leclanche** cell is a well-known and widely used cell of this type. Its positive element (negative electrode) is zinc, usually in the form of a rod; the electrolyte is a saturated solution of ammonium chloride, NH_4Cl (sal ammoniac), and the negative element is carbon, surrounded by manganic oxide, MnO_2 (black oxide, or peroxide, of manganese), which is the depolarizer. This being in the form of a coarse powder, it is usually contained in a porous cup, which allows free access of the electrolyte to the depolarizer and negative element.

Fragments of crushed coke (or carbon in other forms) are often mixed with the manganic oxide to decrease the resistance of the contents of the porous cup.

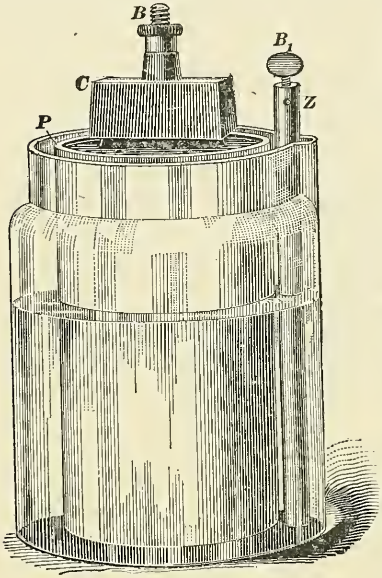


FIG. 1050.

Fig. 1050 shows the usual form of this type of cell. The porous cup *P* contains the manganic oxide and the carbon electrode, which projects from the top of the cup, and to which a binding-post *B* is attached.

The glass jar is circular, with a contracted top, in which a slight recess is formed to contain the zinc *Z*. The top of the zinc is provided with a binding-screw B_1, which serves as the negative terminal of the cell, *B* being the positive.

The top of the jar is coated with paraffin to prevent the crystals of sal ammoniac "creeping" over the top of the jar as the liquid evaporates.

2676. The cell illustrated in Fig. 1050 is of the following dimensions:

Jar,	$4\frac{1}{2}$ in. diameter, 6 in. high.
Zinc,	$\frac{3}{8}$ in. diameter, $6\frac{1}{2}$ in. high.
Porous cup,	3 in. diameter, $5\frac{1}{2}$ in. high.
Carbon,	6 in. × $1\frac{3}{4}$ in. × $\frac{5}{16}$ in., about.

The weight of the zinc rod is about 3 ounces, about two-thirds of which is below the level of the liquid. There are about 16 ounces of peroxide in the porous cup, and it requires nearly 6 ounces of ammonium chloride to make sufficient solution for this size of cell.

For each ounce of zinc consumed in the cell, 2 ounces of manganic oxide and 2 ounces of ammonium chloride must also be consumed; so, from the amount of these materials contained in the cell, it follows that there is enough peroxide in the porous cup to last while four zincs are being consumed, while the ammonium chloride will not last longer than $1\frac{1}{2}$ zincs. As the zincs are usually replaced when eaten away to about $\frac{1}{8}$ in. or $\frac{1}{16}$ in. diameter, the solution need not be replaced until two zincs have been consumed, and the contents of the porous cup will last as long as five or six zincs. The consumption of zinc in the Leclanche cell is about 23 ampere-hours per ounce of zinc, and as about $1\frac{3}{4}$ ounces of each zinc rod may be consumed, the life of each zinc is then about 40 ampere-hours. The E. M. F. of this type of cell is about 1.48 volts, and its internal resistance about 4 ohms.

2677. It is usual to seal the carbon and depolarizer into the porous cup by some compound, such as sealing-wax, leaving small tubes or holes, by which whatever gas not absorbed by the depolarizer may escape. This sealing necessitates the entire renewal of the porous cup, with contents, when the depolarizer is exhausted; to obviate this expense, some makers use a carbon porous cup and place the zinc inside, at the center, the space between the zinc and carbon being filled with peroxide.

2678. A form of Leclanche cell, made by the Law Battery Co., also replaces the clay porous cup by one made of carbon, but in this case the zinc is outside the carbon, as in the regular form. The carbon cup is made with a screw cover, also of carbon, which renders the replacing of the depolarizer a simple matter. This construction reduces the cost of maintenance of the cell, but increases the first cost.

2679. Another widely used form of Leclanche cell is the **Gonda** Leclanche, which uses no porous cup whatever; the manganic oxide is mixed with granulated carbon and some gummy substance, and compressed into cakes under great pressure. These cakes are attached to the sides of the carbon plate, and act in the same manner as the depolarizer in the regular form.

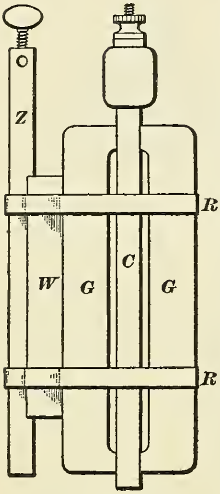


Fig. 1051.

Fig. 1051 shows the construction of the elements of such a cell. The two cakes of depolarizer (called *gondas*) *G*, *G* are clamped one on each side of the carbon plate *C* by the soft rubber bands *R*, *R*, which also serve to hold the zinc rod *Z* in place. The zinc lies in a groove in a block of wood or clay *W*, which serves to keep the zinc away from the gondas. This block is sometimes done away with by supporting both zinc and carbon from a plate of insulating material, which also acts as a cover to the jar. In still other forms, the depolarizer is molded into a cylinder, in the center of which the zinc is supported.

A second zinc electrode is sometimes used in this latter form, consisting of a cylinder of sheet zinc encircling the cylindrical gonda, a common terminal being connected to both zincs.

The liquids and action of the gonda form are the same as in the regular Leclanche cell.

2680. Commercial sal ammoniac often contains a considerable amount of impurities, in the shape of other salts, which materially reduce the life of the electrolyte; not sufficiently, however, to warrant the cost of using the chemically pure salt, as prepared by chemists.

2681. Ammonium chloride has been found to be the only salt which works well with manganic oxide as a depolarizer, so the many other forms of cell that have been constructed, using this depolarizer, differ materially from the Leclanche type only in the mechanical arrangement of the parts.

2682. The principal chemical actions in this type of cell are the formation of zinc chloride and ammonia, and the reduction of the amount of oxygen combined with the manganese. Besides these, there are other more complicated reactions which occur, but which do not affect the E. M. F. of the cell materially.

2683. Another solid depolarizer which is used in important commercial cells is cupric oxide, CuO. The **Lalande and Chaperon** cell uses an iron or copper negative element surrounded with a layer of cupric oxide. The positive element is zinc, the electrolyte a solution of *potassium hydrate* (caustic potash). On closing the external circuit, the potassium hydrate solution attacks the zinc, forming a compound oxide of potassium and zinc, known as *potassium zincate*, and liberating hydrogen, which combines with the oxygen of the cupric oxide, forming water, and depositing metallic copper on the cathode.

If the surface of a solution of caustic potash is exposed to the air, it will gradually form *potassium carbonate;* to prevent this action, cells of this type are either entirely enclosed or the surface of the liquid is covered with a thin layer of heavy oil.

Fig. 1052 shows one form of **Lalande and Chaperon** cell, in which the iron vessel *V* forms the negative elements, the positive terminal being a lug *A* cast on the side of the vessel. The *cupric* oxide *B* is in a layer at the bottom of the vessel. The zinc *D* is suspended from a rod *K*, which passes through a rubber stopper *G*, terminating in a binding-post *F*. The rubber stopper is provided with a valve *H*, which allows such gases as are evolved to escape.

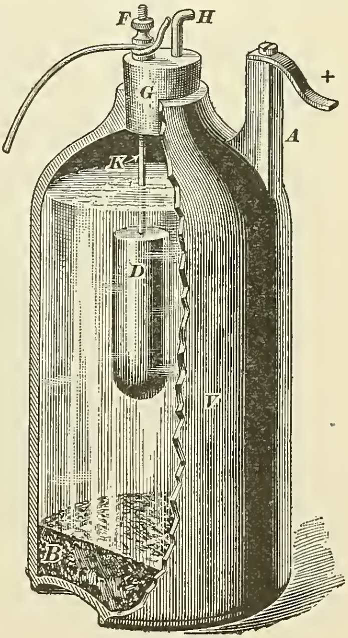


FIG. 1052.

Several other forms, of greater or less capacity, are manufactured. The E. M. F. of this type of cell is about .7 volt, and its internal resistance is usually low.

2684. The **Edison-Lalande** cell is a modification of the Lalande-Chaperon. The cupric oxide is molded under pressure into plates of the requisite size, being first mixed with magnesic chloride, which, when the molded plates are heated, serves to bind the mass together. These plates are held in copper frames, which enclose the edges of the plates. The positive element in this cell is zinc, and the electrolyte a solution of potassium hydrate, as in the Lalande-Chaperon cell. Two plates of zinc are used in most of the forms of this cell, one on each side of the cupric oxide plate.

A form of this cell is shown in Fig. 1053, which represents a 150-ampere-hour cell. The cupric oxide plate *C* is suspended in a copper frame *F*, *F* between the two zinc plates *Z*, *Z*, which are hung from each side of a lug on the porcelain cover of the jar. The sides of the copper frame of the oxide plate are carried up through the cover supporting the plate, and form terminals *B*, *B*, either of which may be used as the positive terminal of the cell. The copper frame is protected from the action of the liquid where it passes up through by tubes of insulating material *T*, *T*. A binding-post B_1, on the bolt which supports the two zinc plates, serves as the negative terminal.

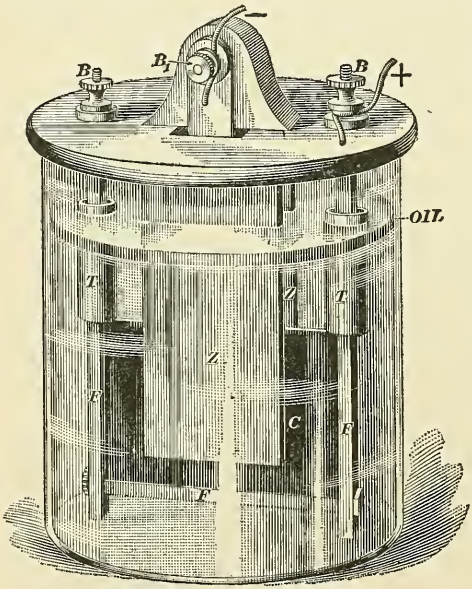


Fig. 1053.

A heavy paraffin oil is used in this cell to prevent the action of the air on the solution; the oil layer is represented in Fig. 1053.

The cell shown is 5¼ in. × 8¼ in., outside dimensions, and will give a current of 3 amperes at a potential of about .7 volt for 50 hours, which is equivalent to about 100 watt-hours, with one "charge" of zinc, caustic potash, and oxide.

The internal resistance of the above cell is about .07 ohm; the weight of the oxide plate is about ¼ pound. This type of cell is made in various sizes, ranging from a 15 ampere-hour cell for telephone and similar work, to 900 ampere-hour cells for running lamps, small motors, etc.

2685. There are several oxides of lead which have been used as depolarizers in single liquid cells: plumbic oxide, PbO, known as **litharge,** which is in the form of a yellowish powder; **peroxide of lead,** PbO_2, and a combination of the oxide and the peroxide, Pb_3O_4, known as **minium,** or *red lead*, which is a brilliant red powder.

2686. As seen from its formula, the peroxide contains the most oxygen, and is rather the best depolarizer; for example, in the zinc, dilute sulphuric acid, and carbon cell, surrounding the carbon with lead peroxide increases the E. M. F. to 2.2 volts; the action of the sulphuric acid on the peroxide, however, forms a small quantity of lead sulphate, which is insoluble, and increases the internal resistance of the cell somewhat.

Lead peroxide is extensively used in accumulators (storage-batteries) as a depolarizer.

2687. It is also used in a cell (made in Europe) which is interesting from its use of identical electrodes; that is, both anode and cathode are of carbon, arranged as follows: The cathode is a cylindrical rod of carbon surrounded with lead peroxide, which is kept in place by a canvas bag. The anode is a perforated carbon cylinder, made to slip over the cathode and its surrounding canvas. The whole is then put in a glass jar and surrounded by fragments of crushed coke; the jar is then half filled with a strong solution of sodium chloride. The lead peroxide is reduced to lead by the action of the hydrogen; the oxygen (due to the decomposition of the water) combines with the carbon anode. This process goes on slowly, so that if much current be drawn from the cell it will polarize by the formation of the oxygen on the surface of the anode.

If used for furnishing feeble currents, this cell will last a long time; its E. M. F. is about .6 or .7 volt.

2688. All cells using the above-mentioned solid depolarizers may be regenerated by passing a current from some other source through them in the opposite direction to that of their own current; the effect of such a current is the decomposition of the various substances formed by the original action of the cell and their recomposition into the original substances of which the cell was composed. If the mechanical construction of the cells is such that these substances return to their original position in the cell, they will again act as a voltaic couple from which a current may be obtained.

2689. This constitutes a **storage,** or **secondary battery,** or **accumulator.**

It is evident that such a cell is nothing but a primary voltaic cell, which, when exhausted, may be restored by the passage through it of a current from an external source; there is no real *storage* of electricity, so the name *storage-battery* is hardly correct; the last name, accumulator, is more appropriate to the action of such a cell.

Accumulators will be treated of more fully later.

2690. The principal chlorides used as depolarizing agents are the chlorides of mercury and of silver.

If the carbon of a zinc, ammonium chloride, and carbon cell be placed in a porous cup and surrounded with a paste of mercurous chloride, the chemical action is as follows: The ammonium chloride attacks the zinc, forming zinc chloride, and freeing ammonia and hydrogen, which attack the mercurous chloride and reform ammonium chloride, leaving free mercury at the negative pole.

The ammonium chloride solution is thus kept up at its full strength until the mercurous chloride is entirely exhausted, and the hydrogen is recombined as fast as formed. Such a cell has an E. M. F. of 1.45 volts, which is maintained as long as the depolarizer lasts, if excessive currents are not used.

2691. The *chloride of silver* is used in a similar manner. Cells employing this depolarizer use as a negative element a silver wire or plate coated with silver chloride. The positive element is usually zinc, and the electrolyte a dilute solution of one of the chloride salts.

With ammonium chloride, the E. M. F. is 1.03 volts; with zinc chloride, 1.02 volts, and with sodium chloride (common salt), 0.97 volt.

Silver chloride cells are quite extensively used in medical and testing work, on account of the constancy of their E. M. F. As in this work only very feeble currents are required, this type of cell is usually made small and of compact form, especially as the use of the silver element would make a large cell very expensive. The chemical action is of the same order as that of the mercurous chloride cell just described; that is, the chlorine part of the electrolyte is continually replaced from the depolarizer.

2692. The various sulphates of mercury which are used as depolarizers are the mercuric sulphate, the mercurous sulphate, and a sulphate containing a still higher percentage of mercury, known as *turbith* (or turpeth) *mineral.* Either sulphate may be used in the zinc, dilute sulphuric acid, and carbon cell without materially affecting the E. M. F., which, under these circumstances, is 1.3 to 1.5 volts.

These sulphates, being slightly soluble, are usually employed in the form of a paste, made with water or the exciting liquid. In ordinary work the mercury sulphates are not extensively used, not only on account of the high cost of these salts, but because of their poisonous qualities.

Still, these sulphates are excellent depolarizers, and are used in standard cells.

2693. The **Latimer-Clark** cell, in which the electrolyte is a paste of mercurous sulphate, formed with a solution of zinc sulphate, and the elements are zinc and pure mercury, is largely used as a standard in laboratory work, its E. M. F. being extremely constant, if proper precautions are taken in its construction. With chemically pure zinc and mercury, and a very carefully prepared electrolyte, the E. M. F. of a standard Clark cell at 15° C. is 1.434 volts. This E. M. F. varies very slightly with the temperature, the temperature coefficient being .077% per degree Centigrade, so that the E. M. F. at any temperature may be expressed by the following formula:

Let t = temperature in degrees Centigrade at which measurement is made;
E = electromotive force of cell;

then, $E = 1.434\,[1 - .00077\,(t - 15)]$ volts. **(472.)**

Thus, for example, formula **472** gives 1.4507 volts for this cell at the temperature of freezing water, 0° C., and 1.4155 volts at 32° C.

The greatest accuracy is demanded in the construction of this cell and in the determination of its temperature coefficient, because the cell is used as a standard in the measurement of unknown electromotive forces.

These cells are used as standards of E. M. F. only. They do not supply anything but very minute currents; so they are made of conveniently small size, and the most approved forms have a carbon or graphite resistance of about 10,000 ohms connected permanently in series with the cell, to prevent its accidental short-circuiting and consequent failure. These cells are very valuable on account of their constancy, but the element of temperature which enters in makes them somewhat difficult to use with great precision, as thermometers are, as a rule, inexact, their measurements depending largely on their physical condition.

2694. By substituting oxide of mercury for the sulphate, and using a weak (10%) solution of zinc sulphate as the electrolyte, the temperature coefficient is, it is claimed, only .01% per degree C. This is the **Gouy** standard cell, which has an E. M. F. of 1.39 volts at 12° C.

2695. A cell has been designed by Mr. Edward Weston, which, it is claimed, has no temperature coefficient whatever within reasonable limits. This cell uses for the positive element the metal cadmium in the form of an amalgam, and for the negative, sulphate of mercury mixed with pure mercury. The electrolyte is a solution of some cadmium salt, preferably the sulphate. The E. M. F. of this form of cell is 1.019 volts, nearly. The mechanical construction of this cell makes it well suited for general use as a standard cell, it being entirely sealed into and enclosed by a solid casing.

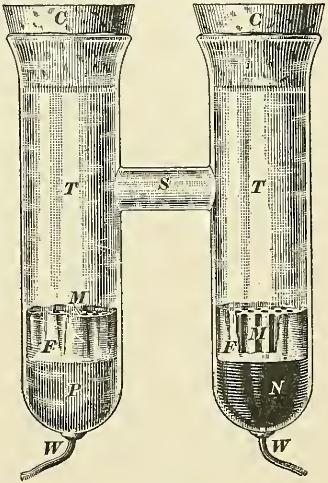


FIG. 1054.

The cell itself is similar to one of the usual forms of standard cells, consisting of two short glass tubes, open at the end, and connected together near the top by a short tube, as represented in Fig. 1054, in which *T*, *T* are the two tubes connected together by the short tube *S*.

In the bottom of the tubes are the elements *P* and *N*, to which connection is made by means of the wires *W*, *W*, which are sealed into the glass. These wires are led to binding-posts conveniently mounted on the case. The space above the elements is filled with the electrolyte, and the top of the tubes fitted with corks *C*, *C*, which are afterwards sealed in place, preferably with some resinous compound. The elements, being in a semi-liquid condition, are each kept in place by a piece of cloth *F*, with a perforated cork *M* laid over it. When this is forced down the tube to the surface of the element, the cloth keeps the element in place, and the cork holds the cloth, the perforations allowing free access of the liquid to the elements.

This is the general form in which most standard cells are made, although the various makers usually introduce slight changes in the mechanical construction.

2696. The **Baille and Fery** cell is also used as a standard cell. Its action is similar to those just described, the depolarizer being lead chloride, deposited in crystalline form on a lead cathode. The positive element is amalgamated zinc, and the electrolyte a solution of zinc chloride. The E. M. F. of this cell is .5 volt, and its temperature coefficient is low, being about .02% per degree C.

CELLS IN WHICH AN ELEMENTARY SUBSTANCE IS APPLIED TO THE CATHODE AS A DEPOLARIZER.

2697. This class of cells is not large, and has no extended commercial application, at least in this country. The principal elements used for depolarizers are those comprised in that group known as *halogens;* that is, chlorine, bromine, iodine, and fluorine. All these elements will combine with hydrogen directly, forming acids; of these the formation which liberates the greatest amount of energy is that of hydrogen and chlorine (HCl, hydrochloric acid); this element (chlorine), therefore, is most used in this class of cells, as it results in a high E. M. F. Chlorine being normally in the form of gas, it is sometimes generated by chemical action in suitable apparatus outside the cell, and allowed to pass through the cell or battery of cells near the cathodes, acting as a depolarizer and forming hydrochloric acid.

In other cases, the materials whose chemical reactions produce chlorine are brought together at the cathode, and the chloride produced acts as in the previous method.

As a rule, some or all of the other products of the chemical actions must be removed as fast as produced, to make room for a fresh supply of chemicals; in any case, as stated in Art. **2606,** the supply of depolarizing element is independent of the output of the cell, and must be regulated by hand. On the whole, cells of this class would be expensive to construct and maintain, and capable only of limited and special application.

2698. Strictly speaking, cells which have been included in the class given in Art. **2602,** whose carbon cathode is made of large surface and very porous, should be included in this class (see Art. **2617**); but their depolarization is very incomplete, and is rather accidental than a pronounced feature of the design; hence, they are placed in the former class.

DRY BATTERIES.

2699. This name is applied to cells, usually belonging to the class mentioned in Art. **2605,** in which the electrolyte is carried in the pores of some absorbent material, or combined with some gelatinous substance, so that the cell may be placed in any position without spilling the liquid.

2700. These cells are usually made in small sizes, with zinc and carbon elements, the zinc usually forming the outside of the cell, being made into a sort of cylindrical can, in the center of which is the carbon, surrounded by its depolarizing compound. The space between them is filled with some absorbent material, such as "mineral wool," asbestos, sawdust, blotting-paper, etc., and the whole is then soaked in the exciting liquid; or the exciting liquid is mixed with a hot solution of some gelatinous body, such as isinglass or "Irish moss," which mixture is poured into the cell; on cooling, it forms a soft jelly. The first method of preparation is most used.

2701. It is evident that only a comparatively small amount of liquid can come in contact with the zinc at one time, so the current output must be small; in fact, they are not adapted for anything but intermittent work. It is quite necessary, however, that they have a depolarizer, as otherwise they must be made open to allow the hydrogen to pass off, which would also allow the small amount of water they contain to evaporate; to prevent this latter action, these cells are sealed with some resinous compound.

2702. Owing to the presence of the absorbent material, the actual amount of liquid in these cells is comparatively small; consequently, they are soon exhausted. The sealing, being seldom perfect, often allows the water to evaporate, in which case the cell ceases to act; a cell of this description may often be made to work when apparently exhausted by drilling a small hole in the seal and injecting a little water.

2703. The materials used in dry batteries are usually kept secret by their manufacturers; they all, however, answer to the above description as to construction, and the best types employ the same materials as the Leclanche battery; that is, a zinc anode, ammonium chloride electrolyte, manganic oxide depolarizer, and carbon cathode.

In spite of its defects, this form of cell is extremely convenient on account of its portability, and in many cases can be profitably used.

2704. Silver chloride cells (see Art. **2691**) are made in a sealed form, and have all the advantages of a dry battery; the materials of the battery are enclosed in a capsule of semi-flexible material, which allows of the necessary contractions and expansions of the apparatus. In this form these cells are very convenient for testing and similar purposes.

THE APPLICATION OF PRIMARY BATTERIES.

2705. Although the cost of electricity generated by chemical action is greater than that generated by dynamo-electric machinery, there are many cases in which, from lack of motive power, or from the small amount of current required, primary batteries may be successfully used. In such cases, the cost of materials consumed in producing the electrical energy is entirely offset by the little attention required and the constancy of the source of supply; and in many cases where current is used intermittently, the cost of the current from a battery in which the materials are consumed only as the current is used would actually be less than the cost of the power for driving an equivalent dynamo all the time.

2706. The most important applications of primary batteries are to telegraph, telephone, and electric fire-alarm systems, where a constant but small current is required more or less continuously, although in large central offices, where the necessary current represents a considerable amount of energy, dynamos are replacing the batteries to some extent, on account of the saving in space. For this (telegraph, telephone, and fire-alarm) work, gravity batteries of the Daniell type are more commonly used, as they possess the advantages of long life and little attention.

2707. For telephone work, the currents used are very minute indeed, and almost any good cell in which there is no local action and in which the depolarization is complete (at least for small currents) will give good results. The E. M. F. required is 1.5 to 2 volts; consequently, in some cases single cells which give about this E. M. F. may be employed.

2708. In fire-alarm work a steady current of (usually) .04 ampere is used, the potential varying with the length of the circuit. Gravity Daniell cells are used largely in this work, the zincs being made large and heavy to insure long life and, consequently, little attention.

2709. Several systems of *block signaling* on lines of railroads also employ electrical devices of such a character that gravity Daniell cells are well suited for furnishing the current for their operation, and are quite extensively used for such purposes.

2710. There are a great number of devices which require the application of a current intermittently; some, such as electric bells and other signals, electric gas-lighting apparatus and the like, are used infrequently and irregularly, and the amount of electricity required is small, so that almost any voltaic cell will do, depolarizing or not, provided there is no local action to cause waste when not in use; therefore, cells with liquid depolarizers (see Art. **2604**) are not well adapted to this work, as in the long periods in which these cells are not called upon to furnish current the two liquids will mix and usually cause local action.

2711. The cells most used for this work are the various zinc-carbon batteries, both of the class described in Art. **2602**, with non-depolarizing electrolytes, and of the class described in Art. **2605**, with solid depolarizers; of the latter, some form of Leclanche cell usually gives the best results. In hotels and large buildings where the bell or signal service is practically continuous, depolarizing cells are required, such as large Leclanche cells, bichromates (with separate fluids), if of good modern construction, Edison-Lalande, and the like.

2712. Electric currents are much used in physicians' and surgeons' offices; currents of a few milliamperes in strength, but of from 75 to 100 volts E. M. F., are applied for curative purposes, while currents of 10 to 20 amperes in strength are used for heating cautery loops in surgical operations, requiring an E. M. F. of from 4 to 8 volts. Miniature incandescent lamps, usually operated from the battery which furnishes current for the cautery, are also used to examine the interior of the body.

2713. The first appliance obviously requires a large number of cells of a small size; for occasional use, and where first cost is not such an object as compactness, a battery of small silver chloride cells is very convenient, while for more frequent use, requiring larger cells, some cheaper form of depolarizing cell is used.

Obviously, if the cells selected have a high E. M. F. (say 2 volts), a less number will be required than if the cells are of a low E. M. F.; however, as in some instances the regulation of the current is obtained by switching in or out some of the cells, this regulation will be more uniform and gradual if the E. M. F. of each cell is low.

2714. For furnishing the larger currents for cautery work, large cells should be selected, those which are so arranged as to have a minimum internal resistance being best. As the use of porous cups in a cell increases the internal resistance largely, cells which employ them are not well suited for this work.

Bichromate cells are very convenient for this purpose, as their internal resistance is low and the E. M. F. high and steady. It is usually convenient to use the form of bichromate cell in which the elements are raised from the liquid when the cell is not in use, as the purpose for which the current is used involves personal and immediate attention to all parts of the apparatus.

2715. The most extensive application of cells of the Bunsen type is to electroplating and similar work, and cells of large size are made especially for this purpose.

Such work being usually carried on in establishments especially fitted up for the purpose, the various unpleasant features of the Bunsen cell, which make them objectionable for many purposes, may be readily provided for, and their high and constant E. M. F. utilized.

2716. The minor applications of primary batteries are almost innumerable. A study of the requirements of such cases will usually determine the best type of cell to use, but attention should also be paid to the mechanical construction of the cells selected, as on this point often depends their life and suitability for the work they are called upon to do.

The binding-posts should be firmly and substantially fixed to the elements, and should be thoroughly protected from possible contact with the electrolyte, as the resulting action will so corrode the joint between the two as to destroy the contact, besides possibly eating away the connecting wires and breaking the circuit.

Of the material of the positive element, as much as possible should be below the level of the liquid, as when that is consumed the balance must be thrown away, and this may represent a considerable loss.

Altogether, the cell should be substantial and compact, not liable to local action, and arranged so that its parts may be readily renewed with the least possible waste.

2717. In general, it must be remembered that the consumption of material in a primary cell (assuming no local action) is proportional to the output in ampere-hours; the *energy* output depends not only on the amount of materials consumed, but on the E. M. F. of the cell and its internal resistance, so that, other things being equal, the higher the E. M. F. of a cell and the lower its internal resistance, the greater its output for a given cost of materials.

2718. As stated, the most economical metal to use for the positive element is zinc, and the amount of zinc consumed in a cell may be readily determined from the output in ampere-hours and the chemical equivalent of zinc (again assuming no local action); but to find the total cost of the energy, to this must be added the cost of the depolarizer consumed, if any, and the cost of labor in renewing the materials and caring for the cells.

2719. The substances resulting from the chemical actions which take place often have a market value; usually, however, the expense of collecting or preparing such substances for sale will be greater than the price they will bring, so that in ordinary cases this should not be taken into account.

2720. It is evident that all the E. M. F. of a cell is not available to send a current through the external circuit, but that a part is expended in overcoming the internal resistance.

If the resistance of the external circuit is very great, this drop is of little importance; while if the external resistance is very small, the internal resistance practically determines the amount of current flowing.

2721. The various methods of connecting up the cells of a battery, in parallel, series, or parallel series, are given in Art. **2250.**

If several cells, all of the same size and kind, are connected in series, their total internal resistance will equal *the resistance of one cell multiplied by the number of cells*, and their total E. M. F. will equal *the E. M. F. of one cell multiplied by the number of cells;* if they are all connected in parallel, their total resistance will be equal to the *resistance of one cell divided by the number of cells*, while their total E. M. F. will be equal to that of a single cell. From this it follows that if the external resistance is very small, increasing the number of cells in series will not increase the current in the external circuit appreciably, as the resistance increases nearly as fast as the E. M. F.; while if the external resistance is great, increasing the number of cells in parallel will not appreciably increase the current flowing, as the total resistance is not much altered, while the E. M. F. remains the same.

2722. For a given external resistance and a battery of a given number of cells, the maximum current will flow when the cells are so grouped that their internal resistance just equals the external; so that, in installing a battery, the resistance of the circuit and of the cells should be ascertained, and the cells grouped accordingly. This may be proved, numerically, as follows: $C = \frac{E}{R}$. Let m cells be in series in l rows, or a total of $m \times l$ cells. Let E be the electromotive force and R the internal resistance of each cell, and r the resistance of the outside circuit. Substituting in above formula, $C = \frac{m\,E}{(m\,R \div l) + r}$ (a). Then, C is greatest when $\frac{m\,R}{l} + r$ is smallest; that is, when m and l are chosen such that $\frac{m}{l}R$, the total internal resistance.

equals or approximates to r. Let us assume $m \times l = 12$, $E = 2$, $r = 3$, $R = 2$. Substituting in formula (a), and taking the following values of l,

$$l = 1 \quad C = \frac{12 \times 2}{(\frac{12}{1} \times 2) + 3} = \frac{24}{24 + 3} = .9 \text{ ampere.}$$

$$\text{Total internal resistance} = \frac{12 \times 2}{1} = 24 \text{ ohms.}$$

$$l = 2. \quad C = \frac{6 \times 2}{(\frac{6}{2} \times 2) + 3} = \frac{12}{6 + 3} = 1.3 \text{ amperes.}$$

$$\text{Total internal resistance} = 6 \times 2 \div 2 = 6 \text{ ohms.}$$

$$l = 3. \quad C = \frac{4 \times 2}{(\frac{4}{3} \times 2) + 3} = \frac{8}{2\frac{2}{3} + 3} = 1.4 \text{ amperes.}$$

$$\text{Total internal resistance} = 4 \times 2 \div 3 = 2\tfrac{2}{3} \text{ ohms.}$$

$$l = 4. \quad C = \frac{3 \times 2}{(\frac{3}{4} \times 2) + 3} = \frac{6}{1\frac{1}{2} + 3} = 1.3 \text{ amperes.}$$

$$\text{Total internal resistance} = 3 \times 2 \div 4 = 1\tfrac{1}{2} \text{ ohms.}$$

It is thus seen that the largest current is obtained when the internal resistance approaches nearest to the value of the external.

Ordinarily, in telephone, telegraph, and fire-alarm work the external resistance is high, while for ringing bells, gas-lighting, and similar work the resistance is low; batteries for these purposes should be grouped accordingly.

2723. The internal resistance of a cell can not be measured in the same way as the resistance of a piece of wire, that is, by sending a measured current through it from some external source, measuring the drop in volts and calculating the result from Ohm's law; for the E. M. F. of the cell itself would either add to or subtract from (depending on the polarity of the current) the drop due to the current, and, hence, the calculated results would be at fault.

2724. A simple way to measure this internal resistance is to cause the cell itself to furnish a current through some known resistance. Then, by measuring the E. M. F. at the terminals of the cell with a voltmeter, when the current is flowing and on open-circuiting the cell, the difference between the two readings will show the drop in volts due to the flowing of this current against the internal resistance of the cell.

For example, if a cell gives an E. M. F. of 1.5 volts on open circuit, and on being connected to an external resistance of 2 ohms the E. M. F. at the terminals drop to 1.25 volts, the drop in the cell is obviously .25 volt. The current is $C = \frac{E}{R} = \frac{1.25}{2} = .625$ ampere; therefore, the internal resistance of the cell is $R = \frac{E}{C} = \frac{.25}{.625} = .4$ ohm.

ACCUMULATORS.

2725. A storage-battery, or, preferably, an accumulator, is an apparatus consisting of certain materials so arranged that when they have undergone chemical action, due to the influence of a current of electricity, the combination has acquired the properties of a voltaic cell, and is enabled to discharge into a closed circuit a current of electricity approximately the same as the original charging current.

Many forms of primary batteries may, when exhausted, be more or less regenerated by passing through them a current, from some external source, in the opposite direction to the current they themselves produce. It is customary, however, to consider as accumulators only those cells whose original construction is similar to an exhausted battery; that is, they can not be used as sources of electricity until they have been *charged* by passing a current through them.

2726. A great deal of confusion exists as to the use of the terms *positive* and *negative* in speaking of the plates of a secondary cell; for in charging the cell the current is in the reverse direction to that which flows when the cell is acting as a voltaic cell and discharging. It is customary, however, to speak of the plate at which the current enters the cell (while charging) as the *positive* plate. In fact, whether charging or discharging, his plate is at a higher potential than the other, which justifies the above use of the term, although with respect to the chemical actions in the cell the positive and the negative plates are reversed in the two operations.

2727. Accumulators may be divided into two general classes: (1) **lead accumulators,** and (2) **bimetallic accumulators.** The larger proportion of cells now in use are of the first class.

2728. Lead Accumulators.—The original lead accumulators, as made by Planté, consist of two plates of lead, usually rolled together in a spiral, and separated by strips of rubber or other suitable insulating material; these are placed in dilute (about 10%) sulphuric acid. On sending a current from some external source through this cell, the water becomes decomposed, and the oxygen combines with the positive plate, forming lead oxide or peroxide, while the hydrogen collects at the negative plate.

On disconnecting the source of the applied current, and completing the external circuit of the cell, the water again is decomposed, the oxygen uniting with the hydrogen collected at the negative plate, and also with the lead plate itself, and the hydrogen uniting with the oxygen of the oxide of lead at the positive plate, thus producing a current in the opposite direction to the applied current.

2729. Owing to the fact that the formation of the layer of oxide prevents further oxidation, the amount of chemical change due to the applied current is small, so the secondary current from the cell is of short duration; after this current has ceased, however, the surface of the positive plate is much increased, owing to the removal of the oxygen from the lead oxide, leaving the metallic lead in a spongy form. On again sending a current through the cell a further oxidation of this (positive) plate takes place, and by continuing this process, reversing the current each time it is sent through, both positive and negative plates become porous to a considerable depth, thus very much increasing the surface on which the oxidation can take place. This process might be carried on until the whole plate is reduced to spongy lead; in that case the plate would not hold together, so a sufficient amount of the original plate must be left for mechanical strength. After the plates are so *formed*, they are ready to be used as an accumulator.

2730. This forming process, however, is too long and expensive for commercial success, though it is considerably hastened by roughening the surface of the lead plates with nitric acid before commencing the process; it was soon superseded by the process invented by Faure, of coating the surface of the plates with some substance which by the first charging current is converted into lead peroxide on the positive plate and into spongy lead on the negative. This substance may be lead oxide (litharge), lead sulphate, minium (Pb_2O_3), lead peroxide, or mixtures of these substances.

2731. These substances are applied in various ways; one method is to make a paste of the substance (in this case usually minium), that for the negative plate being made with sulphuric acid, which changes the Pb_2O_3 into $PbSO_4$ (lead sulphate) and water, while that for the positive plate is made with water only. These pastes were originally applied directly to the surface of the plain lead plate; but as they proved to be only slightly adhesive, the plates were prepared by scratching or otherwise roughening the surface, which process has been gradually extended until the lead plates are now cast into *grids*, or latticework plates, in the spaces of which the paste is applied, or forced by hydraulic pressure. Some manufacturers do not use a paste of the active material, but employ the minium, litharge, or lead sulphate in the form of dry powder, forcing it into the grid under such enormous pressures that the powder is solidified.

2732. The grids are usually designed to hold the active material securely in position; to this end they are made with perforations which are not of the same area throughout the thickness of the plate, but wider or narrower in the center, so as to hold the filling of active material by the dovetailing action of their shape, as will be shown later.

2733. After the grids have been filled with active material, they are set up in pairs in suitable vessels, and surrounded by an electrolyte consisting of sulphuric acid diluted to about 1.17 sp. gr., which density corresponds to about 20% of acid in the liquid. A *charging current* is then sent through the cell from some external source; the action of this current decomposes the water, the oxygen of which further oxidizes the lead oxide (litharge or minium) to peroxide, at the positive plate, the hydrogen going to the negative plate, where it reduces the lead sulphate to spongy lead by uniting with the SO_4, forming sulphuric acid. Thus, the active material becomes lead peroxide in the positive plate and spongy lead in the negative. By many investigators this lead peroxide is thought to be *hydrated lead peroxide;* that is, it contains a certain amount of hydrogen and oxygen in excess of the normal peroxide, and is represented by the formula $H_2Pb_2O_5$. This, as well as many of the actions which occur in accumulators, is not clearly established as yet.

2734. Continuing the charging current, when all the active material is thus converted, produces no further effect, except to continue to decompose the water; the resulting gases pass off through the water, giving it a milky appearance.

This phenomenon is known as *gasing* or *boiling*, and is an indication that the cells are fully charged. Continuing the charging current beyond this point, that is, overcharging the cells, does no harm to the plates, but the energy represented by the current is wasted.

2735. On discontinuing the charging current at the gasing point, and completing the external circuit of the cell, a current will flow in the opposite direction to that of the charging current, the resulting chemical action being to reduce the lead peroxide to lead oxide at the positive plate, and the spongy lead to lead sulphate at the negative; a secondary action is the formation of a part of the lead oxide at the positive plate into lead sulphate. The sulphates thus formed are not all of the same proportions; one exists as red, another as yellow, and a third as white crystals, of which the white sulphate is best known, as it is formed when the cell is considerably discharged, and is extremely troublesome. This discharge may be continued until all chemical action ceases, and the E. M. F. consequently falls to zero; but this is not advisable, since, if the discharge is carried beyond a certain point, the red or yellow sulphates, probably by combination with the litharge (PbO), form the white insoluble sulphate, which has a higher proportion of lead than the others; this, being a non-conductor, materially increases the internal resistance of the cell, and when it is removed it usually carries some of the active material with it, as it is very adhesive.

2736. When the cells have been properly charged, the positive plate is of a brown or deep red color, while the negative is a slaty gray.

The presence of the insoluble sulphate is made apparent by the formation of a white coating or glaze over the plates, which are then said to be *sulphated.* If the cells are discharged and left to stand with the electrolyte in place, *sulphating* takes place rapidly.

2737. It will be noticed that sulphuric acid is formed during the charge, and decomposed during discharge; thus the proportions of it in the electrolyte, consequently the density of the electrolyte, vary with the state of charge of the cell; starting with a specific gravity of 1.17, when the cell is fully charged the specific gravity will be found to be about 1.22, indicating the presence of about 25% of sulphuric acid in the electrolyte.

2738. The chemical actions of charging or discharging do not take place simultaneously, as is shown by the variations in E. M. F. under different conditions of charge or discharge, nor are they probably the only actions which occur.

2739. The E. M. F. of this type of cell is approximately 2 volts, being 2.04 when slightly discharged, which gradually falls to 1.90 volts when nearly discharged. Beyond this point, further discharging causes the E. M. F. to fall more rapidly, the decrease after 1.8 volts being very rapid. (See Fig. 1055.)

2740. The rating of accumulators is usually based on their capacity when discharged to an E. M. F. of 1.8 volts; but in spite of this rating, the result of a long series of tests shows that in practice they should not be continuously discharged to below 1.9 volts, as below this point *sulphating* is very liable to occur, and, the nature of the chemical action being changed, it also leads to the distortion of the positive plate, which is known as **buckling.**

As the plates are located very close together in the cells to reduce the internal resistance, buckling is liable to cause the plates to touch, thus short-circuiting the cell.

2741. The cause of buckling seems to be the formation of sulphate in the plugs of active material which fill the spaces of the grids, thus causing the plugs to expand; lead having very little elasticity, the grid is forced out of shape. As usually constructed, the edges of the grid are heavier than the intermediate portion, so that the effect of the distortion is to bulge the plate in the center. If the plates are not discharged too far and too rapidly, the expansion of the active material is gradual, causing the grid to stretch evenly; this makes the plates "grow," or increase in area, sometimes as much as 10 per cent.

2742. The *quantity* of electricity which may be taken from a completely charged cell depends upon the amount (weight) of material altered by the chemical action, as in a primary cell; while the *rate* at which this material is altered, consequently the rate at which the electricity can be taken out (the rate of discharge in amperes), and, to a large extent, the amount of material altered, depends upon the surface of the active material exposed to the chemical action.

2743. Cells of this type are then rated at a certain number of *ampere-hours* capacity, depending on both the weight and the surface area of the active material in the cell, and a certain economical *discharge rate* is also recommended, depending on the surface of the plates exposed to the electrolyte.

If this discharge *rate* be continually exceeded, the chemical action goes on too rapidly, the white sulphate is formed in the active material of the positive plate, finally causing disintegration of the active material and buckling of the plates, even if the discharge is not carried beyond the point (1.9 volts E. M. F.) given above. With the ordinary construction, the normal discharge *rate* is about .0165 ampere per sq. in. of surface of positive plate, and the discharge *capacity* about 4.5 ampere-hours per pound of plate (both positive and negative plate included).

2744. Fig. 1055 shows the manner in which the E. M. F. of an accumulator falls as the discharge proceeds. In this case the cell was connected to a variable external resistance, such that about the normal discharge current, as advised by the manufacturers, was maintained throughout the test in the

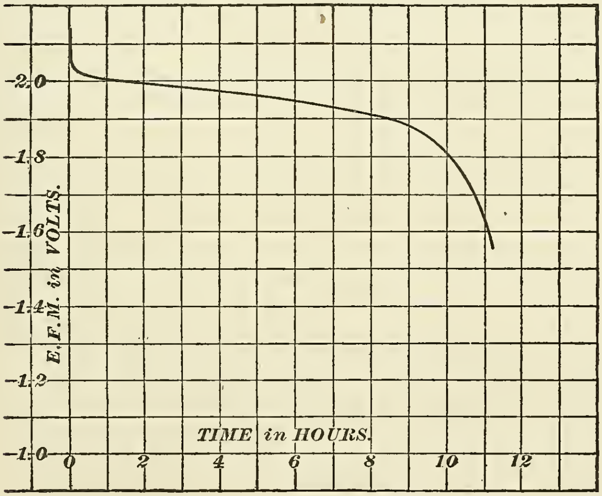


Fig. 1055.

external circuit. The oxidation of the slight layer of hydrogen left on the negative plate from the discharge causes the E. M. F. to be high at first, but as this is quickly disposed of, the E. M. F. falls in the first ten minutes or so to 2.04 volts; on continuing the discharge, the E. M. F. falls slowly and evenly until after about 8½ hours of discharging the E. M. F. falls to 1.9 volts. If the discharge is continued beyond this point, the nature of the chemical action changes somewhat, and the fall of E. M. F. becomes more rapid, at 10 hours being 1.8 volts, and at 11 hours being only 1.63 volts.

2745. This falling off of the E. M. F. is due to the weakening of the acid solution and to the gradual reduction of all the spongy lead on the one plate and the peroxide on the other to sulphate.

As this reduction can only go on at the points where the acid is in contact with the spongy lead or the peroxide, it is evident that the interior portions of the active material are affected much more slowly than the surface, as the acid penetrates the active material only at a comparatively slow rate.

On this account, discharging at slow rates allows the active material to be more uniformly and thoroughly reduced, thus giving a greater output.

This also accounts for the fact that on discontinuing the discharge at any point the E. M. F. will soon rise to practically its original value, 2.04 volts; for unless the cell is entirely discharged there is always some unconverted active material in the interior of the plate, which serves to give the original E. M. F. when reached by the acid. If the discharge is resumed, this acid is soon exhausted, and the E. M. F. rapidly falls to the value it had when the discharge was stopped.

2746. In the above case, the product of the amperes and the hours will give the output of the accumulator in ampere-hours; if the discharge *rate* had been greater, the output in ampere-hours would have been diminished, the discharge being continued until the E. M. F. falls to the same value in each case. Conversely, if the discharge rate had been lower, the output would have been increased.

For example, assume the limiting E. M. F. to be 1.9 volts. In a certain cell, with a discharge current of 30 amperes, the E. M. F. reaches its limit in 10 hours, giving an output of 300 ampere-hours.

If the discharge current were 40 amperes, the limiting E. M. F. would be reached in about 6½ hours, giving an output of only 260 ampere-hours; while if it were 20 amperes, the limiting E. M. F. would not be reached for about 17½ hours, giving an output of 350 ampere-hours.

For the sake of uniformity, the rating of the capacity of accumulators is made on the basis of a discharge current which will cause the E. M. F. to fall to 1.8 volts in 10 hours, although most manufacturers give tables showing the comparative capacity of the various sizes of cells at other rates of discharge.

2747. The *rate* of charge (charging current) for accumulators of this class should be about the same as the normal (10-hour) discharge rate, although much smaller currents, continued for a proportionately longer time, may be used.

2748. Although "storage-batteries" do not store *electricity*, they certainly do store energy by converting the *kinetic energy* of the electric current into *chemical potential energy*, which may be realized as kinetic energy again. The *efficiency* of the accumulator (or of any other means of storing or transforming energy) is the *output* divided by the *input*. This quotient is always less than 1, as the accumulator is not a *perfect* storer of energy; that is, there are certain losses in the transformation of kinetic electrical to potential chemical energy, and *vice versa*, besides the loss of the energy required to force the current through the cell, that is, the loss due to the resistance of the plates and electrolyte.

2749. The input and output of an accumulator may be expressed either in ampere-hours (the *quantity* of electricity) or in watts (the *rate of doing work* of the current). If secondary cells of this class be fully charged at normal rate, after a discharge to 1.8 volts, and then discharged to the same point, also at normal rate, the *ampere-hour efficiency* will be ordinarily from .87 to .93, or 87% to 93%. If charged and discharged to the same point at very slow rates, this efficiency may rise to 96% or 97%.

2750. The *watt efficiency* at normal rates of charge and discharge is lower, being from 65% to 80%, depending on the construction of the cell. In larger cells of modern construction, the watt efficiency is as high as 84%.

2751. The cause of the loss represented by the foregoing figures is, for the ampere-hour efficiency, due to the fact that the charging current must perform several chemical decompositions, of which the elements either do not recombine in the cell or, recombining, do not give up their potential energy in the form of electrical energy. This loss varies with the rate of charge and discharge, as indicated by the figures given, but for a given rate it is practically fixed, the mechanical arrangement of the cells having little effect upon it.

2752. The greater loss shown in the watt efficiency figures is due to the fact that the E. M. F. of charge is higher than that of discharge, due in part to the E. M. F. required to perform the wasteful chemical actions referred to above, but largely to the drop in volts caused by the passage of the current against the resistance of the plates and electrolyte. This drop *adds* to the E. M. F. required to perform the chemical decomposition in charging, and *subtracts* from the E. M. F. due to the chemical recompositions, and its amount depends more on the construction of the cell than does the loss represented by the ampere-hour efficiency, as it varies with the shape and size of the plates, their distance apart, their state of charge (on account of variations of the resistance of the electrolyte as the percentage of acid varies) and other conditions.

This loss due to the internal resistance in well-designed cells usually amounts to about 8%, at normal rates of charge and discharge; the loss is correspondingly less at low rates and more at high rates, being proportional to the *square* of the current flowing.

In a good modern cell exposing about 1,100 sq. in. of positive plate surface, the internal resistance is about .005 ohm when charged. Cells of greater capacity than the above (which is listed as 350 ampere-hours) would have a proportionately lower resistance.

2753. The above efficiency figures, as stated, are given for a discharge to 1.8 volts E. M. F., the usual manufacturers' rating; if the cells are not discharged to so great an extent, both ampere-hour and watt efficiencies are higher.

2754. The E. M. F. required to send a given charging current through a secondary cell varies with the state of charge of the cell. Fig. 1056 shows the E. M. F. required to charge the same cell that gave the discharge E. M. F. curve (Fig. 1055), being in this case charged at the same rate as previously discharged.

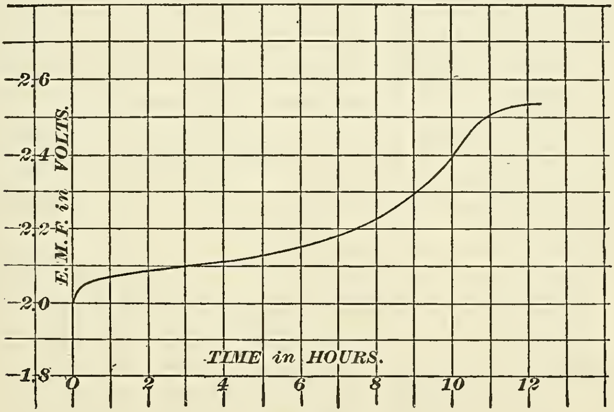


FIG. 1056.

This curve shows that the charging E. M. F., after a quick rise in the first few minutes to about 2.06 volts, gradually rises during the first 6 or 7 hours, after which the rise is more rapid, until after 11 hours of charging it becomes

2.5 volts; at this point gasing begins and the cell is practically charged. On continuing the charging current, the E. M. F. rises a little more, and then remains practically constant at about 2.55 volts; as the only action which now takes place is the decomposition of the electrolyte, giving off gas, further charging would only result in a waste of energy; although long-continued overcharging at a moderate rate will gradually remove any formations of white sulphate that may exist. (See Arts. **2734** and **2736.**)

2755. From this curve it appears that the cell became completely charged in practically 11 hours; as the discharge curve (Fig. 1055) shows that with the same number of amperes the discharge is complete (to 1.8 volts) in 10 hours, the *ampere-hour efficiency* of this cell is $\frac{10}{11}$, or 91%, practically.

2756. If an accumulator of this class is not discharged at an excessive rate nor to more than 1.9 volts E. M. F., the positive plates should last for about 1,200 or more discharges; while if discharged each time to below 1.8 volts, or at excessive rates, the life of the positive plate will not ordinarily be more than 400 or 500 discharges. The negative plates, with good care, will usually outlast four or five positive plates.

Some of the more modern cells of this class will show better results than the above, which, however, are good average figures.

2757. The usual construction of cells of this class is as follows:

The plates and electrolyte are contained in a vessel of approximately cubical form; this vessel is of glass, if the cells are not intended to be portable, the glass allowing the examination of the condition of the plates while the cell is in operation. If the cells are intended to be portable, the vessel is usually made of hard rubber or gutta-percha, or of wood lined with hard rubber or lead. Very large accumulators for central-station use are usually set up in lead-lined wooden tanks.

2758. The plates are usually approximately square, and from $\frac{1}{4}$ inch to $\frac{1}{2}$ inch thick, according to size. To get a large surface area without using single large plates, and to allow of one size of plate being used for cells of various capacities, each cell contains a number of positive and negative plates, arranged alternately side by side a short distance apart. The number of negative plates is always one more than the number of positive plates, so that *each side* of each positive plate has presented to it the surface of a negative. All the positive plates are connected together by a connecting strip, usually at one corner of the plate, and all the negatives are similarly connected. The arrangement of a typical accumulator cell is represented in Fig. 1057, where *N*, *N*, *N*, *N*, *N* are the negative plates and *P*, *P*, *P*, *P* the positive. From a corner of each plate a lug projects; the lugs on the negative plates are joined to a connecting strip, as represented at *T*, and the lugs on the positive plates are similarly joined to a connecting strip *T'*. The joints are made by a process called "burning," which consists in melting the lugs and strip together by a flame of hydrogen. This hydrogen flame absorbs the oxygen from the film of lead oxide with which the lead is usually covered, thus making a clean and solid joint. These connecting strips are extended beyond the limits of the cell, and serve to connect the various cells of the battery together, as shown at *C*, the connection being made by a brass bolt, which clamps the connecting strips together firmly.

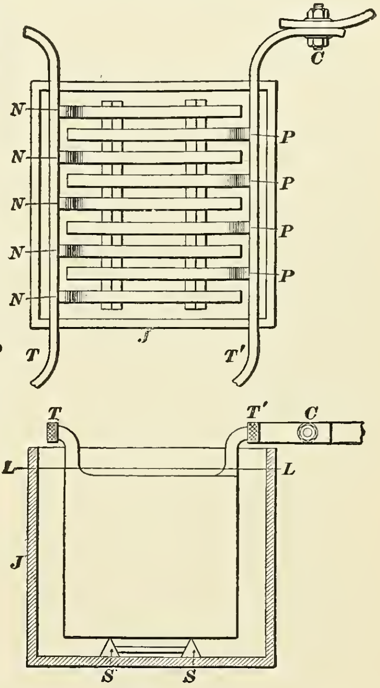


FIG. 1057.

2759. The plates are placed in the jar *J*, resting on a wooden support made from two strips of wood (usually boiled in paraffin) of triangular section *S*, *S*. These support the plates at such a height that any loosened particles of active material fall below the level of the bottom of the plates, thus preventing possible short-circuiting. When in position, the electrolyte is poured in until it reaches the line *L L*, thus covering the plates. To prevent the plates from touching each other, it is usually the practice to separate them by blocks or strips of insulating material, the exact arrangement varying with the different manufacturers.

2760. Owing to the expense of *forming* the plates by the Planté process, cells of the construction invented by Faure, known as "pasted plate" cells, have been very extensively used. Those principally used abroad are known as the *Faure-Sellon-Volkmar* cells, from the company owning the principal French patents.

2761. Sections of the grids principally used by this company are shown in Figs. 1058 and 1059. The first is cast of lead alloyed with a little antimony to give stiffness to the grid, and oxide paste is forced into the openings in the grid (*n*, *n*, *n*). The section, taken at the line *a b*, shows the shape of these openings.

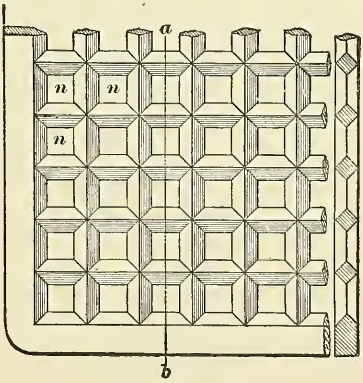


FIG 1058.

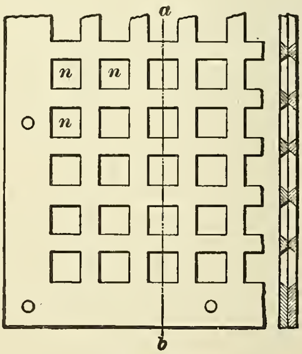


FIG. 1059.

2762. The second grid is made of two plates cast separately and afterwards riveted together with lead rivets. In this grid, as shown by the section, the openings for the paste (*n*, *n*, *n*) are larger in the center of the plates than at the faces, thus securely holding the plugs of active materials.

2763. Grids similar to those shown in Fig. 1057 are used in the E. P. S. accumulator in England and in the cells made by the Electric Accumulator Co., the Julien Co., in the United States, and by other manufacturers.

2764. In Germany, where the accumulator has been most extensively employed, more complicated forms of grids are used. One of these is shown in Fig. 1060; it consists of a double lattice united at the edges of the plate, and kept at a little distance apart, as shown in the section, by small columns at the points where the members of the two lattices cross, as represented at *c c*. This plate is cast at one operation. This form of plate holds a large quantity of active material, and is quite stiff. Even more complicated grids are used, some consisting of three layers of latticework, separated by columns, as in the grid just described.

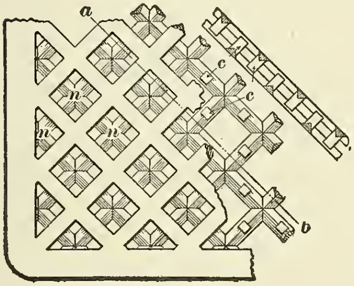


FIG. 1060.

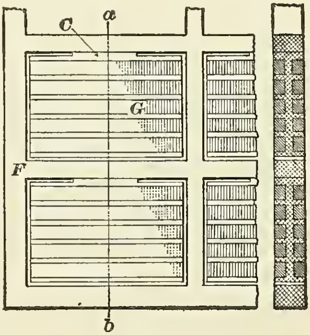


FIG. 1061.

2765. Fig. 1061 represents a section of the **Tudor** grid, a form of pasted-plate grid which has many good features; it is composed of a number of small square or rectangular grooved grids *G*, about 6 inches square, with the active material pasted or forced in the grooves as in the ordinary form (see section, Fig. 1061). Six or more of these small grids are then fastened by a lug on one edge, as at *C*, to the bars of a cast lead supporting frame *F*, which has openings between the bars slightly larger than the small grids which they enclose. The small grids are thus free to expand or contract without interfering with the plate as a whole, thus preventing to a large extent buckling and disintegrating of the plate, and any damaged grid may be replaced without disturbing those remaining.

2766. Accumulators employing this form of grid are largely used in central stations in Germany, and also form one of the largest accumulator installations in the United States, that of the Edison Electric Illuminating Co., of Boston, Mass., which consists of two sets of 70 cells each, each set having a capacity of about 3,500 ampere-hours. Other forms of grids are also made by the same company, and are also known as Tudor grids.

2767. Fig. 1062 illustrates the grid used in the **Sorley** cell, made in the United States. It is made of strips of lead *s*, *s*, *s* of uniform width and thickness, which are bent into the shapes shown, and are held in place by other strips around the edge of the plate. These strips are led out at the upper edge to form a terminal. The oxide paste is forced into the openings between the strips at *n*, *n*, *n*, as in the cast grids. The advantage claimed for this type of grid is that it allows of free contraction and expansion of the active material.

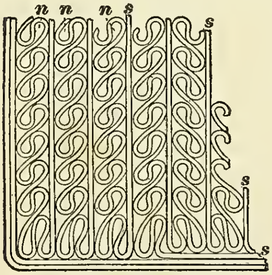


Fig. 1062.

2768. A form of grid which is cast around the plugs of active material is represented in Fig. 1063. This grid was invented by **Reckenzaun** for use in street-car propulsion; the active material is prepared in cylindrical plugs, shown at *c*, which are laid in a corrugated mold, and the melted lead alloy poured in around them. They are thus held quite firmly in place, while exposing a considerable surface to the electrolyte. As can be seen from the section, taken along the line *a b*, the cylindrical form of the plugs holds them in place, even if the plate be bent considerably.

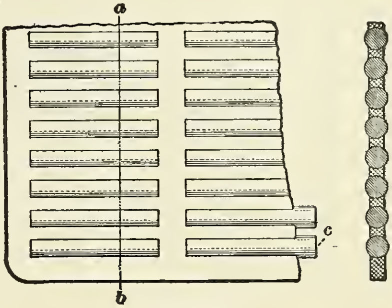


FIG. 1063.

2769. None of the pasted-plate cells, however, is as substantial as those in which the active material is formed from the plate itself, as in the Planté cell.

The principal objection to the Planté process being the length of time required to alter the surface of the plates from a smooth to a spongy condition, attempts have been made to construct plates which are porous at the start, such as compressing lead dust, or fine threads of lead made by blowing a stream of air through melted lead, etc., deeply grooving or even slitting the plates to increase the surface; none of these processes has resulted in a plate which is substantial enough for commercial use.

2770. A form of cell in which it is claimed the plates combine the cheapness of preparation of the pasted plate with the greater solidity and longer life of the Planté plate is the **chloride** accumulator made in this country by the Electric Storage Battery Co., of Philadelphia.

2771. The plates of this type of cell are made as follows: A mixture of zinc chloride and lead chloride is melted and run into molds, which form it into cylindrical pellets or pastilles, which have a bevel ∧ shaped edge, being thus larger in the center than at the faces. These pellets are placed in a second mold, being held in position by steel pins, and an alloy of lead and antimony is melted and forced in between the pellets under heavy pressure. When this cools it forms a plate, binding all the pellets of zinc and lead chloride together.

2772. This plate can not be used in this form in an accumulator; a number of these are first set up in a bath of dilute zinc chloride with plates of zinc, to which the lead plates are connected. These plates then act as the elements of a primary battery, and the resulting chemical action dissolves out the zinc chloride from the pellets, and converts the lead chloride into metallic lead, which assumes a crystalline form. The plate is now practically a continuous lead plate, solid and dense in some parts and porous in others.

2773. The plates in this condition are suitable for negative plates; those required for positive plates are then set up with plain lead plates in a bath of dilute sulphuric acid, and a forming current sent through them from the prepared plates to the plain.

This current causes the porous parts of the plates to be formed into lead peroxide and sulphate; the plate is now the equivalent of a pasted plate, and is an improvement through having its active material firmly bound in place in the compressed grid. Fig. 1064 shows a part of one of these plates; the section, taken along the line *a b*, shows the shape of these plugs. The holes in the plugs are caused by the pins by which they are supported in the mold.

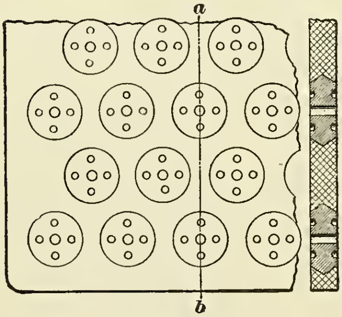


FIG. 1064.

2774. The requisite number of these prepared plates are then set up together to form a cell, alternate positives and negatives being connected to common conductors, as in other types of cells. (See Fig. 1057.)

The plates are each surrounded by a sheet of asbestos paper, and are separated from each other by a thin wooden strip so thoroughly perforated with large holes that it really fills little of the space between the plates; this wooden strip serves as a distance-piece, keeping the plates a certain fixed distance apart.

2775. The E. M. F. and action of this form of accumulator are the same as that of the Faure (pasted) type or the Planté. It is claimed by the manufacturers that, from the solidity of the construction, buckling and loosening of the active material are practically impossible, so that the cells may be discharged to a low E. M. F. or at high rates without serious injury. Its output per pound of element is greater than that usually assigned to lead accumulators, being about 5 ampere-hours per pound of plates (both positive and negative) at normal discharge rates.

2776. Most of the larger installations of accumulators in central stations in this country have been of this type of cell, and they are in use in France on street-cars, and also in England. The majority of German installations are of the pasted-plate type.

2777. There are, as in primary cells, a great number of forms of accumulators in use, both of the Planté type and the Faure; they differ from those described only in details of construction, such as the arrangement of the plates, vertically or horizontally, the form of the grids, etc., and need not be described here.

BIMETALLIC ACCUMULATORS.

2778. In this class of cells the elements consist of two different metals, the electrolyte being a salt of one of the metals.

There have been several combinations of materials proposed for cells of this type, but the only cells which have actually been used to any extent are the zinc-lead, copper-lead, and copper-zinc cells.

2779. The zinc-lead cell usually consists of plates of zinc and lead in a solution of zinc sulphate. On sending a charging current through this cell (the zinc being the negative plate) the zinc sulphate is decomposed, depositing zinc on the zinc plate and forming free sulphuric acid with the hydrogen of the water, which is also decomposed, its oxygen uniting with the lead plate, forming peroxide of lead. On open circuit and while charging, the free sulphuric acid in the solution slowly attacks the deposited zinc, reforming zinc sulphate, so that the efficiency of this form of cell is low, and it will not retain a charge more than a few days. The E. M. F. is high, being about 2.35 volts.

2780. The more modern forms of this cell employ a tinned-iron plate, amalgamated, or a lead plate, in place of the zinc plate. On charging the cell the zinc is deposited on the surface of the tinned-iron or lead plate, where it acts as the negative plate on discharge. (See Art. **2726.**)

2781. By substituting copper sulphate for zinc sulphate, and copper plates for the zinc or other negative plates in this type of cell, the acid formed during charge can not attack the copper, so that this loss is obviated; the E. M. F., however, is but 1.25 volts under these circumstances, so the watt output is materially reduced.

2782. Owing to the variations in the composition of the electrolyte, the internal resistance of cells of the types before described is variable, being lowest when charged and increasing during discharge as the sulphuric acid forms sulphate of copper or zinc.

2783. The copper-zinc accumulators are in greater commercial use than the other forms of bimetallic cells, the best known being the **Phillips-Entz** accumulator, which was made by the Waddell-Entz Electric Company. This accumulator employs the same active materials as the Lalande-Chaperon or Edison-Lalande primary cell (see Arts. **2683** and **2684**), modified in mechanical construction to adapt them for accumulator use.

2784. The positive plate is made of porous copper on a solid foundation, prepared in the following manner: A copper wire is surrounded with a paste made of finely ground copper oxide and sulphur; around this is woven a netting of fine copper wire, and the whole is then heated nearly to red heat, which causes the sulphur to unite with the oxygen of the copper oxide and pass off as gas, leaving the copper on the central wire in a very porous state. This cable is covered with a thin layer of loosely woven cotton thread, which forms a porous partition, and is then wound or braided into a mat or plate, forming the positive plate, the negative plate being a thin sheet of steel, thoroughly amalgamated; a number of these plates are mounted, alternately positive and negative, in a jar made of sheet steel, and surrounded by the electrolyte, which is a solution of *potassium zincate* and *potassium hydrate*. (See Arts. **2683** and **2689.**)

The jar is covered with an air-tight steel cover, to prevent the carbon dioxide (carbonic acid gas) in the air from coming in contact with the potassium hydrate solution; this cover is provided with a gas-valve to allow the gases formed in the cell to pass off.

2785. On charging the cell as thus constructed, the chemical reactions are complicated, but result in the deposition of the zinc from the potassium zincate on the steel plate and the sides of the jar, and the oxidation of the porous copper. On discharge the action is the same as in the Lalande-Chaperon primary cells; that is, the zinc is dissolved, the potassium zincate is reformed, and the copper oxide reduced to metallic (spongy) copper.

2786. The efficiency of this type of accumulator is about the same as that of the lead accumulators, while its output is very much greater, weight for weight, the ampere-hour output being about 5 times that of a lead cell, or about 20 ampere-hours per pound of plates. The E. M. F. of this form of accumulator being much lower than that of the lead accumulator, averaging 0.75 volt during discharge, the comparison on a basis of watt output is not so favorable; still, the zinc-copper accumulator will show an output of about 15 watt-hours per pound of plates, while the lead accumulators seldom exceed an output of from 7 to 10 watt-hours per pound of plates, the latter figure being seldom reached at normal rates of discharge.

2787. The efficiency and internal resistance of the copper-zinc accumulator vary quite largely with the temperature, on account of the considerable variations in the density of the electrolyte; on this account the cells are ordinarily charged and discharged at a temperature of about 54° C. (130° F.), at which point the resistance is about the same as in a similar lead accumulator.

2788. These cells are not much affected by the rate of discharge, there being no such occurrence as sulphating or buckling; but on account of the difficulty of depositing the zinc in a solid form, the charging must be done at a slow rate, and the action of the cells is improved by intermittent charging. The E. M. F. required to charge one of these cells varies from 0.90 volt at the start to 1.05 volts at the finish.

2789. In spite of the porous partition (cotton thread) which surrounds the positive plate, local action is liable to occur, on open circuit, so that these cells will not retain their charge for more than a few days, while a lead accumulator will scarcely lose 25% of its charge in as many months.

2790. On account of these features the copper-zinc accumulator can be successfully used only in installations where it is charged and discharged daily, thus preventing local action, and when it can have the necessary appliances, care and attention in charging, to insure proper charging rate, temperature, etc.; so, in spite of its large output per unit of weight, it can hardly come into general use. However, for traction work, that is, for use on street-cars and other vehicles in constant use, where the accumulators must be able to stand variable and very frequently heavy discharge rates, and must also be as light as possible, this form of accumulator possesses especial advantages, and is, consequently, better suited to the work than the lead accumulator.

2791. Other forms of bimetallic accumulators have been proposed, and in some cases used, among which may be classed several forms of primary cells, such as the Daniell, Leclanche, and others, which may be "regenerated" by passing a current through them; these have never been of commercial value, and do not require further attention.

THE USES OF ACCUMULATORS.

2792. In central stations furnishing electric current for lighting and other purposes, the demand for current varies very largely at different periods in the day; for example, a lighting station in a large city would probably be called upon to furnish, from 7 to 8 P. M., 10 times the amount of current that was required from 7 to 8 A. M., and in smaller stations the disproportion is even greater. As economy of operation demands that the engines and dynamos be worked at or near their full capacity, especially if the engines be compound or triple expansion, these conditions can both be met only by dividing the machinery into a large number of small units, or by having some system of storage of the electrical energy. In the first case, the small units require more attention, and are much less efficient than larger ones, and most modern large stations have their machinery divided into a few large units, employing large compound or triple-expansion engines.

2793. In these stations accumulators are being introduced on a large scale, and are installed according to one of two plans, as follows:

1. The dynamos and engines are not capable of carrying the full current required at certain parts of the day, for example, in the evening, but are of a size sufficient to furnish the current for the *average* rate required during the 24 hours. In this case, accumulators are installed which have a capacity sufficient to furnish the required excess of

current over the average. At times when the output of the station is less than the average rate, the current is used to charge the accumulators, thus keeping the output of the engines and dynamos at its maximum, which is the condition of greatest economy in operation. On account of the loss in charging and discharging the accumulators, the machinery must really have a capacity slightly greater than the average output of the station; but in any case the total amount of machinery, including engines, boilers, and dynamos, that must be installed is far less than if accumulators were not used, as in such case the total capacity of the machinery must evidently equal the maximum output of the station.

2. The second plan is to install accumulators of sufficient capacity to furnish all the current of the station for a part of the day when the output is less than the average; in this case, the engines are shut down for a considerable part of the day, the accumulators furnishing the entire output of the station during this time; when the demand for current begins to increase, the machinery is started up, and then furnishes the entire output of the station for the balance of the day, charging the accumulators when the station output is less than the capacity of the machinery. In this case the capacity of the accumulator plant is relatively less than in the former, and as the cost of accumulators is high, this may cause a saving over the first plan, although the mechanical efficiency of the station may be somewhat lower than in the first case.

2794. The result of applying accumulators to a large station is shown in Figs. 1065 and 1066. In both, the continuous line represents the actual output in amperes of a certain large station in New York, for a certain day.

2795. If this station were designed to use accumulators according to the first plan, the result would be about as represented in Fig. 1065. Here the dotted line represents the output of the dynamos (in amperes); the difference between the ampere output of the dynamos and that of the station is either absorbed or given out by the accumulators, as the station output is less or greater than the dynamo output. From the curve it appears that the accumulators are absorbing current (that is, being charged) from about 11.45 P. M. to about 4.30 P. M. (of the next day), while during the balance of the 24 hours the accumulators are giving out current, that is, discharging. The output of the dynamos is nearly constant, at about 850 amperes, the

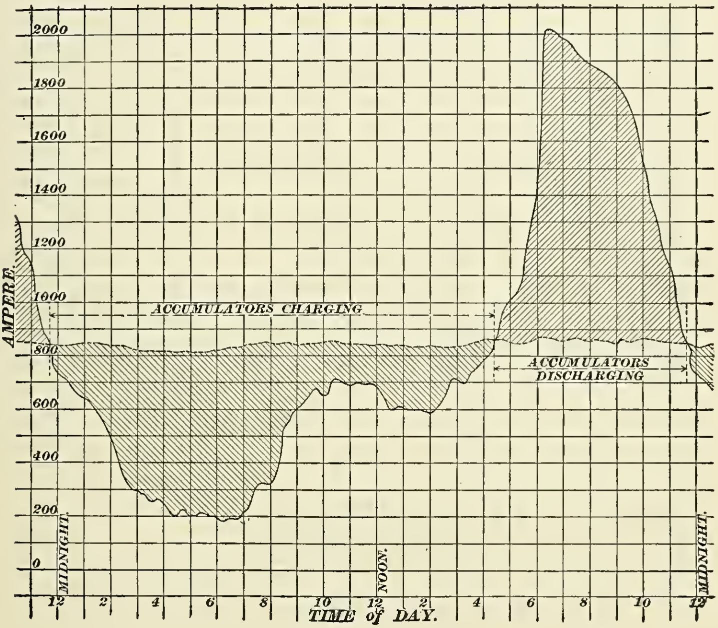


FIG. 1065.

output for the 24 hours being about 20,400 ampere-hours. The output of the station is 20,000 ampere-hours, the *average* output being about 835 amperes. This shows a loss in charging of 400 ampere-hours, which, allowing for slight overcharging, etc., is about right, as the capacity of the accumulator plant is 5,000 ampere-hours, about. This station would probably have installed three dynamos, each of about 450 amperes output, two of which would be kept running all the time. To allow of cleaning, inspection, etc., one machine would be replaced from time to time by the machine which had been previously idle, so that all three machines would come in for an equal amount of work. In case of accident to one, the other two would be kept running until repairs were made; the accumulators would then furnish the current for such brief shut-downs of the dynamos as would be necessary if made at periods of light load.

2796. If the accumulators were installed according to the second plan, the output curve would be represented by Fig. 1066. Here all the dynamos would be shut down from 2 until 8 A. M., the previously charged accumulators furnishing the output of the station during that time. At 8 A. M., in this case, the dynamos would be started, supplying both

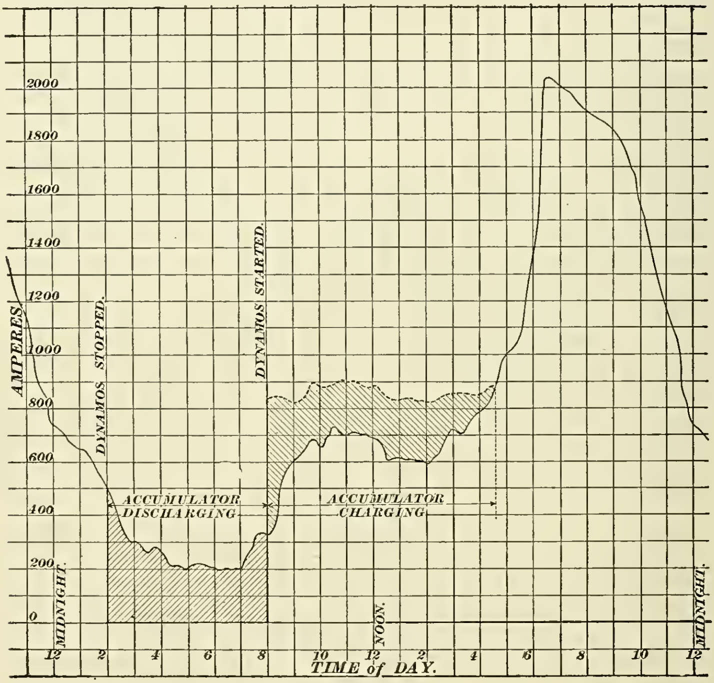


FIG. 1066.

the output of the station and sufficient current to recharge the accumulators. When the accumulators are fully charged, they are disconnected from the circuit until required the next day. In this case, this is done at about 4.40 P. M., the output of the station from this time being furnished by the dynamos, more being connected in circuit to furnish the extra output during the evening. In this particular case, the station would probably have installed five dynamos, each of a capacity of about 500 amperes; two of these would be of sufficient capacity to run the station from 8 A. M. until about 5 P. M., at which time two more would be switched in; the first two would then be shut down when the output was reduced sufficiently to permit; in this case one would be shut down at about 10 P. M. and the other at about 11.30. The extra dynamo is provided to use in case one of the others becomes disabled. The station output is, as before, 20,000 ampere-hours; the dynamo output is about 20,150 ampere-hours, the loss in charging being 150 ampere-hours. The capacity of the accumulator plant, in this instance, is evidently much smaller than before, being but about 1,600 ampere-hours; the dynamo plant is rather more than proportionately larger, as the machines do not run under so uniform a load as in the previous case.

In this arrangement the dynamos do not operate quite so economically as in the first, but the accumulators operate more economically, being charged and discharged at fairly uniform rates, while in the previous case the accumulators are discharged at a more rapid rate than they are charged, and the *maximum* discharge rate is much higher than the *average*. This either results in a short life for the accumulators, consequently a high allowance for depreciation, or necessitates a larger accumulator plant than the output in ampere-hours would seem to require, which makes the first cost high; again, in the first case, the firemen, engineers, etc., would be required to be in attendance during the whole of the 24 hours, which would probably be done by having three "shifts," or three separate gangs of men, while in the second case no firemen or engineers are required during the time (from 2 until 8 A. M.) that the engines and dynamos are shut down, so two "shifts" of men would be sufficient; hence, it would appear that for the station which gave this particular output curve the second plan of installing the accumulators would be preferable.

2797. Whether or not it would pay to install accumulators in any particular station depends on various circumstances, but it can generally be determined by the output curve, actual or calculated, from which may also be determined the size of the dynamos and accumulator plants, and the proper time for charging and discharging the battery, which features will vary largely in different stations.

2798. In railway power stations, where the wide variations in the output occur from second to second, instead of from hour to hour, as in lighting stations, accumulators would serve to greatly steady the load on the generators, acting on somewhat the same principle as does a heavy flywheel on an engine; the accumulators would remain at about the same state of charge continually, if properly installed, supplying current to the line when the demand for current is heavy, and absorbing energy when the demand grows light.

2799. Accumulators would be especially useful if the source of power was a water-wheel, since they would make the load on the water-wheel so nearly uniform that its regulation would be good, which is not the case if the load is irregular.

2800. Electrically operated street-cars, in which the source of the current required is a battery of accumulators, carried upon the cars, have many advantages over the trolley system, overhead or underground; the disadvantages, however, are so serious that very few commercially successful systems of this kind have been operated in this country, although several lines are running in France and in other parts of Europe.

2801. The advantages of this system are the absence of the overhead wires or underground conduit of the trolley roads, the complete independence of each car, and the ability of the dynamos which charge the batteries to run uniformly at their full output.

2802. A sufficient number of accumulators are usually carried on one car to run it about 30 miles, or for about six hours, with one charging. Such a battery weighs about 4,000 to 4,500 lb., increasing very materially the weight of the car, which ordinarily weighs, with passengers, about 10,000 lb. The power required to propel this extra weight must be provided, and the wear of the tracks and car-trucks is increased.

2803. In order that the cars shall not stand idle while its battery is being charged, several sets (about 3 sets to each car) are provided, which makes an expensive equipment, and the cost of handling the heavy batteries, when moving them into and out of the cars, is considerable.

2804. The chief disadvantage, however, is the rapid deterioration of the plates. On starting the car, and in ascending steep grades and rounding curves, the accumulators are called upon to furnish currents far in excess of their normal discharge rate, which, added to the continual jolting to which they are subject, makes the disintegration of the positive plates very rapid indeed; and only when the plants have been under the charge of skilled experts has accumulator traction been at all successful in this country.

2805. The Waddell-Entz accumulator would seem to be especially suited to traction work, on account of its light weight and capacity for high rates of discharge; but to operate efficiently, this accumulator must be charged and discharged under special conditions (see Art **2790**), which makes its cost of operation high.

2806. Accumulators have been very successfully applied to launches and other small boats, in which the propeller is driven by a suitably connected motor; the battery is located under the seats or in lockers, and is usually of sufficient size to furnish current for running the boat at a speed of seven or eight miles an hour for about 40 miles; higher rates of speed can be obtained, but the total distance covered is then lessened.

2807. In general, accumulators have been more or less successfully applied (*a*) where it is desired to supply a variable demand for current and at the same time keep the output of the source of the power approximately constant; (*b*) where it is desired to utilize an electric current at a point where it is objectionable or impossible to obtain the current directly from dynamo machines; (*c*) where it is desired to obtain a comparatively small but continuous current from a source of a considerable current, which can be utilized only a short time and at infrequent intervals, and (*d*) where a perfectly steady current is required for certain applications, where the current from dynamo machines, which is always slightly irregular, would be unsuitable.

2808. Under (*a*) would be classed the various lighting and power station installations, the principles of which have been described (Art. **2792**). An extension of the plan of such stations has been adopted abroad, in cases where a considerable demand for current from a (direct current) central station occurs in some particular district, at a distance from the station. It is evident that if this demand is met by sending the current directly from the station, the wires for carrying the current must be large enough to carry the *maximum* current required, although this maximum only continues for a few hours in each day. If an accumulator plant be installed in this district, the wires from the station need only be large enough to carry the *average* current required for that district, the battery furnishing the additional current during the period of heavy load and charging when the load is less than the average, just as in the first plan for installing accumulators in central stations. (See Arts. **2792** and **2793.**) Aside from the saving in the wire, there are other advantages of this method of current distribution, which will be treated of in another section.

2809. Another application which comes under this same head, although carried out on a much smaller scale than the examples given, is made in places where a current of considerable strength is required only occasionally, with considerable intervals between; primary cells might be directly applied in such cases, but on account of their high internal resistance a considerable number of cells or a few very large cells would be required to furnish the necessary current, or else local action would soon render them useless; if, however, primary cells, say of the "gravity" or other type giving a constant E. M. F., be used to charge secondary cells, the charging can go on continuously day and night at a slow rate, and at any time the secondary cells may be drawn upon for a considerable current far beyond the capacity of the primary cells themselves. This method is often adopted in surgeons' offices, where a considerable current is occasionally required for heating cauteries and for similar work. (See Arts. **2712** and **2714.**)

2810. Under class (*b*) would be included such applications of accumulators as in propelling street-cars and small boats, the main features of which have been given. (See Arts. **2800** and **2806.**)

2811. Under this same head would also be classed the transporting of electrical energy by means of charged accumulators; these are usually charged at some central station, and are then carried to the point where it is desired to use the current. Thus far this has been done only on a comparatively small scale, principally for furnishing current to the motors which drive the phonographs and kinetoscopes and similar machines, which are so generally on exhibition, often in localities where electric-light circuits are not available, or are not of the right character.

2812. Under class (*c*) may be included several of the more important of the minor applications of accumulators, as follows: The electric lighting of railroad-cars, the charging current for the accumulators being obtained from dynamo machines driven from one of the axles of the car, which source of power is obviously intermittent and irregular; the lighting of houses, at a distance from electric-light stations, such as country residences, where the power for driving the charging dynamos is obtained from windmills or the action of the waves or tides, which sources of power are very variable. Special devices are usually used in such plants which automatically disconnect the dynamo from the accumulators when the source of power has stopped, or is insufficient to furnish the requisite current for charging. The lighting and furnishing of small amounts of power to the offices or such other departments of mills or factories as are obliged to be in operation when the main engine or other source of power is shut down, the accumulators being charged during the day, when the main engine is running, also comes under this head.

2813. The amount of current required for lighting an ordinary house is comparatively little, and a very small engine and dynamo would readily furnish it; but the noise and trouble of operating such a small plant at the time when the current was required would make it objectionable. In such a case an accumulator plant may be installed, of sufficient capacity when charged to furnish the current for lighting for several days or even weeks; then, by installing an engine and dynamo of proper size, the battery may be charged once a week or month, as the case may be, with comparatively little trouble and expense, and the time for charging may be chosen so that the noise or other features would not be objectionable. Such a plant would be classed under this same head, (*c*).

2814. Under (*d*) would be classed the special application of accumulators in testing and in telephone work. Their action, as regards the steadiness of the current, is no more favorable than that of a good primary cell; but for a given output the accumulator is more compact, requires less attention, and its elements do not need to be replaced when exhausted, but renewed by a charging current. Consequently, accumulators are coming into use for telephone central stations, where three or four good-sized cells may replace several hundred primary cells; they are usually installed in conjunction with suitable charging apparatus, usually small dynamos, the operation of charging being gone through with whenever necessary.

2815. The specific applications of accumulators cover many more cases than have been given; but they may be all classed under these several heads.

THE INSTALLATION OF ACCUMULATORS.

2816. As stated in Art. **2757,** accumulators are usually of approximately cubical form, and the jars are usually of glass or hard rubber, in ordinary sizes; for special applications, such as portable cells and batteries for street-car and launch use, special jars are provided to suit the conditions. For ordinary installations for lighting purposes, the glass jar is best suited, as it permits the examination of the interior of each cell at any time, and any cells in which the active material shows signs of buckling or disintegrating may be attended to before the fault becomes serious.

2817. Accumulators should be placed on racks or shelves, and if the number of cells be large, it is usually advantageous to place them in several tiers. Plenty of room should be allowed between the tiers, to allow of making connections, taking out or replacing plates or the electrolyte, etc.

2818. If the cells are located in a room where the air is warm and moist, water will collect on the surface of the jars and shelves, and will cause an appreciable leakage of the current; to obviate this, each cell should be supported on a small shelf, which should rest on porcelain or glass insulators, and the jars of adjacent cells should not be allowed to touch.

2819. If exhaust steam is to be had for heating purposes, a double line of pipes, running under the shelves on which the cells rest, will be beneficial, as the heat will cause a circulation of the electrolyte, and will decrease its resistance. Such an arrangement is very necessary for cells of the Waddell-Entz type (see Art. **2783**), but for ordinary lead accumulators the advantage is not sufficient to warrant any great outlay for such heating.

2820. The space to be allowed for a battery depends on the make and type, and may usually be found in the catalogues of the manufacturers. Lead accumulators of ordinary size will usually have a capacity of 2.5 to 4 ampere-hours per square inch of floor space that they occupy; for large accumulators in lead-lined boxes, such as are installed in central stations, this value may be increased to 5. A good average figure for cells of 200 to 500 ampere-hours capacity is 3 ampere-hours per square inch of floor space occupied.

The output per cubic inch of volume is rather more constant, being about .25 to .3 ampere-hour. The weight of a battery of accumulators is considerable, and the shelves or other supports intended to hold them should be made amply strong, for if they sag or bend, the glass jars are liable to be broken.

2821. The electrolyte of an accumulator will not freeze until exposed to a temperature of about − 11° C. (about 13° F.); freezing should be avoided, as it is very liable to break the jars.

2822. The number of cells required for any given installation depends upon the E. M. F. desired; ordinary lighting plants are usually designed for an E. M. F. of 50 to 55 or 100 to 120 volts. The number of accumulators required may be found by dividing the E. M. F. required by the average E. M. F. of the cell during discharge, which is usually taken (for lead accumulators) as 1.9 volts; a 55-volt installation would then require $\frac{55}{1.9} = 29$ cells (obviously, fractional cells are an impossibility).

2823. When only partially discharged, the E. M. F. of such a battery would be higher than that required; this may be reduced by placing a suitable resistance in series with the battery, and adjusting this as the E. M. F. diminishes, to keep a constant E. M. F. at the lamps, or arranging the connections so that one or more cells may be cut into circuit from time to time, to effect the same result.

Manufacturers of electrical apparatus furnish devices which will automatically perform the above operations as the E. M. F. of the battery changes.

2824. To allow for possible accidents to one or more cells, and for the drop in the wiring and connections, one or two extra cells may be provided, and so arranged that they may be cut into the circuit at any time.

2825. The size of each cell in such an installation depends upon the strength of the current which it is desired to use, and the length of time (number of hours) the current is required; thus, to furnish a current of 10 amperes for 10 hours with one charging would require cells of a capacity of $10 \times 10 = 100$ ampere-hours each.

2826. The current required for operating incandescent lamps, the average number of hours per day that they are lighted, etc., will be treated of later, and from the values given, the size and number of accumulators which should be installed to furnish the current for lighting a building may be determined.

2827. The method of procedure in setting up the ordinary forms of cells is about as follows:

Having prepared the shelves or supports for the cells, unpack and thoroughly clean the jars, and place them in position on the shelves, with the support for the plates (*S*, *S*, Fig. 1057) in place. The plates should then be thoroughly cleaned of the sawdust in which they were packed, and placed in position in the jars. They should rest evenly on the supports which raise them from the bottom of the jar, and the blocks or strips of insulating material which separate the plates should be properly placed in position between them. In some makes of cells pieces of rubber or glass tubing are used for this purpose.

2828. The plates should be so placed in the jars that the connecting strips will come into the proper position for connecting the positive plates of one cell to the negative of the next, and so on.

The joints between the connecting strips should then be made bright and smooth, for which purpose a fine file or sandpaper is best; the connecting bolts should be set up with a good pressure, so that these bright surfaces will be squeezed together firmly, insuring good contact.

2829. The electrolyte should be prepared in a lead-lined or stoneware tank, and the sulphuric acid should be slowly poured into the water and thoroughly stirred until the solution is of the proper density (1.17 sp. gr.). It is well to note that water should never be poured into sulphuric acid; as the two liquids combine with considerable heat, the small quantity of water which first reaches the acid is instantly converted into steam, resulting in an explosion, which, by scattering the acid about, is liable to cause serious injury. Therefore, in preparing the electrolyte, *always pour the acid into the water.*

In case a considerable quantity of solution is to be prepared, blocks of *pure* ice (manufactured ice is best) may be used in place of water. The heat generated by the dilution of the acid is then absorbed in melting the ice.

2830. When all connections are made and preparation for charging completed, the electrolyte should be poured into the cells until the plates are covered to a depth of half an inch or so, and as soon as possible thereafter the cells should be charged. If allowed to stand uncharged in the acid, the plates are liable to become sulphated (see Art. **2736**). Care should be taken that the charging current is of the right polarity; that is, that the current flows *from* the positive *to* the negative plates through the cell; if this is reversed, the cells will be reversed, and a great deal of trouble will be experienced in getting them back to the proper condition.

The first charging should be long continued and at a low rate, to remove any sulphate that may exist.

2831. In charging an accumulator, as has been shown, only a small part (about 8%) of the E. M. F. required to force the current through the cell is expended in overcoming the resistance of the plates and electrolyte; the remainder is expended in overcoming the E. M. F. of the chemical action of the cell. It follows, then, that if the *applied* E. M. F. be just equal to the E. M. F. of the cell no current will flow (see Art. **2504**), so that the E. M. F. of the cell itself may be considered as a *counter* E. M. F., opposing that of the charging current. To apply Ohm's law $\left(C = \frac{E}{R}\right)$ to this case, the E must be considered as representing the *algebraic sum* of the applied and the counter E. M. F., or $E =$ applied E. M. F. − counter E. M. F. This is merely another way of stating that the E. M. F. *required to drive the charging current through the cell* is only that required to overcome its ohmic resistance; but to this must be added an E. M. F. equal and opposite to the E. M. F. of the cell itself, due to the chemical affinity of the substances of which it is composed.

In charging, then, if from any cause the E. M. F. of the charging current be changed by a small amount, the charging current will be altered in a much greater degree, depending on the ratio between the applied E. M. F. and the difference between the applied and counter E. M. F. For example, consider a cell which has been discharged until its E. M. F. is 1.925 volts (on open circuit). The resistance of the cell is .005 ohm and its normal charging current is 35 amperes. The drop due to this current is $35 \times .005 = .175$ volt; the applied E. M. F. must then be $1.925 + .175 = 2.10$ volts, to cause 35 amperes to flow. Now, if the applied E. M. F. drops to 2.0 volts, it is evident that, the counter E. M. F. being the same, the drop is equal to $2.0 - 1.925 = .075$ volt, and the current which would cause this drop when flowing against .005 ohm resistance is $C = \frac{E}{R} = \frac{.075}{.005} = 15$ amperes. Thus, a drop in the applied E. M. F. of $\frac{.1}{2.1}$, or about 5%, causes the current to fall off more than 50%.

This shows the necessity for having the source of the charging current so arranged that the E. M. F. may be closely adjusted in order that the charging current may be maintained at its proper value.

In all accumulator plants of any considerable capacity the source of the charging current is a dynamo, and the methods of attaining this adjustment therewith will be given later. In any case, the maximum E. M. F. of the source of the charging current must be higher than the highest counter E. M. F. that the battery can give.

2832. With every accumulator plant a *hydrometer* (see Art. **995**) should be included, as the electrolyte should be kept at the proper density. The state of charge of the cell can be approximately determined by the density of the electrolyte (see Art. **2737**).

2833. The volume of the electrolyte will gradually diminish during the operation of the cell, due to evaporation and to the evolution of gas when the cell is charged; this loss should be made up by occasionally adding pure water, or acid, if the density as indicated by the hydrometer is too low.

2834. A portable voltmeter should also be provided, which shall have a capacity such that the E. M. F. of a single cell may be accurately measured, so that if the action of any cell seems to be irregular, its condition may be determined by measuring its E. M. F. and comparing it with that of the other cells. Some instrument makers furnish portable voltmeters with two scales, one a tenth or a twentieth the value of the other; these are very convenient for accumulator work, as by selecting the proper scale the

E. M. F. of the entire battery or of single cells may be accurately determined.

2835. Accumulators should not be installed near any apparatus which has bright metal surfaces, such as an engine, as the fumes from the acid will corrode such surfaces unless they are protected by a coating of grease or varnish; all connectors and other brass pieces used around the cell should be coated with varnish or grease for the same reason. Vaseline is especially applicable for this purpose.

This effect may be largely prevented by providing the cells with covers, which also prevents evaporation to a large extent. Covering the liquid with a layer of heavy oil has also been proposed, but this plan involves a great deal of troublesome dirtiness, as the bubbling of the escaping gas causes the oil to spatter.

APPLIED ELECTRICITY.

(CONTINUED.)

THEORY OF THE DYNAMO.

3015. In order that a current may flow through a circuit, and thereby be available for doing work, it is necessary that a difference of potential be established between two points in that circuit, and in order that the resulting current may be maintained, it is necessary to maintain the difference of potential between these points. (See Art. **2237.**)

This difference of potential may be established and maintained in a variety of ways, some of which—the chemical action of certain liquids on certain other substances, for example—have already been explained.

Generating an E. M. F. in a conductor by moving the conductor in a magnetic field in such a direction that it cuts the lines of force of the field, is by far the most extensively used method of establishing the required difference of potential, and a machine for generating and maintaining an E. M. F. by the movement of one or more conductors across the lines of force of a magnetic field is called a **dynamo.**

3016. The amount of the E. M. F. generated in a moving conductor depends upon the rate at which the conductor cuts the lines of force. (Art. **2449.**) With practicable values for the length of the conductor, for its velocity, and for the extent and density of the magnetic field, the E. M. F. which can be generated in a single conductor is not sufficient for most of the applications of the electric current. However, by joining a number of conductors in series, in such a manner that the E. M. F.'s generated in them are added together, any desired E. M. F. may be obtained.

The *character* of the E. M. F. of a dynamo depends upon the grouping of the various conductors with respect to the magnetic field, and to the method of connecting these conductors with the external circuit.

The part of the dynamo in which the E. M. F. is generated, consisting of the conductors, the means of supporting and moving them, and the device for connecting them with the external circuit, is called the **armature,** and the conductors, in their various arrangements and connections, constitute the **armature winding.**

The effect of the various methods of arranging and connecting armature windings upon the character of the E. M. F. produced may best be studied by developing them from the simplest form, a straight conductor moving in a straight line through a uniform magnetic field, as will be explained later.

3017. The principal sources of magnetic fields are permanent magnets, such as the earth, masses of lodestone, or hard-steel magnets, and electromagnets.

As the E. M. F. generated in a moving conductor is proportional to the density of the field in which it moves, other conditions remaining the same, it is obvious that a field of considerable density is desirable. For this reason the earth's field or that of permanent magnets, either natural or artificial, is seldom used in dynamo-electric machinery, since the density of such a field is low compared with that possible to obtain with suitably designed electromagnets, so that, in spite of the fact that electrical energy must be continually expended in order to keep up the magnetizing force of the electromagnet, this last form is almost universally used.

The design of electromagnets for dynamo-electric machinery will be discussed later; for the present it is sufficient to consider that the magnetic field exists in the space between one or more pairs of poles, the balance of the magnetic circuit not being considered.

GENERATION OF E. M. F.

3018. If any straight conductor $a\ b$, the direction of whose length is at right angles to the lines of force, is moved in a uniform magnetic field in a direction at right angles to the lines of force of the field and to its own length, and at such a uniform velocity that at the end of unit time (1 second) it reaches the position $a'\ b'$, as represented in Fig. 1123, the following conclusions result:

The *velocity* of the conductor may be represented by the length of the line $a\,a'$ (or $b\,b'$), as the conductor moves over this distance in unit time. The total number of lines cut by the conductor is evidently that number enclosed in the area $a\,a'\,b'\,b$; the lines $a\,a'$ and $a\ b$ (or $b\,b'$ and $a'\,b'$) being at right angles to each other, the area enclosed by $a\,a'\,b'\,b$ is the product of the *length of the conductor* $a\ b$, and the *length of its path* $a\,a'$ (or $b\,b'$). Formula **447,** given in Art. **2449,** $E = \frac{N}{10^8\,t}$, may be modified to fit this case, as follows: Let $\mathbf{B}$ = the density of the magnetic field, L = the length of the conductor, and M = the length of its path, or its velocity. Then, the area moved over by the conductor (in unit time) is $L\,M$, and the total number of lines cut by the conductor is $\mathbf{B}\,L\,M$. Then, substituting this value for N in the above formula, $E = \frac{\mathbf{B}\,L\,M}{10^8\,t}$. As in this case $t = 1$, unit time being assumed, this may be written

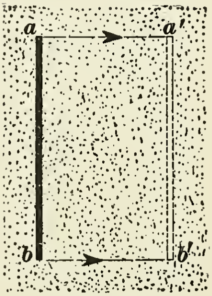


FIG. 1123.

$$E = \frac{\mathbf{B}\,L\,M}{10^8}, \qquad \textbf{(473.)}$$

which gives an expression whereby the E. M. F. generated in a conductor moving in a magnetic field *under the conditions given above* may be found.

The density $\mathbf{B}$, being the number of lines of force per unit of area, i. e., per square inch, or per square centimeter, it is evident that the product $L\ M$ must be expressed in the same units in order that the equation $N = \mathbf{B}\ L\ M$ should hold true.

EXAMPLE.—Suppose the conductor $a\ b$ in Fig. 1123 to be 1 foot 8 inches long, and that it is moved in a magnetic field whose density is 50,000 lines of force per square inch at such a (uniform) velocity that at the end of 1 minute it would have moved 2,250 feet. What is the E. M. F. generated in the conductor?

SOLUTION.—As the density is expressed in lines of force per square inch, the other dimensions must be reduced to the same unit, i. e., inches.

The length of the conductor L is then 20 inches, and the velocity, or distance through which it would move in 1 second, $= M = \frac{2,250}{60} =$ 37.5 ft., or $37.5 \times 12 = 450.0$ in.

Hence, from formula **473**, $E = \frac{\mathbf{B}\ L\ M}{10^8}$, where $\mathbf{B} = 50,000$, $L = 20$, and $M = 450$, $E = \frac{50,000 \times 20 \times 450}{10^8} = \frac{450,000,000}{10^8} = 4.5$ volts. Ans.

3019. The formula given in Art. **3018** does not hold good as it stands if the conditions governing the direction of the motion of the conductor are not as before stated, which are, that the conductor must lie in a plane at right angles to the lines of force and move in a direction at right angles to its own length and to the direction of the lines of force. It is evident that a conductor might readily be moved in a direction which would not conform to all or any of the above conditions; the formula, to be generally applicable, must then be modified to suit such cases.

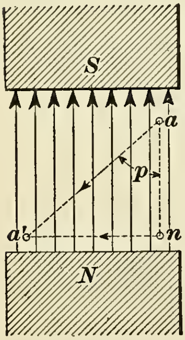


FIG. 1124.

3020. Fig. 1124 represents a case where a conductor a lies in a plane at right angles to the lines of force (so that we are looking along its length, and consequently see only the round section, as shown), and is moved in a direction at right angles to its own length, but at the angle p, which is not a right angle, to the direction of the lines of force flowing between the poles N and S of a magnet. If the conductor move from a

to a' in unit time (say one second), the area swept over by the conductor in unit time is a rectangle, and the area is measured by the product $L\ M$ of the length of the conductor L and the length of its path in unit time, or its velocity, M; but the total number of lines cut by the conductor is *not* the product of the density $\mathbf{B}$ and the area $L\ M$, since the density is measured on a plane at right angles to the lines of force, and the area $L\ M$ is at an angle to this plane.

From an inspection of Fig. 1124, it will be seen that the conductor will cut exactly the same number of lines of force if moved from *any* point on the line $a\ n$ (which is parallel to the lines of force) to the point a'; in other words, whatever the value of the angle p, the number of lines of force cut by the conductor in moving from a to a' will be the same.

By making this angle a right angle, as at n, the path of the conductor along the line $n\ a'$ will be at right angles to the lines of force, and all the conditions prescribed in Art. **3018** will be fulfilled.

The length of the line $n\ a'$ is, however, not equal to the length of the line $a\ a'$; but as the former length must be used in calculating the total number of lines cut, and the latter is the length which is known, an expression for the length $n\ a'$ in terms of the length $a\ a'$ must be found. From the construction of the figure, the triangle $a\ n\ a'$ is a right-angled triangle, with the length of the hypotenuse $a\ a'$ and the adjacent angle p given; the length of the side $n\ a'$ opposite the angle p is found by trigonometry to be $a\ a' \sin p°$, which is the desired value.

3021. Calling the length of the conductor L and the length of its path M, as before, it follows from the above that the total number of lines of force cut by the conductor is given by the formula

$$N = \mathbf{B}\ L\ M \sin p°. \qquad \textbf{(474.)}$$

With given values of $\mathbf{B}$, L, and M, it is evident that N is a maximum when $p° = 90°$, as then $\sin p° = 1$, and $\mathbf{B}\ L\ M \sin p° = \mathbf{B}\ L\ M$, corresponding to the case given in Art. **3018**, while if $p° = 0°$, then $\sin p° = 0$ and $\mathbf{B}\, L\, M \sin p° = 0$, which means that if the conductor be moved in a direction at an angle of 0°, i. e., parallel to the lines of force, no lines will be cut by it and no E. M. F. generated.

A method of considering the relation between the length $n\,a'$ and the length $a\,a'$, which is very useful in some cases, is to regard the length $n\,a'$ as the *projection* of the length $a\,a'$ on a plane at right angles to the lines of force. The application of this method will appear in other parts of this section.

3022. In a similar manner other variations from the conditions given may be considered. Fig. 1125 represents the case where the conductor $a\,b$ lies in a plane at right angles to the lines of force, and is moved in the same plane, but in a direction $b\,b'$ at an angle s, which is not a right angle, to its length $a\,b$. The shape of the area swept over in moving from $a\,b$ to $a'\,b'$ is evidently a rhomboid, of which the area is equal to the product of the base $b\,b'$ and the altitude $a\,n$. This altitude being perpendicular to the base $b\,b'$, the triangle $a\,b\,n$ is a right triangle, and, by trigonometry, side $a\,n = a\,b \sin s$. Consequently, the total number of lines of force cut by the conductor $a\,b$ of length L in moving over a distance $b\,b' = M$ through a field whose density $= \mathbf{B}$, is

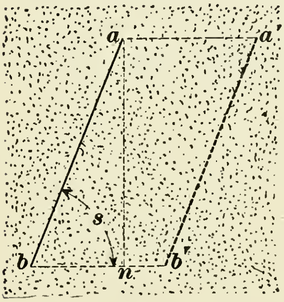


FIG. 1125.

$$N = \mathbf{B}\, M\, L \sin s. \qquad \textbf{(475.)}$$

Again, with given values of $\mathbf{B}$, L, and M, the value of N is a maximum when $s = 90°$, for $\sin 90° = 1$ and $\mathbf{B}\, M\, L \sin s = \mathbf{B}\, M\, L$; and where $s = 0°$, $\sin s = 0$ and $\mathbf{B}\, M\, L \sin s = 0$, which means that if the conductor is moved at an angle of 0°, i. e., parallel to its own length, no lines of force are cut by it, and no E. M. F. is generated in the conductor.

In this case, again, the length $a\,n$ is the *projection* of the actual length of the conductor $a\ b$ on a plane at right angles to the direction of its path.

3023. Fig. 1126 represents the plan and elevation of the case where a conductor $a\ b$ is situated in a magnetic field at an angle r to the lines of force, as represented in the elevation, and is moved through the field in the direction $a\ a'$ or $b\ b'$ at right angles to the lines of force and to its own length, as represented in the plan.

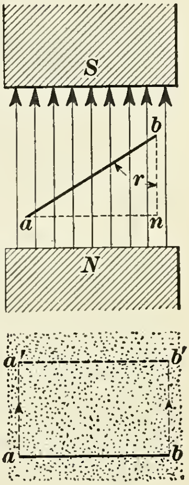


FIG. 1126.

The area swept over by the conductor is equal to the product of its length $a\ b$ (see elevation) and the length of its path $a\ a'$ (see plan), but, as before, the product of this area and the density of the lines of force is not equal to the total number of lines of force cut, as the area is not measured at right angles to the lines of force.

The number of lines of force cut, however, is measured by the product of the density, the length of the path of the conductor, and the *projection* of its length on a plane at right angles to the lines of force. This projection is represented by $a\ n$, Fig. 1126, and the triangle $a\ b\ n$ being a right triangle, side $a\ n = a\ b \sin r$, as before, and the total number of lines cut is

$$N = \mathbf{B}\ M\ L \sin r. \qquad \textbf{(476.)}$$

With given values of $\mathbf{B}$, L, and M; N will again have a maximum value when $r = 90°$, as $\sin 90° = 1$ and $\mathbf{B}\ M\ L \sin r = \mathbf{B}\ L\ M$; and when $r = 0°$, and $\sin r = 0$, $\mathbf{B}\ M\ L \sin r = 0$, which means that if the conductor is located in a plane at an angle of 0°, i. e., parallel to the lines of force, no lines of force will be cut; hence, no E. M. F. will be generated by a movement of the conductor.

3024. For any case where the conditions governing the motions of the conductor differ in more than one respect from those given in Art. **3018,** a formula may be constructed by combining formulas **474, 475,** and **476.** Thus, the total lines of force cut by any conductor of length L situated in a uniform magnetic field of density **B**, lying in a plane at an angle of $r°$ with the lines of force, and moved with a velocity M through the field in a direction at an angle of $s°$ with its length, and at an angle of $p°$ with the lines of force, will be given by the formula $N = \mathbf{B}\, L \sin r° \sin s° M \sin p°$, and the E. M. F. resulting from this motion will be given by the formula

$$E = \frac{\mathbf{B}\, L \sin r° \sin s° M \sin p°}{10^8}. \qquad \textbf{(477.)}$$

It is evident that with given values of **B**, L, and M, the value of N, hence of E, will be a maximum when the angles r, s, and p are each equal to 90°, while if any of these angles is equal to 0°, the value of N and E will be 0.

It follows, then, that to get the maximum E. M. F. with a given length of conductor, these angles should all be as near 90° as possible, which is the case in almost all dynamos, as will be pointed out.

3025. Thus far a field of uniform density has been assumed; but from the statements which have been made the effect of variations in the density of the field may be readily found.

It should be remembered that as the E. M. F. generated in a moving conductor is proportional to the *rate* of cutting lines, it is not necessary that the conductor should actually move over any particular area in order that an E. M. F. be generated in it; it is only required that the conductor move at such a velocity that *if* that velocity were maintained for one second, the conductor would cut a certain number of lines of force, as measured by the area which would be swept over. This area is obviously the same whether it encloses lines of force or not; so if at any one point in a conductor's path the density is known, the number of lines of force which *would be cut* by the conductor in moving over that area *if* the density were uniform at its known value would evidently be the product of the known density and the area, and the E. M. F. generated *at the instant* when the conductor is passing through the part of the field where the density is of the given value may be found from the formula.

3026. The considerations just mentioned apply if the velocity is not constant, for if the velocity at any instant is known, the area which *would be* moved over by the conductor in one second *if* the velocity were constant at the known value, measures the number of lines which would be cut in one second, and hence the *rate* of cutting or the E. M. F. generated.

In actual practice the velocity of conductors in any particular case is almost invariably constant, while the density of the field is seldom uniform.

Example.—A conductor 3 feet 3 inches long is dropped vertically through a magnetic field whose lines of force are horizontal, but of varying density. At a certain point a the velocity is known to be 40 feet per second, and at the same point the density of the magnetic field is known to be 28,000 lines of force per square inch. What E. M. F. is generated in the conductor when at the point a?

Solution.—The velocity of the conductor at this point is $40 \times 12 = 480$ in. per sec. The conductor being 3 ft. 3 in. = 39 in. long, if moved at this velocity for one second would sweep over an area of $480 \times 39 = 18{,}720$ sq. in., which area would enclose $18{,}720 \times 28{,}000 = 524{,}160{,}000$ lines of force, if the density were uniform at the known value throughout the area. From the formula $E = \frac{N}{10^8}$,

$$E = \frac{524{,}160{,}000}{10^8} = 5.2416 \text{ volts. Ans.}$$

THE EFFECT OF CURRENT IN THE CONDUCTORS.

3027. Thus far, only the production of the E. M. F. has been considered. If this E. M. F. is allowed to act on a closed circuit, so that a current will flow, certain effects will be produced, which must be taken into account. First, the passage of the current through the conductor implies a loss or drop of potential equal in value to the product of the current and the resistance of the conductor. (Art. **2315.**) The difference of potential between the terminals of the conductor in which the E. M. F. is generated is then less than that E. M. F., by an amount equal to the drop. Calling e the E. M. F. generated, or the internal E. M. F., E the difference of potential between the terminals, R_i the internal resistance of the source of e, and C the current flowing, then, $E = e - C R_i$. The total amount of energy expended in the circuit w is evidently the product of C and e; i. e., $w = C e$. Of this, $C \times C R_i = C^2 R_i$ is expended within the source of the E. M. F. itself, leaving $C e - C^2 R_i = C E = W$, the energy expended in the external circuit, or the *output.* It is evident that as $C^2 R_i$ is entirely expended in heating the conductors in which the E. M. F. is generated, it is wasted as far as any practical application is concerned, and should, therefore, be made as small as possible, in order that $C E$ can be as large a proportion as may be of the total energy developed, $C e$.

On this account, the internal circuits of dynamo machines are made of copper, that being the metal which has the greatest conductivity for a given cost and bulk.

3028. In addition to this drop of potential, the presence of the current introduces reactions between the magnetic field in which the conductors are moved and the field due to the current itself. (Arts. **2438** and **2439.**)

These reactions result in a tendency for the conductor to move, relative to the lines of force of the field, in a direction at right angles to its own length and to the direction of the lines of force.

The amount of the force is proportional to the amount of current and also to the density of the magnetic field, measured in a plane at right angles to the direction of the lines of force. If the length of the conductor lies in this plane, the force acting *on each centimeter of its length*, when the field is of unit density (one line of force per square centimeter) and a current of one absolute (C. G. S.) unit is flowing through it, is one *dyne.* From this it follows that, calling

A the current in absolute units, $\mathbf{B}$ the density of the field in lines of force per square centimeter, and L the length of the conductor that is within the limits of the field in centimeters, the force on the whole conductor in dynes,

$$f = A\,\mathbf{B}\,L. \qquad (478.)$$

3029. This force acts in a direction at right angles to the length of the conductor and to the direction of the lines of force; it is evident, however, that f may be resolved into components in any other direction, by a method similar to that used in finding the E. M. F. generated in a moving conductor. If this is done, it will be found that similar results are obtained; namely, that the component of the force f in any direction is equal to the value of f given by the above formula multiplied by the sines of the angles which the direction of the component makes both with the length of the conductor and with the direction of the lines of force, and of the angle which the length of the conductor makes with the lines of force. From this it follows that if any one of these angles is equal to 0°, the component to that direction is also 0; that is, there is no tendency for the conductor to move in a direction parallel to its own length or parallel to the lines of force, nor in any direction if the length of the conductor is parallel to the lines of force.

It will be seen that the maximum component of the direction of the force is in the same direction as the motion required to produce the maximum E. M. F. in the conductor; and, further, in any other direction the force is reduced in the same proportion as the E. M. F. would be reduced from the maximum by movement in the same direction.

3030. If, then, the E. M. F. generated in a moving conductor is allowed to cause a current to flow, the reaction of the current in the magnetic field will cause the conductor to tend to move in a direction opposite to its motion in generating the E. M. F. (See Arts. **2439** and **2440.**) In order to move the conductor, it is necessary, then, to apply to it a force (neglecting inertia, friction, etc.) equal to the component of the reaction between the current and the field that is in the direction in which the conductor is moved.

The product of this force and the distance through which the conductor is moved is evidently the mechanical work done upon the conductor; hence, the product of the force and the distance through which the conductor is moved in unit time is equal to the rate at which energy is expended in moving the conductor. In C. G. S. units, then, $w = A\,\mathbf{B}\,L\,M$, where w is the rate of doing work in *ergs* per second; A, $\mathbf{B}$, and L have the same values as before, and M is the distance through which the conductor is moved in one second, in centimeters.

As 10^7 ergs per second equal 1 watt, dividing both sides of the equation by 10^7 gives the power directly in watts, or $W = \frac{A\,\mathbf{B}\,L\,M}{10^7}$. By dividing the upper term of the fraction by 10, C, the current in amperes, may be substituted for A, and the formula will then read $W = \frac{C\,\mathbf{B}\,L\,M}{10^8}$.

Formula **477,** which gives the E. M. F. generated in the moving conductor, may, on the assumption here made that the angles between the path of the conductor and its length, and the direction of the lines of force, are all 90°, be written $E = \frac{\mathbf{B}\,L\,M}{10^8}$, in which $\mathbf{B}$, L, and M have each the same value as in the above formula; which, therefore, may be written $W = C\,E$.

This means *that the work done in moving a conductor through a magnetic field* (*neglecting friction and inertia*) *is equal to the work done by the resulting E. M. F. and current;* which also follows from the law of conservation of energy.

3031. From this it will be seen that it is necessary to supply mechanical power to the armature of a dynamo in order that it may supply a current, and the manner in which this power is expended should also be clear.

In commercial apparatus, it is evident that aside from the power required to move the conductors, an additional amount of power must be supplied for overcoming the friction and all other sources of loss that may exist in the mechanism which is used for moving the conductors; further, the amount of electrical energy that appears in the external circuit is less than the total energy generated, by the amount expended in heating the conductors in which the E. M. F. is generated, as has already been pointed out. The ratio of the energy appearing in the external circuit, or the output of the dynamo, to the total amount generated, is called the **electrical efficiency** of the dynamo; the ratio of the output to the input, the input being the total amount of energy mechanically applied to the conductors to move them, including all losses in the mechanism used, is called the **commercial efficiency** of the machine. Both values are usually given in per cent. of the input; it is evident that this percentage must be less than 100, and in commercial machines it ranges from 75% to 95%, or higher, depending upon the size and design.

In finding the efficiency, both output and input must be reduced to the same units.

3032. If, instead of mechanically moving the conductors through the magnetic field, they be located therein with their lengths at an angle to the lines of force, and an E. M. F. from some external source be applied to the terminals of the conductors, a current will flow through them, and the reaction between the field produced by the current and the field in which it is located will produce a tendency of the conductors to move relatively to the magnetic field, just as when the current is produced by the E. M. F. generated in the conductors themselves. This tendency is exerted in a direction at right angles to the lines of force and to the length of the conductor, producing a force acting in that direction, which may be resolved into components acting in any other direction.

If the conductor be free to move in any direction, then this force will cause it to move, provided its component in that direction is greater than 0, and the rate at which work will be done by the conductor in moving will be equal to the product of the value of the force in the direction of motion and the distance moved in unit time, i. e., the velocity. This apparatus is then an **electric motor,** capable of doing external mechanical work.

3033. The motion of the conductors through the field under these conditions will set up in them an E. M. F. which is opposite in direction to the E. M. F. which is sending the current through the conductors. (See Arts. **2439** and **2440.**)

It is evident, then, that in order to keep up the strength of the current in the moving conductors, the E. M. F. applied to their terminals must be equal to the E. M. F. generated, plus the drop or fall of potential in the conductors.

3034. It has already been shown that the energy represented by the product of the force on the conductor and the velocity of the conductor is equal to the energy represented by the product of the current flowing and the E. M. F. generated by the motion of the conductor. Calling e the E. M. F. generated in the winding and C the current flowing, as before, the mechanical work done by the moving conductor is $e\,C$. The energy represented by $C^2 R_i$ (R_i being the internal resistance of the conductors), as in the case previously considered, appears only as heat. The sum of these two, then, equals the total amount of energy which must be put into the apparatus to move the conductors; that is, $e\,C + C^2 R_i = E\,C$, E being the E. M. F. applied to the terminals of the winding. (Compare this with Art. **3030.**)

3035. It appears, then, given a conductor, or several conductors, situated in a magnetic field, with their lengths at an angle to the lines of force, that the conductors may be moved by the application of a mechanical force so that an E. M. F. will be generated in them, and this E. M. F. may be utilized in sending a current through an external circuit; this is the dynamo, in which the mechanical energy supplied is converted into electrical energy. Or, an E. M. F. may be applied to the terminals of the conductors, causing a current to flow through them, which will cause them to move through the field; this is the motor, in which the electrical energy supplied is converted into mechanical energy. It will be seen that precisely the same features are required for both kinds of apparatus, and the same actions go on in both, the distinction being that what is the *output* of one is the *input* of the other. If the conductors had no resistance, and there was no friction or any other loss in the mechanism used to transmit the mechanical energy to or from the conductors, then the input would equal the output, and the efficiency would be 100%. This is manifestly impossible, so that the input must exceed the output by the amount of energy lost in heating the conductors and in the mechanism used to move the conductors, or that is moved by the conductors; that is, the efficiency is *always* less than 100%, in either dynamos or motors, and the various losses are of the same kind in both cases. This subject will be taken up more in detail later.

The development of the various systems of armature winding from the principles that have been given will now be taken up.

GRAPHICAL REPRESENTATION OF E. M. F. OR CURRENT.

3036. The value of the E. M. F. generated in a moving conductor during successive instants may be graphically represented by a "curve" on cross-section paper, the method usually adopted being to make the ordinates represent the E. M. F. and the abscissas either intervals of time or, what usually amounts to the same thing, distances passed over by the conductor. In the case of a conductor moving in a straight line at a constant velocity through a uniform magnetic field of unlimited extent, the E. M F. generated at any instant is constant, and would, therefore, be represented by a straight line parallel to the axis of abscissas, and at a certain distance from that axis, depending upon the value of the E. M. F. generated and the scale selected to represent it. Thus, the graphical representation of the E. M. F. generated in the conductor, as given in the example in Art. **3018,** would be a straight line parallel to the axis of abscissas and at a distance from it equal to 4.5 volts on the scale of the ordinates, as shown in Fig. 1127.

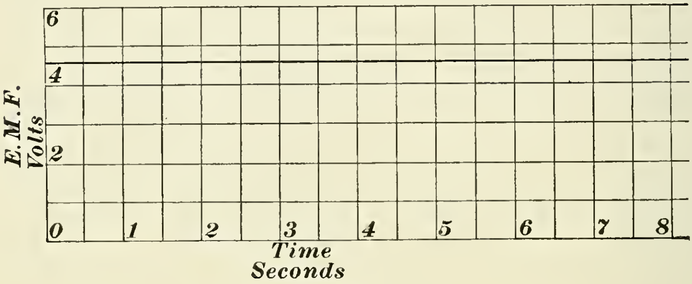


FIG. 1127.

The direction of the E. M. F. may be found from the rule given in Art. **2442,** the E. M. F. being considered as having the direction in which the current which it would produce is considered to flow.

By applying this rule, it will be seen that if *either* the direction of the lines of force or the direction of motion of the conductor be reversed, the direction of the E. M. F. is also reversed, but if *both* be reversed, the direction of the E. M. F. is unchanged.

3037. If the *direction* of the motion of a moving conductor is instantly reversed, but the *velocity* maintained constant, the E. M. F. generated in that conductor will be reversed in direction, but unchanged in value. In order to represent this condition graphically, it is necessary to make some distinction whereby the change in the direction of the E. M. F. will be indicated. This is done by plotting the E. M. F. curve on both sides of the axis of the abscissas; assigning to one side the values of the E. M. F. when in one direction, and to the other its values when in the opposite direction, both to the same scale. It is customary to plot the curve of the E. M. F. or current when in a *positive* direction with respect to some given part of the circuit *above* the axis of the abscissas, so that this part of the curve is considered as + in direction.

Thus, if the conductor giving the curve represented in Fig. 1127 had been moved in one direction for 2 seconds, then instantly reversed and moved in the opposite direction at the same velocity for 2 seconds, then reversed again and so on, the curve of the E. M. F. would be as represented in Fig. 1128.

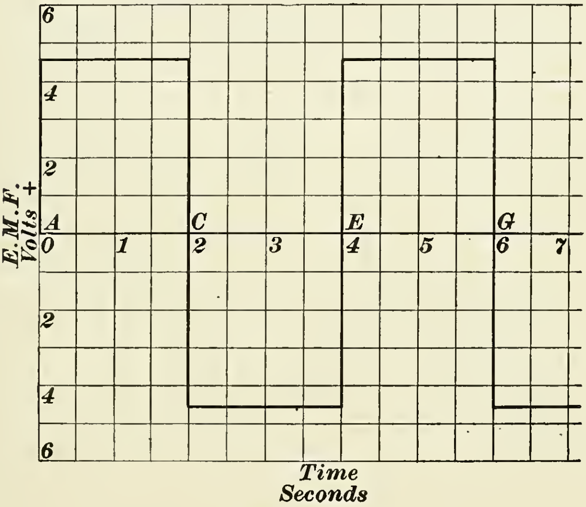


FIG. 1128.

Here from *A* to *C* the curve is the same as Fig. 1127, 4.5 volts in one (the +) direction; at *C* (the end of the 2d second) the direction of the motion, also the E. M. F., is reversed, and is represented by the line drawn on the other side of the axis of the abscissas from *C* to *E*, where the E. M. F. is again reversed, and so on.

As at the instant the E. M. F. is reversed it passes through all the intermediate values between 4.5 volts in one direction and 4.5 volts in the other, a line must be drawn to indicate all these values, as shown; the reversal being assumed to be instantaneous, this line coincides with the ordinate which passes through the abscissa which represents the time at which the reversal took place, as at 2 seconds, 4 seconds, 6 seconds, etc.

3038. If the change from the maximum E. M. F. in one direction to the maximum in the other were not instantaneous, this line would not coincide with one of the ordinates; for example, assume that this same conductor was not moved at a constant velocity, but with a uniform *acceleration* of such an amount that, starting from zero, the velocity at the end of the 1st second was the same as the constant value assumed in the previous case; then the E. M. F. generated at successive instants during this 1st second would be represented, as in Fig. 1129, by a straight line commencing at 0 volts at 0 time and rising to 4.5 volts at the end of the 1st second.

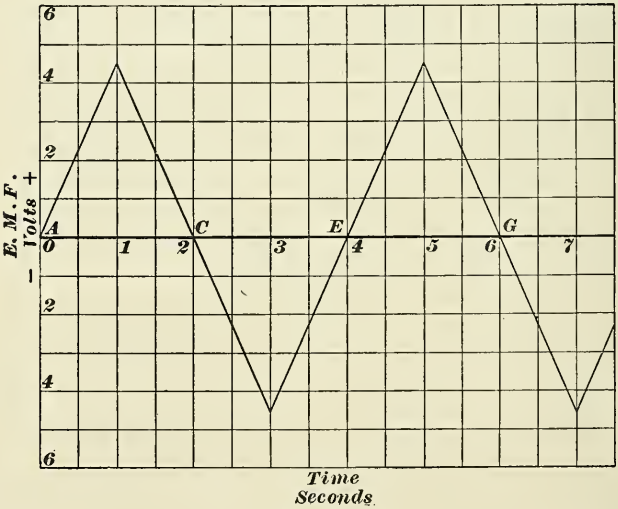


FIG. 1129.

If from this point the velocity of the conductor is retarded at the same rate as it was previously accelerated, its velocity at the end of the 2d second would be 0 and the curve of the

E. M. F. generated during the 2d second would again be a straight line, commencing at 4.5 volts at the end of the 1st second and falling to 0 volts at the end of the 2d second.

If the conductor is now moved in the opposite direction and the same cycle of acceleration and retardation gone through with, the curve of the E. M. F. generated during this cycle of motions will be of exactly the same shape as that for the previous cycle, but will lie on the opposite side of the line, as is represented by the part *C E* of the curve shown in Fig. 1129. Continuation of this cycle of motion gives a series of repetitions of this curve, as represented in Fig. 1129.

3039. A similar curve would result if the velocity of the conductor had been kept constant, as in the case in Art. **3037,** but the density of the field had been uniformly varied along the path of the conductor from 0 to a maximum, and then to 0 again.

Both of these cases assume that the change from accelerating to diminishing velocity, or from increasing to decreasing density, is instantaneous, as indicated by the sharp peaks of the E. M. F. curve. In any apparatus as actually constructed these changes would be more gradual, which would result in more or less rounding the peaks of the curve.

3040. If a conductor is moved in a straight line through a succession of magnetic fields of alternate polarity, as represented in Fig. 1130, the curve of the E. M. F. generated will be of similar character to those shown in Figs. 1128 and 1129; that is, the E. M. F. will be maintained in one direction as long as the conductor is moving through a field of

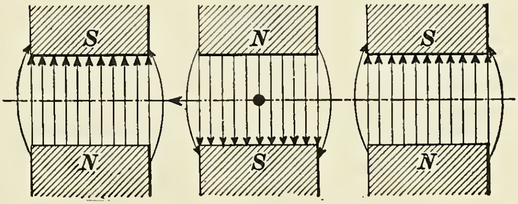


FIG. 1130.

one polarity, but as soon as the direction of the lines of force is reversed, the E. M. F. is also reversed. (Art. **3036.**) The actual shape of the waves of the curve will depend, as in the previous cases, upon the variation in the uniformity of the density of the field or of the velocity.

The manner of finding the E. M. F. generated in any conductor moved in any *straight* line through a magnetic field of any condition of density should now be clear.

It is evident, however, that motion in a straight line or in any irregular line is not in general desirable for dynamos; motion in a straight line can not be indefinitely continued, for that would require a field of unlimited extent, and sudden changes in the direction of motion of the conductor do not usually permit of good mechanical construction. To avoid both the infinite field and the sudden changes in the direction of motion, the conductor may be moved in a circular path, which is mechanically convenient and allows of the use of a field of limited extent; the effect of such motion will now be considered.

3041. In the case of a conductor moving in a circular path through a field of uniform density, if the lines of force are at every point radial to the path of the conductor, as represented in Fig. 1131, the angle which the direction of motion of the conductor makes with the lines of force at every instant is constant; the E. M. F. generated in the conductor is, therefore, constant as long as a constant velocity is maintained.

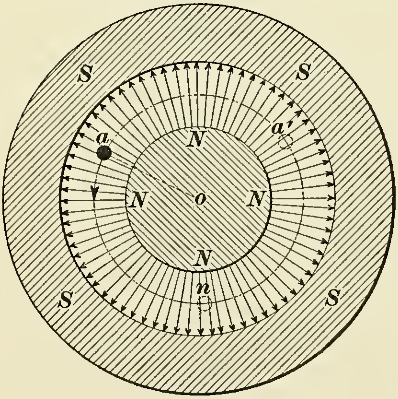


FIG. 1131.

In order to obtain the distribution of the lines of force required in the above case, one pole of the magnet (S S S S) must be made in the form of a hollow cylinder, concentric with and enclosing the path of the conductor (*a n a'*) and the other pole (*N N N N*). In the space between these two poles the lines of force will be radial in direction and uniform in density if the rest of the magnetic circuit is properly designed.

3042. If the direction of the length of the conductor is radial, instead of parallel to the axis, as in the above case, by making the lines of force parallel to the axis and uniform in density all along the path of the conductor, the angle between the lines of force and the direction of motion of the conductor will at all instants be the same; hence, the E. M. F. will be of the same character as before. This arrangement of the lines of force may be obtained by making the pole faces circular in shape and placing them in two parallel planes with the center of the pole-pieces coinciding with the axis of rotation of the conductor, as represented in Fig. 1132.

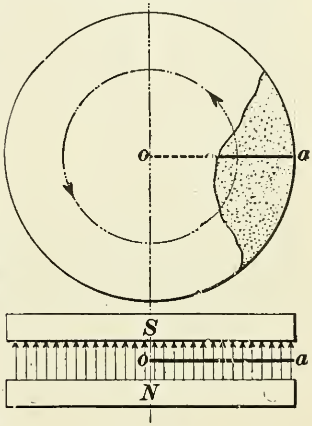


FIG. 1132.

The arrangement of the balance of the magnetic circuit is not indicated in this figure. A part of the north pole-piece is broken away in the elevation to show the conductor *a o*.

THE SINE CURVE.

3043. Fig. 1133 illustrates a case of circular motion which differs in many features from the first two considered. Here the lines of force are parallel to each other and at right angles to the axis of rotation; consequently, the angle between the direction of motion and the direction of the lines of force changes at every instant. From this it follows that the E. M. F. also varies during successive instants. Although the direction of motion of the conductor changes, at any one point it may be considered to be along a straight line tangent to the circle at that point, and a certain length may be assigned to this line which will represent the velocity of the moving conductor. Thus, at the instant when the conductor is at the point *a* (Fig. 1133) of its path, the line representing its direction of motion at that point makes an angle of 0° (is parallel) with the lines of force; hence, no E. M. F. is generated at this point. As the conductor moves along its path, this angle becomes greater until at a point 90° from *a* the angle is also 90°, and the E. M. F. generated is a maximum. Further continuation of the motion for another 90° decreases the angle until it is again 0°, and the E. M. F. is again zero.

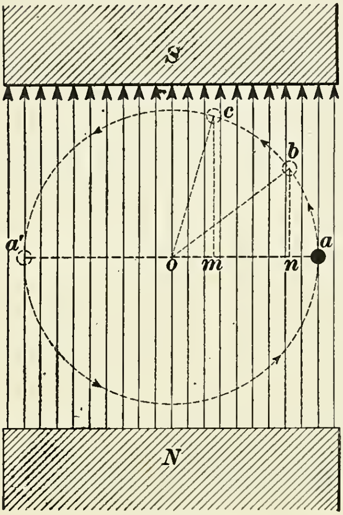


Fig. 1133.

The remainder of the revolution repeats this cycle of changes of the angle, but, as the conductor is moving in the opposite direction relative to the lines of force, the E. M. F. is reversed in direction. From this it follows that the curve of this E. M. F. would form a series of waves on each side of the axis of the abscissas, as in the cases described in Arts. **3037** and **3038.**

3044. The shape of the waves of this curve may be found by calculating the E. M. F. generated at successive intervals of time; this E. M. F. will evidently be proportional to the sine of the angle which the direction of motion makes with the lines of force (see Art. **3020**), so that by constructing at any point on the circle a right-angled triangle which includes the above angle, the side opposite this angle may be used as the length of the ordinate representing the E. M. F. generated at that point.

This process is represented in Fig. 1134, where *b* represents a certain point in the path of the conductor *a* at which it is desired to know the E. M. F. generated. The line *b r*, tangent to the circle at the point *b*, represents the direction of the motion of the conductor at this point, and makes the angle *p* with the direction of the lines of force, represented by the line *r t*. The line *t b*, at right angles to *r t*, is, then, the sine of the angle *p*; hence, it is a measure of the E. M. F. generated in the conductor at the point *b*, and by repeating this process for successive points around the circle a series of values will be obtained which may be used as ordinates of the E. M. F. curve, as stated.

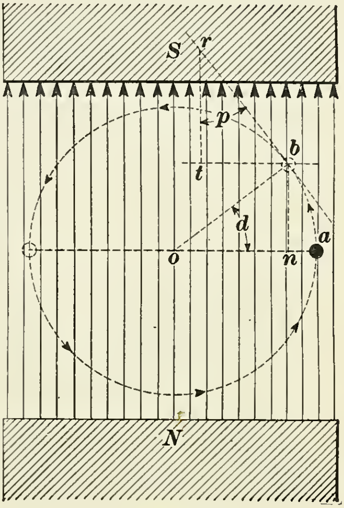
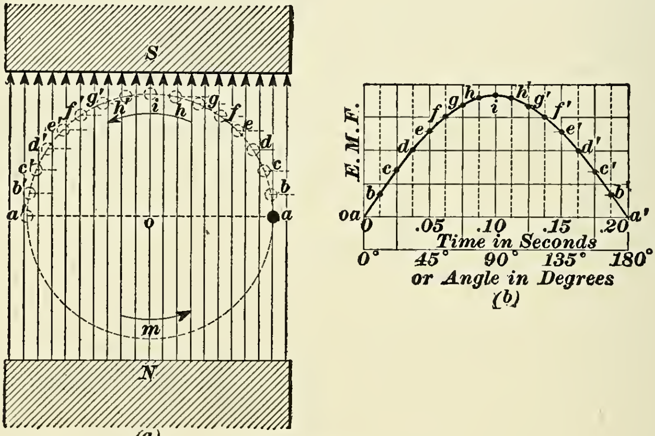


FIG. 1134.

From the construction of the figure, the line *o b* is perpendicular to the line *b r*, and the line *o a* is perpendicular to the line *r t*; hence, the angle *d* included between the lines *o a* and *o b* is equal to the angle *p* included between the lines *b r* and *r t*, and the E. M. F. is, therefore, proportional to the sine of the angle *d*, or to the length of the line *b n*. This length (*b n*) may or may not *equal* the E. M. F. when laid out on any scale, but will always be *proportional* to it.

Hence, the E. M. F. generated at any instant in a conductor moving in a circular path through a magnetic field of uniform density whose lines of force are at right angles to the axis of rotation and parallel to each other is *proportional to the sine of the angle through which the conductor has been rotated from a point where its direction of motion is parallel to the lines of force.*

3045. If the velocity of the conductor is uniform (as is assumed above), the conductor will move through equal angles in equal intervals of time; hence, the abscissas of the E. M. F. curve may represent either the intervals of time required for the conductor to move through the various angles or the angles themselves. The ordinates may represent the sines of the same angles, the E. M. F. being proportional to them. This curve may be conveniently drawn by laying off on a circle a series of points representing successive positions of the conductor after equal intervals of time, as represented at *a*, *b*, *c*, ... *a′* in Fig. 1135 (*a*), *a a′* being a diameter at right angles to the lines of force. The vertical height of any of these points above the diameter *a a′* is, then, the sine of the angle through which the conductor has been moved, and these heights may be projected on the ordinates of properly arranged cross-section paper, as represented in Fig. 1135 (*b*), giving the curve *a*, *b*, *c*, ... *a′*, as shown. The curve for the remainder of the revolution, *a′ m a*, may be laid out in the same way, and would evidently be of the same shape, but would be on the opposite side of the axis of the abscissas. This form of curve is called a **sine curve,** or **sinusoid,** from its method of con-



(*a*) (*b*)

FIG. 1135.

struction, and possesses several peculiar features, as will be pointed out.

3046. Value of E. M. F.—From the explanation given in Art. **3024,** it will be seen that the E. M. F. generated in the conductor in the above case at any instant may be found from formula **477**; $r°$ and $s°$ being in this case each equal to 90°, sin $r°$ and sin $s°$ are each equal to 1, and may be omitted, making the formula read

$$E = \frac{\mathbf{B}\, L\, M \sin p°}{10^8}.$$

It is evident that during one complete revolution the conductor passes over a distance equal to $2\,\pi\,r$, r being the radius of its circular path, or $o\,a$ (Fig. 1135).

Then, if the speed of the conductor is S revolutions per minute, its velocity is equal to $2\,\pi\,r\,\frac{S}{60}$ $\left(\frac{S}{60}\right.$ being the revolutions per second), and this value may be substituted for M in the above formula, which would then read

$$E = \frac{\mathbf{B}\, L\, 2\,\pi\, r\, \frac{S}{60} \sin p°}{10^8}.$$

All the lines of force cut by the conductor in each revolution are included in the area obtained by multiplying together L and $2\,r$. This being the case, the total number of lines cut by the conductor, which is represented by N, is equal to $\mathbf{B}\, L\, 2\, r$.

From this, $\frac{N}{2\,r} = \mathbf{B}\, L$, which value may be substituted for $\mathbf{B}\, L$ in the previous formula, which then reads as follows:

$$E = \frac{\frac{N}{2\,r}\, 2\,\pi\, r\, \frac{S}{60} \sin p°}{10^8}.$$

Simplifying this, it becomes

$$E = \frac{N\,\pi\, S \sin p°}{10^8 \times 60}.$$

When E is at its maximum value, E_m, $p^\circ = 90^\circ$ and $\sin p^\circ = 1$; hence,

$$E_m = \frac{N \pi S}{10^8 \times 60}. \qquad \mathbf{(479.)}$$

THE AIR-GAP.

3047. If the field through which the conductor moves in its circular path, having a radius $o\,a$, exists between two parallel pole faces, as represented in Fig. 1135, it is evident that the distance between the pole faces must be a little greater than $2\,o\,a$, in order that the conductor may not touch the pole faces in its rotation.

This distance between the pole faces introduces an air-gap of great length compared to its area, thus requiring a considerable expenditure of M. M. F. (magnetomotive force) in forcing the field across the gap.

The average length of this air-gap may be reduced by making the pole faces concentric with the path of the conductor, but as in this case the length of the air-gap is no longer uniform, the density of the field in the gap will not be uniform; and, further, the increased density is situated at a point where it has the least effect; i. e., at the point where the conductor is moving in a direction nearly or quite parallel to the lines of force, as represented in Fig. 1136.

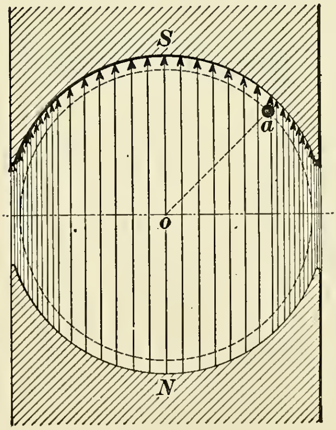


Fig. 1136.

3048. As there is actually required for the movement of the conductor only a thin cylindrical space near the pole faces, the length of the air-gap may be reduced very largely by filling in the space not required for the movement of the conductor with a cylindrical core of iron. The distribution of the lines of force will now be materially different from that shown in Fig. 1136; the length of the air-gap being practically uniform and much shorter, the direction of the lines of force will be in nearly the shortest distance across the gap, i.e., will be nearly radial, and the density will be practically uniform and, with the same M. M. F., much higher.

The actual distribution of the lines of force will be about as represented in Fig. 1137. It will be noted that some of the lines of force do not pass into the core at all, and are even entirely outside the path of the conductor. These *leakage lines* (see Art. **2415**) are always present in any magnetic circuit which includes an air-gap, although they have not thus far been represented. Their influence on the design of the magnetic circuit may be neglected for the present, but will be fully discussed later.

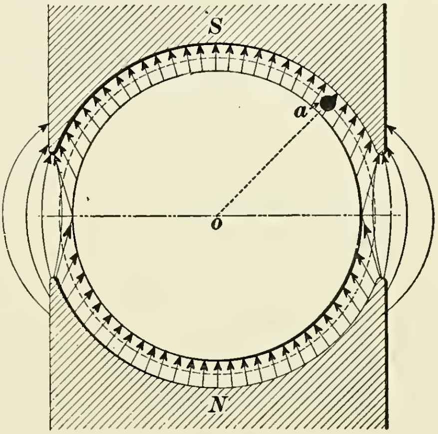


FIG. 1137.

3049. If the lines of force in the air-gap were absolutely radial and of uniform density throughout, the direction of the motion of a conductor moving through this air-gap would at any instant be at right angles to the lines of force, and (assuming a constant velocity) the E. M. F. would be constant in value, but, of course, reversed at the end of each half revolution. The E. M. F. curve would, therefore, be similar to that shown in Fig. 1128, Art. **3037.**

This differs from the case described in Art. **3041,** in that the direction of the motion of the conductor with respect to the direction of the lines of force is not constant, but is reversed at the end of each half of a revolution, which causes the E. M. F. to be reversed, as stated.

The actual distribution of the lines being different from this just described, the curve is actually more like the sine curve (Fig. 1135), but with a more flattened top, consequently a more rapid increase from zero to values near the maximum.

ARMATURE CORE LOSSES.

3050. A very convenient method of moving the conductor through the magnetic field is to mechanically attach it to the surface of the cylindrical core, which may then be rotated around its axis by any convenient means. The motion of the core does not of itself affect the distribution or the density of the lines of force ; but in order to maintain the motion of the core, certain losses, due to *hysteresis* and to *eddy currents* circulating in the core, must be overcome.

3051. The hysteresis is due to the continual change in the direction of the lines of force through the core, as it rotates, amounting to one complete reversal in each half revolution ; the amount of the hysteresis loss depends upon the quality of the iron of which the core is made, the density of the lines of force in the core, the number of reversals of magnetism per unit of time, and the amount of iron affected. (Art. **2413.**)

3052. The eddy-current loss is due to the fact that, the iron of the core being a conductor, the E. M. F. generated in it by its rotation in the magnetic field causes currents to circulate through the mass of metal in the core ; these currents do not differ from the currents flowing in the conductors attached to the surface of the core, but as they do not appear in the external circuit, they represent so much wasted energy.

3053. The E. M. F. of these eddy currents is necessarily low ; but if the core is a solid mass of metal, the resistance offered to these currents is extremely small, so that the small E. M. F. may cause enormous currents to flow, which would thereby be the source of a great loss of energy.

The direction of these currents may be found by applying the rule given in Art. **2442,** when it will be seen that if the direction of the current in a section of the core under one pole is from front to back, under the other pole it will be from back to front, so that these currents will circulate around the core, as represented in Fig. 1138, in which only the lower half of the core is represented, the paths of the eddy currents being indicated by the lines with the arrow-heads.

In order to reduce the value of the eddy currents as much as possible, it is evidently necessary to reduce their E. M. F. and to increase the resistance of their path. This is usually accomplished by building up the core of a number of thin iron disks, as represented in Fig. 1139, arranged parallel to the lines of force and at right angles to the axis of rotation, and insulated from one another.

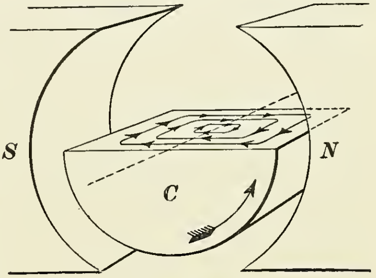


FIG. 1138.

Instead of being one single conductor of large section, the core is now made up of a number of conductors of less section and shorter length; the E. M. F. generated in each conductor is, therefore, much less, and the current produced thereby is relatively much smaller than in the case of the solid core, so that the loss of energy is reduced.

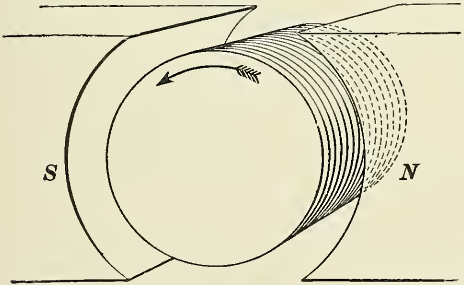


FIG. 1139.

3054. This process of dividing the core into thin plane sections is called **lamination,** the separate sections forming the **laminae.** Lamination does not affect the magnetic qualities of the core, since all the sections are continuous in the direction of the lines of force.

Building up the core of lightly insulated iron wire will also prevent eddy currents, but as in this case the iron of the core is not magnetically continuous, the reluctance of the core as a whole is much greater than that of the iron of which it is composed. The laminated structure is, therefore, most extensively used.

3055. As in the case mentioned in Art. **3042,** the direction of the length of the conductor may be radial instead of parallel to the axis of rotation. In that case the cylindrical core would take the form of a disk, and the lines of force would enter and leave the core at the end faces instead of the cylindrical face, as represented in Fig. 1140, *a b* being the conductor supported on the core *C*, which rotates about the axis *o o'*.

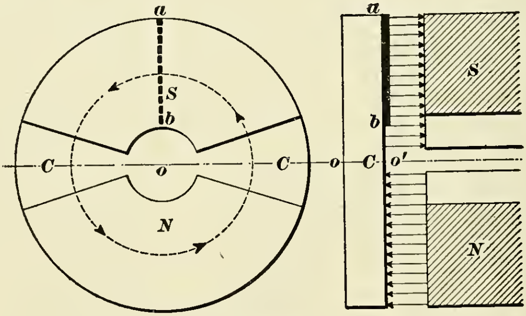


FIG. 1140.

The curve of the E. M. F. generated under these circumstances would be similar to that mentioned in Art. **3049,** i. e., similar to a sine curve, but with flatter peaks to the waves.

From an inspection of the figure, it will be seen that if the core were made up of disks arranged at right angles to the axis of rotation as in the previous case, the laminæ would not act to reduce the E. M. F. available for the production of eddy currents; so this type of core is usually constructed of a long strip of thin iron, wound in a helical form, with strips of insulating material between the layers. This makes a core of the requisite form that is magnetically continuous in the direction of the lines of force, and is free from excessive eddy currents.

CHARACTER OF COMMERCIAL CURRENTS.

3056. Any electric current in commercial use may be classified either as a **direct current** or an **alternating current.** The abbreviations for these are D. C., direct current, and A. C., alternating current.

A *direct current* may be defined as *a current which always flows in the same direction through the conductor or circuit.*

A direct current may be **continuous,** so-called, or **pulsating.** A strictly continuous current is one in which the E. M. F. has an absolutely constant value during succeeding intervals of time, which would, therefore, cause a perfectly steady current to flow through a circuit of constant resistance.

The curve of a continuous current would then be a straight line parallel to the axis of the abscissas, as represented in Fig. 1127. Incandescent dynamos or constant potential dynamos are familiar examples of machines that furnish continuous currents.

3057. The pulsating current is practically unknown in the commercial world, and although the current from a Thomson-Houston arc machine may be called a pulsating current, yet it is never mentioned as such. As will be explained farther on, a pulsating current is one which always flows in the same direction, but the electromotive force constantly varies, so that the current consists of distinct impulses or rushes of current.

In Fig. 1141, (*a*), (*b*), and (*c*) represent three possible curves of pulsating currents; in (*a*) the fluctuations of the E. M. F. or current occur between a maximum and zero, while in (*b*)

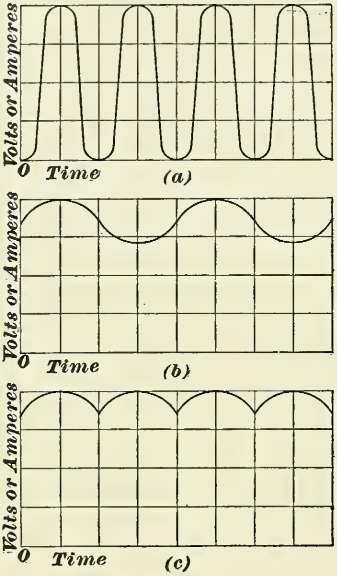


FIG. 1141.

the minimum is about .7 of the maximum; (c) represents a slightly different type of curve, in which the minimum is about .85 of the maximum. It will be seen that either of the last two quite closely approaches a strictly continuous current.

3058. An *alternating current* may be defined as *a current which is continually reversed in direction;* consequently, its E. M. F. and current *alternate* between two opposite maximum values; the curve of the E. M. F. or current would, therefore, lie on both sides of the axis of the abscissas. A dynamo which generates an alternating E. M. F. is called an **alternator.** The E. M. F. generated in a conductor whose direction of motion with respect to the direction of the lines of force is periodically reversed would, therefore, be an alternating E. M. F., as shown by the curves in Figs. 1128, 1129, and 1135.

If the conductor so moved cuts the lines of force in each direction in the same time and at the same rate, the curves of the E. M. F. generated by the motion in each direction will both be of the same shape, and will lie equally on both sides of the axis of the abscissas, and a continuation of the

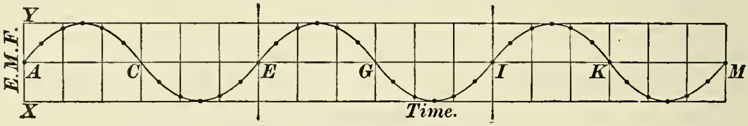


FIG. 1142.

cycle of motions, under the same conditions, will give an E. M. F. curve that is merely a series of repetitions of the curve representing the E. M. F. generated during the first cycle of motions. Such an E. M. F. (or its resulting current) is called a **cyclic, periodic,** or **harmonic** alternating E. M. F. (or current). A curve showing a form of such an E. M. F. is given in Fig. 1142, the axis being $A\ M$; the positive impulses are above, at Y, and the negative impulses at X. The curve crosses the axis at the points C, E, G, etc.

GENERAL PRINCIPLES OF ARMATURE WINDINGS.

3059. It should be clear that rotary motion of a conductor in a magnetic field may be divided into two general classes: (1) where the arrangement of the field with regard to the path of the conductor is such that the direction of the motion is always the same, relative to the direction of the lines of force, and (2) where the arrangement is such that the direction of motion is periodically reversed, relative to the direction of the lines of force.

A further distinction between these two classes is that in the first class each line of force is cut *only once in each revolution*, while in the second class each line of force is cut *twice in each revolution.* This has given rise to the names **unipolar** and **bipolar** (or **multipolar**) induction for the two classes; i. e., the E. M. F. generated in a conductor so arranged as to come under the first class would be said to be due to *unipolar induction*, etc. These terms have been extended to the machines themselves, so a machine in which the E. M. F. is generated by unipolar induction is called an *unipolar dynamo;* if the E. M. F. is generated by bipolar induction, it is called a *bipolar dynamo*, and so on. This application of the term *unipolar* is hardly correct, since an "unipolar dynamo" must necessarily have two poles. Its application to induction, however, is more accurate, because, aside from its influence on the design of the magnetic circuit, the presence of more than one pole is not necessary in considering this class of induction; that is, as each line of force is cut only at one point, it does not matter what course it takes after being cut. With bipolar or multipolar induction, it is necessary that the lines from each magnet be grouped together in the same manner at the two separate points of their own path at which they are cut by the conductor, which is most conveniently done by making these points the surfaces of the poles of the magnet. Still, the distinction is usually applied to the machines themselves.

3060. It has been shown that with either unipolar or bipolar induction the conductor may occupy one of two radically different positions; namely, the direction of its length may be *parallel* or *radial* to the axis of rotation. In either case, with unipolar induction, as illustrated in Arts. **3041** and **3042,** Figs. 1131 and 1132, it is evident that the E. M. F. generated in the conductor is a *direct* E. M. F. in the sense of being continuous in direction, while with bipolar induction, as illustrated in the cases given in Art. **3043** and those following, the E. M. F. generated in the conductor is an *alternating* E. M. F.

3061. In order to electrically connect a stationary external circuit with the moving conductor, some form of sliding or rubbing contact is necessary, which usually takes the form of stationary strips of copper, carbon, or other conducting material called **brushes,** which form the terminals of the external circuit, and which rest upon bare metallic surfaces which are electrically connected to the conductors and mechanically attached to but insulated from the shaft by which the armature core, conductors, and collecting devices are driven. In case it is desired to make continuous connection throughout the revolution with the conductor, these bare metallic surfaces are made continuous, i. e., in the form of rings, and the device is then called a **collector,** while if it is desired to make the connection between the conductors and the external circuit during a part of a revolution only, the bare metallic surfaces are made segmental. It is never the case that the external circuit is entirely disconnected from all the conductors of the armature at the same time, so that if any particular conductor is disconnected from one of the terminals of the external circuit at any time during the revolution, another must be substituted. This results in a collecting apparatus consisting of a series of separate metallic segments arranged in cylindrical form, each connected to some part of the winding, forming a device called a **commutator.**

From the nature of the device, the character of the difference of potential which appears between the terminals of the external circuit (the brushes), if a *collector* is used, is the same as the character of the E. M. F. generated in the conductors; i. e., it is subject to the same fluctuations in value or reversals of direction.

If a *commutator* is used, this is not necessarily the case; in fact, is not likely to be, since the connection with any particular conductor is not maintained throughout that conductor's cycle of motion, so that the character of the E. M. F. generated is not reproduced in the difference of potential existing between the brushes. This is an important distinction, since it is the character of this difference of potential which *directly* determines that of the current in the external circuit, and not the character of the E. M. F. generated.

3062. It has been pointed out that, in order to generate a sufficiently high E. M. F. for commercial applications, a number of conductors must be used, so connected together that their E. M. F.'s will add together to the desired amount. These conductors may obviously be located in the same magnetic field, and rotated under the same conditions; then, the E. M. F. of each will pass through exactly the same cycle, with a phase difference depending upon their relative positions in the field at any instant. From a study of these features, the proper methods of connecting the conductors to each other and to the external circuit to attain any desired result may be deduced.

Fig. 1143 represents 16 conductors, a, b, c, ... p,

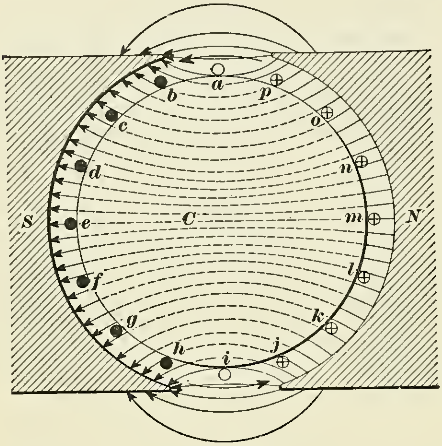


FIG. 1143.

equally spaced around the periphery of the core *C*. The direction of the lines of force being from *N* to *S*, and the direction of motion being as indicated by the arrows, the direction of the E. M. F. in the conductors under the *N* pole face is from back to front, while in those under the *S* pole face it is from front to back, as will be seen if the hand rule (Art. **2442**) is applied. This is indicated by marking the conductor with a + or a solid black dot, as shown. Conductors in the positions *a* and *i*, being in such a position that no lines of force are cut by them, have no E. M. F., and are not marked.

3063. In connecting two or more conductors in series, it is evident that the maximum E. M. F. will result when the E. M. F.'s of the two conductors coincide in phase, for, otherwise, at a part of their cycle the E. M. F.'s in the two conductors would oppose each other; the same result may be obtained if the phases are displaced 180°, for then the two conductors will each have a maximum E. M. F. generated in them at the same instant, and although these two E. M. F.'s would be represented in a clock diagram as acting in opposite directions, the conductors may be so connected that the E. M. F.'s will add together. This is represented by the diagram, Fig. 1144, where *a b* and *c d* represent two conductors, in which the E. M. F.'s generated are in opposite directions, as indicated by the arrow-heads, but by

FIG. 1144.

connecting the ends *b* and *c* together the difference of potential between *a* and *d* is equal to the sum of the two E. M. F.'s.

Applying these principles to the case illustrated in Fig. 1144, it is evident that the proper conductor with which any one of the conductors—for example, conductor *e*—should be connected in series is either the conductor diametrically opposite it, in this case conductor *m*, or either of the conductors immediately adjacent to it, in this case either conductor *f* or *d*.

NOTE.—The figure being very much out of proportion, the angular distance between these adjacent conductors would seem to be sufficient to cause a considerable difference in their phase; in practice, however, the angular distance between adjacent conductors would be very small, and the difference in phase of the E. M. F.'s generated in them almost inappreciable.

3064. Applying the principle illustrated in Fig. 1144, opposite conductors would be connected by conductors extending across one of the end faces of the core. But in connecting adjacent conductors in series a different method must be followed, since the ends which are similarly situated on the core must be connected together, as illustrated in Fig. 1145, where *a b* and *c d* represent two conductors, in each of which the E. M. F. generated is in the same direction, as indicated by the arrow-heads. In order to connect these two conductors in series, so that the E. M. F.'s will add, the ends *b* and *d* (or *a* and *c*) must be connected together, as represented, in which case the difference of potential between *a* and *c* (or *b* and *d*, if *a* and *c* are connected) would be equal to the sum of the two E. M. F.'s generated.

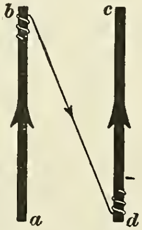


FIG. 1145.

3065. Now, in the armature, the above connection manifestly can not be made across the end faces of the drum; neither can it be made directly across the cylindrical face, for in the latter case an E. M. F. would be set up in the connecting wire, opposite in direction to the E. M. F. of the conductors. The only reasonable way in which this style of connection can be made is to make the armature core in the form of a *cylindrical ring*, and pass the connecting wires through the hole in the center of the ring, as illustrated in Fig. 1146. Here the conductor *e* is connected to the conductor *d* by a wire passing down the back face of the ring, through the hole in the center at *x* and up the front face to *d*.

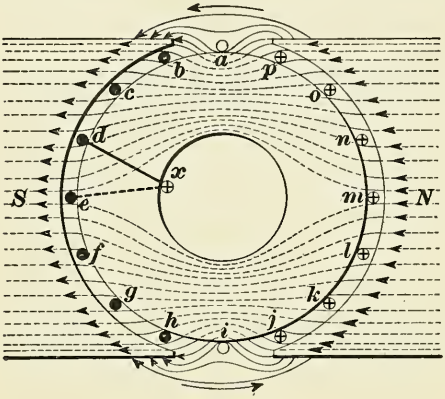


FIG. 1146.

The lines of force pass from pole to pole through the iron of the core, as represented in Fig. 1146, and hence are not cut by this connecting wire *x*, which, therefore, has no E. M. F. generated in it.

3066. These two general methods of connecting conductors in series are called **drum winding** and **ring winding,** respectively, from the shape of the cores used. The first practical use of the ring winding was due to Gramme, hence it is often called the **Gramme winding.** It was invented by Paccinotti. The drum winding was originated by Siemens.

3067. In building up a drum armature core, the disks of which it is composed may be slipped directly on the driving shaft, forming a solid mass of metal; but in the ring core it is necessary to provide a support for the ring-shaped disks, which shall have sufficient strength to drive the armature core and at the same time provide a sufficient opening between the shaft and the inside of the core to admit the connecting wires of the winding. Such a support is called a **spider,** and usually consists of two castings, made with a central hub bored out for the shaft, from which hub a number of thin arms radiate and support the armature core, the connecting wires being wound in between these arms.

3068. It will be seen that if the conductors are arranged *radially* on the end faces of the core, with the pole-pieces facing these surfaces (see Art. **3055**), the same two systems of winding may be followed when connecting the conductors in series. In this case the connecting wires are arranged on the cylindrical surface (or surfaces in a ring winding) instead of the radial surfaces. To get the best results, these armatures are made in the form of a disk; the distinctive features of the ring or drum winding are not altered by this change in the form of the core, but the mechanical construction is materially different.

In order to distinguish between the two forms of cores, those in which the lines of force enter and leave the cores at the cylindrical surface are called **cylinder** armatures, whether the winding be ring or drum, and those in which the lines of force enter and leave at the end face (or faces) are called **disk** armatures. (See Arts. **3041** and **3042**.)

3069. Fig. 1147 illustrates these two methods of connecting conductors in series for cylinder amatures, (*A*) being the ring and (*B*) the drum winding. In each the upper half of the core is removed, showing the loop formed by the conductors and the connections between them. In order to connect the free ends of the loop to the collecting device, or to other conductors, other connecting wires are added, as represented at *1*, *2*. It will be seen that in either form of winding the active conductors and the connecting wires form a coil of one or more turns. In practice these coils are usually formed from a single piece of insulated wire, of suitable length, wrapped around the core a sufficient number of times to make the coil of the requisite number of turns. Each coil so wound covers a certain fraction of the surface of the armature core; in the case of the drum winding, this fraction of the surface is divided into two parts that are on opposite sides of the core, while in the ring winding it is altogether on one side. The amount of surface of the core covered by the coil may be called the *width* of the coil.

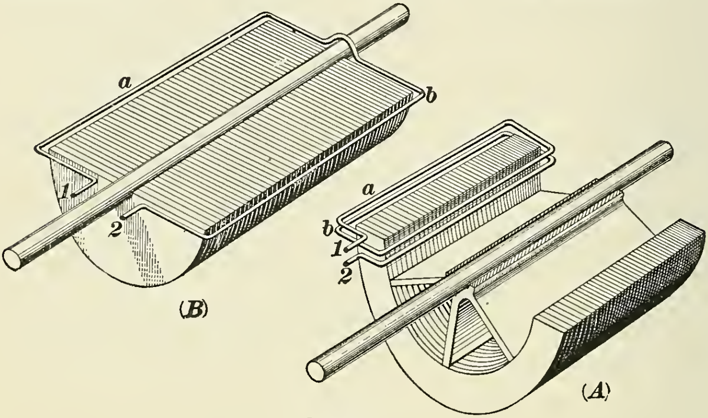


FIG. 1147.

DIRECT-CURRENT ARMATURE WINDINGS.

UNIPOLAR ARMATURES.

3070. In order that an E. M. F. which acts continually in one direction may be generated in a moving conductor, it is necessary that the direction of motion of the conductor be always the same with reference to the direction of the lines of force of the magnetic field in which the motion takes place. Motion in a straight line is here obviously impossible, since it could not be continued for any length of time; motion in a circular path is, therefore, the only kind that answers the requirements.

In Arts. **3041** and **3042** two methods of moving a conductor in a circular path in a constant direction relative to the lines of force are described and illustrated. These are examples of *unipolar induction.* (See Art. **3059.**) In either of the above methods, it is evident that a number of conductors may be used, distributed along their circular path, and in each the same E. M. F. will be generated. In order to obtain a high E. M. F., it would then be desirable to connect these various conductors in series, in such a way that all their E. M. F.'s would be added together.

If this is attempted, it will be found that, owing to the fact that the lines of force form closed loops, it is impossible to *permanently* connect the active conductors in series in any manner so that the connecting wires will not cut the lines of force in such a way as to set up in them an opposing E. M. F. of exactly the same value as that generated in the armature conductors proper. The final effect of connecting any number of armature conductors in series is, therefore, at most only the E. M. F. of a single conductor.

The only way, then, in which the conductors may be connected is by means of sliding contacts, whereby the connecting wires may be stationary with respect to the moving armature conductors.

It is evident that this method is of limited application, since the connections for a large number of conductors would become too complicated.

3071. With unipolar induction, then, the maximum E. M. F. possible is that of a single conductor; it is evident, however, that if a number of separate conductors are used, they may all be connected in *parallel*, which, while it does not increase the E. M. F., does increase the possible current output, since it decreases the internal resistance of the armature winding.

A number of such conductors connected in parallel are equivalent to a single wide conductor; in the case illustrated in Fig. 1131, this would be equivalent to a tube, of a thickness sufficient to allow it to rotate freely between the poles *N* and *S*, while in the case illustrated in Fig. 1132 it would be equivalent to a disk rotating between the poles *N* and *S*.

It is in one or the other of these forms that the armatures of unipolar dynamos, which have a limited application in cases where a large current at a low potential is required, are constructed.

OPEN-COIL BIPOLAR ARMATURES.

3072. The E. M. F. generated in the separate coils of an armature winding which is revolved between the opposite poles of a bipolar magnet is naturally alternating in character, since its direction when passing through one field is opposite to that which it has when passing through the other.

When connected to the external circuit by means of collector rings, this alternating E. M. F. is impressed directly on the external circuit; but by using a suitably arranged commutator, the connections between the coils of the armature winding and the external circuit may be reversed at proper intervals, so that the current in the external circuit will be uniform in direction.

A simple way of accomplishing this result with a single coil is shown in Fig. 1148, in which a coil of three conductors, *a b c*, is wound on a ring core and connected to the two commutator segments S' and S'', each of which covers nearly one-half the circumference of the commutator. On

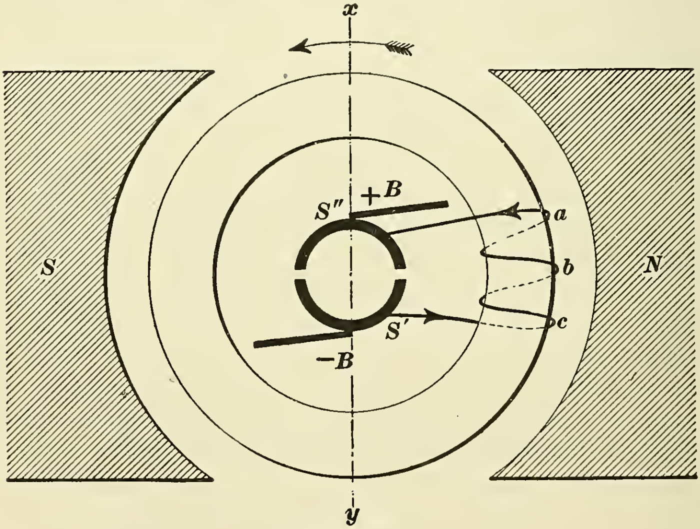


FIG. 1148.

these two segments rest the two brushes $+B$ and $-B$, they being placed opposite each other and making contact with the segments S' and S'' on the neutral line $x\,y$.

3073. When the coil is in the position shown, it being rotated in the direction indicated by the arrow, the E. M. F. generated in the coil will be acting in the direction indicated by the arrow-heads, thus making the top brush positive. When the coil reaches the neutral space, the brushes will each momentarily make contact with both commutator segments, by bridging the space which separates them; but as in this position there is no E. M. F. generated in the coil, this has no effect. On further motion of the coil under the opposite pole-piece, by which its E. M. F. is reversed in direction, the top brush comes in contact with segment S' and the bottom brush with segment S''. Since the direction of the E. M. F. in the coil has been reversed, this reversal of the connection between the brushes and segments results in keeping the difference of potential between the brushes in the same direction as before.

It is evident that this difference of potential is not at all constant, but varies from a maximum to zero and then to maximum again; the curve of its various instantaneous values would be a series of waves, all on one side of the base line. In other words, such an arrangement as has been described would cause a *pulsating* current to flow in the external circuit. (See Art. **3056.**)

3074. Another coil can be wound on the core directly opposite the first, and connected in series or in parallel with it. The width of the coils can not be greater than the width of the neutral spaces, without causing opposing E. M. F.'s during parts of a revolution; consequently, only a part of the surface of the armature can be utilized, and at best there is a part of the time that the E. M. F. of the winding is zero.

However, other pairs of coils may be wound on the surface of the core, in positions intermediate between those of the original pair; these pairs may then each have its own commutator and brushes, and as the maximum and zero of values of the E. M. F.'s of the new windings occur at different periods of time from those of the first pair, the E. M. F.'s may be combined so as to prevent the E. M. F. acting in the external circuit from falling to zero.

3075. Fig. 1149 shows the arrangement, for both ring and drum winding, of two sets of coils *A A'* and *B B'*, each set containing four active conductors, those of one set occupying a position on the core 90° from those of the other. Both sets are supplied with their two-segment commutators, which for convenience are represented as being concentric, *A A'* being connected to *a* and *a'*, and *B B'* to *b* and *b'*. Brushes *1* and *2* rest on segments *a* and *a'*, and brushes *3* and *4* rest on segments *b* and *b'*.

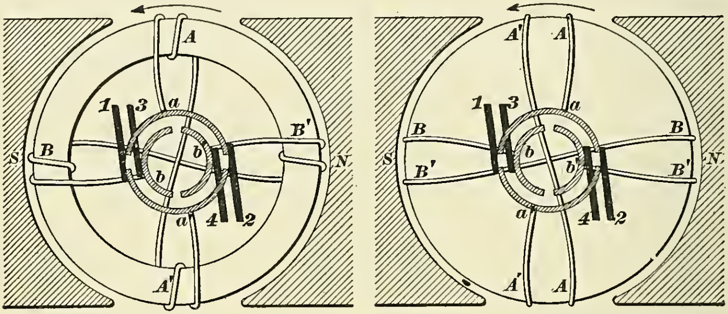


FIG. 1149.

3076. The maximum E. M. F. of each of these sets is the same, but that of the one occurs $\frac{1}{4}$ revolution ahead of the other, so that the curves representing the instantaneous values of the E. M. F.'s of these two sets of coils for one revolution would be about as represented in Fig. 1150, where curve *1* is the E. M. F of coils *A* and *A'*, and curve *2* is the E. M. F. of coils *B* and *B'*, for a complete revolution, starting from the position of the coils represented in Fig. 1149.

If the two sets of coils are connected together in series by means of an external connection between, say, brushes *2* and *3*, then the difference of potential between brushes *1* and *4* at any instant is equal to the sum of the E. M. F.'s of the two sets of coils at that instant. The result of the addition for the entire revolution is represented by the dotted curve *3*, Fig. 1150.

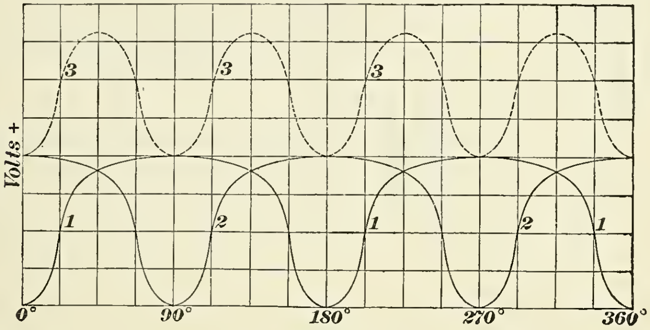


FIG. 1150.

It is readily seen that for about one-fourth of each wave the E. M. F. of one set of coils is nearly at its zero value, and, therefore, contributes but little to the total E. M. F. of the armature; the resistance of this set of coils must, nevertheless, be overcome, since it forms a part of the circuit.

3077. Instead of connecting the sets of coils in series, they may be connected in parallel; but with the coils connected as shown in Fig. 1149, this would result in having the more active set of coils short-circuited by the set that is less active, which would very materially reduce the difference of potential between the brushes. There is, however, a part of the revolution when the E. M. F.'s of the two sets of coils are nearly enough the same to allow of their being connected together in parallel, and by widening the gap between the ends of the segments of the commutator, each set of coils may be entirely disconnected from its brushes during the part of its revolution when its E. M. F. is much lower than that of the other set.

This arrangement of the commutator segments for the windings shown in Fig. 1149 is represented in Fig. 1151, in which *a* and *a'* are the segments connected to coils *A* and *A'*, and *b* and *b'* are the segments connected to coils *B* and *B'*, as before.

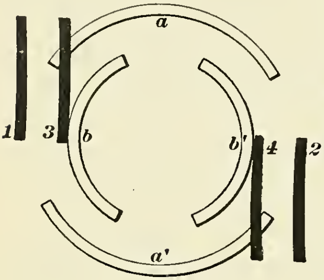


FIG. 1151.

3078. It is evident that curves showing the difference of potential between either pair of brushes would comprise that part of curves *1* or *2* (Fig. 1150) that represents the E. M. F. of the winding during the time that the brushes are in contact with the commutator segments. The curves in Fig. 1152 represent the difference of potential which would exist between the brushes if the arrangement shown in Fig. 1151 were used with the windings shown in Fig. 1149, curves *1*, *1*, etc., showing the difference of potential between brushes *3* and *4*, and curves *2*, *2*, etc., showing the difference of potential between brushes *1* and *2*, starting from the position of the commutator represented in Fig. 1151. It is apparent that with this arrangement the windings might be connected in parallel by connecting together brushes *1* and *3* and *2* and *4* (Fig. 1151). In that case, the difference of potential between the brushes would be the E. M. F. of one winding until the other is connected in parallel with it, which connection would cause the difference of potential to drop a little, since the winding which is newly connected has a slightly lower E. M. F. than the other. The result of

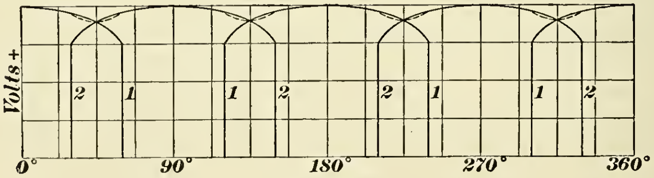


FIG. 1152.

this is that the curve is depressed a little during the time that the coils are in parallel, as represented by the dotted lines in Fig. 1152.

3079. From this curve (Fig. 1152) it will be seen that at the moment when the two sets of coils are thrown in parallel by the brushes, the E. M. F. in the two sets is not the same, that of the set which had just before alone been connected to the brushes being higher than that of the other. A little later, at the moment when one of the sets is disconnected from the circuit by one set of brushes leaving its segments, the coil which is disconnected has a less E. M. F. than the other.

If the coils had little inductance, this would result in the greater E. M. F. of the one set of coils sending a current around through the other set against the E. M. F. generated in it, which current would not appear in the external circuit, and would, therefore, represent so much wasted energy.

3080. This *local current* would evidently be greatest when the difference between the E. M. F.'s of the two sets of coils is greatest; that is, at the moment when the two sets of coils are connected in parallel, and at the moment one of the sets is disconnected from the brushes.

Then, when the one set of coils is disconnected from the other, this local current would be suddenly broken, which would result in sparking.

In an armature as actually constructed, however, the inductance of the coils is sufficient to prevent these local currents; when a coil is first connected in parallel with another, its inductance prevents a sudden rush of current through it, and allows it to take up its share of the current output gradually. As the coil approaches the point where it is to be disconnected from the circuit, and the E. M. F. generated in it becomes less than that of the coil with which it is connected, its inductance serves to keep up its E. M. F., so that its current gradually grows less, until at the time when it is disconnected from the circuit, if that time is properly chosen, its current output is practically zero, and little or no spark results from breaking its connection with the circuit.

3081. In Fig. 1151 the two sets of segments have, for convenience, been represented as concentric; in practice, however, the two sets would be made of the same diameter and placed side by side. If made in this way, the separate brushes *1* and *3* and *2* and *4*, Fig. 1151, may be replaced by two wider brushes, wide enough to bear on either or both sets of segments. This is represented in Fig. 1153, in which *a* and *a'* and *b* and *b'* are the two pairs of commutator segments, and *1* and *2* are the brushes, which are wide enough to bear on either or both pairs of segments, according to the position of the commutator.

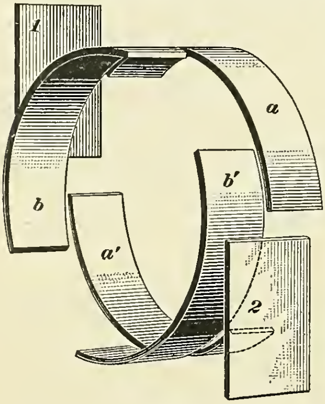


FIG. 1153.

3082. It will be seen that this arrangement of coils and commutator gives a direct but pulsating current, in which the pulsations are not excessive. As has been pointed out, however, the width of the coils should not be greater than the width of the neutral spaces, so that even with two sets of coils the entire armature surface can hardly be utilized.

More than one complete winding, however, can be placed upon the same core, and if each is provided with its own commutator, they may be coupled up in series or in parallel, as desired.

Such a winding as has been described, in which separate sets of coils are used, and which are connected together in various combinations and connected to or disconnected from the circuit during the rotation of the armature, is called an **open-coil winding.**

3083. Only two or three forms of open-coil windings are in commercial use at the present time. That which has been described is used in the **Brush** dynamos, the ordinary sizes of machine using two separate windings, each with its commutator, as represented in Fig. 1154.

In this machine the pole-pieces face the sides of the armature, as represented by the heavy dotted lines. The segments of the two separate commutators are for convenience represented as concentric, with the brushes resting on their edges; whereas, actually, they lie side by side, forming two separate commutators of the same diameter, each having four segments, and the brushes rest on their circumference.

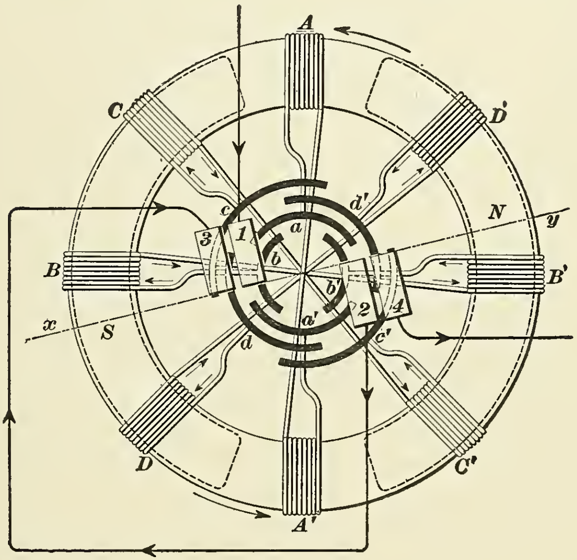


FIG. 1154.

One winding consists of two pairs of coils A and A' connected in series, and B and B' also connected in series, the two pairs being located at right angles to each other, as represented.

This winding is connected to its commutator, coil A to segment a, coil A' to segment a', coil B to segment b, and coil B' to segment b', as represented. Brushes *1* and *2* rest on this commutator, making contact on the line *x y* of maximum action of the coils.

The second winding consists of two pairs of coils *C* and *C'* and *D* and *D'*, located at right angles to each other and half way between the coils of the first winding. These coils are connected in series and to the segments of the second commutator, coil *C* to segment *c*, coil *C'* to segment *c'*, coil *D* to segment *d*, and coil *D'* to segment *d'*, as represented. Brushes *3* and *4* rest upon the segments of this commutator on the same line of maximum action of the coils.

3084. Taking each winding separately, it will be seen that its two sets of coils pass through the following combinations : One set of coils only connected to the brushes ; then the two sets, connected in parallel, both connected to the brushes; then one set only; then both sets in parallel, and so on.

The maximum E. M. F. occurs when the single set of coils is connected, and is directly in the line of maximum action ; the minimum occurs $\frac{1}{8}$ of a revolution ahead of this point, when both sets of coils are in parallel, and are equally distant from the line of maximum action. (See Fig. 1152.)

This being the case, it is evident that as the coils of one winding are half way between the coils of the other, the maximum E. M. F. of one winding occurs at the same instant as does the minimum E. M. F. of the other. On account of this, when the two windings are connected in series, the fluctuations of the current are much reduced.

This connection of the two windings is obtained by connecting the positive brush (*2*, Fig. 1154) of one winding with the negative (*3*, Fig. 1154) of the other, the external circuit being connected between the two remaining brushes (*1* and *4*, Fig. 1154).

In the larger sizes of these machines, three and even four separate windings are used, each with its commutator and all connected in series.

3085. Instead of using overlapping segments, the same results may be obtained with segments which are placed end to end, by making the brushes have a large arc of contact, or, what amounts to the same thing, using two brushes on each side, spaced a distance equal to the desired arc of contact and connected permanently together. This is represented in Fig. 1155, *a* and *a'* being the segments connected to winding *A A'* (Fig. 1149), and *b* and *b'* being the segments connected to winding *B B'* (Fig. 1149). It will be seen that the pairs of brushes *1* and *3* and *4* and *2*, being permanently connected together, act as one wide brush. In the position shown, both sets of coils are in parallel; if the commutator is rotated in the direction of the arrow, segments *a* and *a'* will pass out from under brushes *3* and *4*, leaving only segments *b* and *b'* connected to the winding, and, therefore, only coils *B* and *B'* in circuit. Further rotation will bring segments *a* and *a'* under brushes *1* and *2*, respectively, throwing coils *A* and *A'* and *B* and *B'* in parallel again, and so on. It will be seen that this arrangement gives the same results as that previously considered.

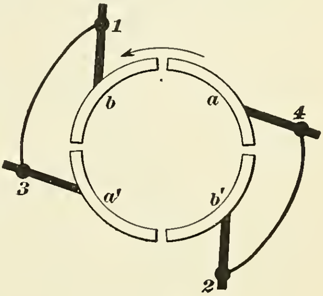


FIG. 1155.

3086. Instead of two sets of coils, three may be used, situated 120° apart on the armature.

In this arrangement, which is used in the Thomson-Houston open-coil dynamos, only one end of each set of coils is carried to a commutator segment, there being, therefore, but three segments; the other end is connected to a common junction of the three ends.

The commutator segments are each a little less than 120° in span, being placed end to end and separated by a small air-gap.

The brushes used are divided into pairs, as described in Art. **3085**; that is, the equivalent of two wide brushes is used, the arc of contact being about 60°, or about half the span of one segment.

Both ring and drum windings are used for the armatures; Fig. 1156 gives a diagram of the connections, etc., of the drum-wound armature. *A A′*, *B B′*, and *C C′* are the three coils, wound on the core in planes making angles of 120° with each other. One end of each of the coils is joined to a metal ring (not represented in the figure) on the back of the armature, which forms a common connection for the three. The other ends are joined to the commutator segments, that of *A A′* to segment *a*, that of *B B′* to segment *b*, and that of *C C′* to segment *c*, as represented. *1*, *2*, *3*, and *4* are the brushes, as before. Those numbered *2* and *4* are usually called the *primary* brushes, and *1* and *3* the *secondary* brushes, to distinguish them.

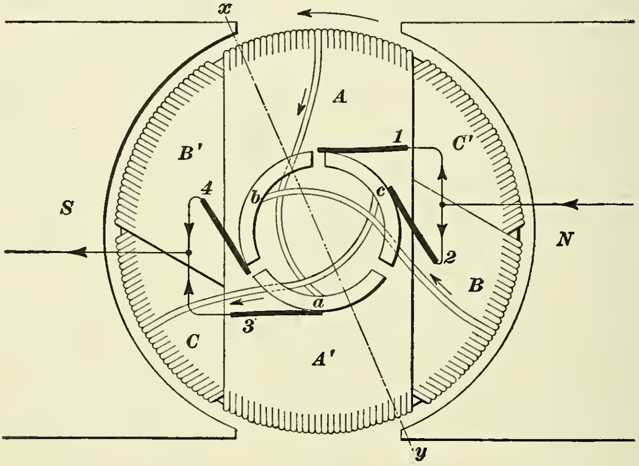


FIG. 1156.

3087. From the diagram (Fig. 1156) it will be seen that coil *A A′*, though half way between the pole-pieces, is partly active, since the neutral line is shifted forwards in a manner which will be taken up later, into the position indicated by the line *x y*. This coil *A A′* is connected in parallel with the coil *B B′* by the two positive brushes, and the two are in series with coil *C C′*. If the armature be considered as moving in the direction indicated by the arrow, it will be seen that as coil *A A′* gets to the position of least action, it is disconnected from the circuit by segment *a* passing out from under brush *3*, leaving coil *B B′* and coil *C C′* in series. However, as the distance between brush *3* and brush *2* is only slightly greater than the span of one segment, coil *A A′* is almost immediately connected in parallel with coil *C C′*, as segment *a* passes under brush *2*, making the following combination: coil *B B′* in series with coils *A A′* and *C C′* in parallel.

As the rotation of the armature continues, coil *C C′* is disconnected from the negative brush *1* and connected to the positive brush *4*, being thus thrown in parallel with coil *B B′*, the two being then in series with coil *A A′*.

Completing the half revolution, coil *B B′* is disconnected from the positive brush *3* and is joined in parallel with coil *A A′* by the two negative brushes *1* and *2*, leaving coil *C C′* connected to the positive brushes.

Further rotation of the armature repeats this series of connections; that is, during every half revolution, one of the coils (*A A′* in the preceding paragraphs) is first in parallel with the coil *behind* it, then momentarily disconnected from the circuit, then connected in parallel with the coil *ahead* of it, then connected in series with the other two, which are then in parallel.

3088. From the diagram (Fig. 1156) it will be seen that when a coil is disconnected from one set of brushes it is very nearly in the position of least action, and the coil with which it was just before connected in parallel has the higher E. M. F. of the two. As explained in Art. **3080**, the self-induction of the coil prevents the higher E. M. F. of the other sending a current through it in opposition to its own E. M. F. at the time when they are connected in parallel; in fact, when a coil is disconnected from its mate it is still supplying some of the current, so that there is a spark at the brushes.

There being but three sets of coils in this machine, a great number of turns must be used in each coil to give the required E. M. F., which gives each set of coils a high inductance. This lessens to a great extent the fluctuations in the E. M. F. acting on the external circuit, which would otherwise, owing to the small number of coils used and the changes in the manner in which they are interconnected, be very considerable.

3089. Since only a very small fraction of a revolution carries a segment from contact with one set of brushes to contact with the other, a slight increase in the arc of contact of the sets of brushes would allow each segment of the commutator to momentarily be in contact with *both* sets of brushes at the same time; the effect of this is evidently to short-circuit the armature, thus reducing the difference of potential between the brushes (momentarily) to zero.

There being two places where the short circuit occurs, i. e., between brushes *1* and *4* and *2* and *3*, and there being three commutator segments, six short circuits occur during every revolution, and if the armature is revolving at 850 revolutions per minute, there are 6 × 850 = 5,100 short circuits every minute; each can, therefore, last only an extremely short time, and the high inductance of the armature coils prevents any excessive flow of current from one to the other through the short circuit. It will be seen that this short-circuiting of the armature does not reduce the maximum value of the E. M. F., but as it introduces periods in each revolution where the difference of potential between the brushes is zero, it does reduce the *effective* E. M. F. acting on the circuit. By varying the arc of contact of the brushes, and thus varying the length of time in each revolution that the armature is short-circuited, the effective E. M. F. of the machine may be varied within comparatively wide limits.

OPEN-COIL MULTIPOLAR ARMATURES.

3090. Thus far the open-coil winding has only been considered with reference to bipolar fields. It is evident, however, that introducing multipolar fields will only result in a duplication of the parts used with a bipolar field for each pair of poles of the multipolar field. Thus, for a winding for a four-pole field equivalent to that shown in Fig. 1149, four coils would be required in each set.

Since each set would have to go through its various combinations of connections during each half revolution, instead of each revolution, it is evident that twice as many commutator segments, of one-half the span, would be required, or, rather, each segment would be divided into two. These two parts of the segment would be situated directly opposite each other, and either four brushes would be used on each commutator, of which the opposite brushes would have to be connected together, or the opposite commutator segments would be permanently connected together, and only two brushes would be used. This latter plan is best, as permanent connections are less difficult to maintain than sliding connections.

3091. A form of multipolar open-coil armature which differs slightly from the above description is used in the Westinghouse open-coil dynamo. A diagram of the connections of this armature is given in Fig. 1157.

The two commutators are represented as concentric, though they are actually side by side on the shaft, and, as in the Brush machine, Fig. 1154, are situated on the end of the shaft outside one of the bearings, the leads to the commutator being brought out through a hole in the shaft, instead of being connected directly, as represented in the diagram.

This type of machine employs a field-magnet with six poles; the armature is drum-wound, but instead of the coils being laid flat on the surface of the core, they are wound around eight projecting teeth on the armature core, there being, therefore, eight armature coils. This armature winding, as in the Brush machine, is divided into two separate windings, each consisting of two pairs of opposite coils, and each connected to a separate commutator. The combination of connections of the various sets of coils is similar to that of the Brush machine; that is, the set of coils in the position of least action is disconnected entirely from the circuit; those near the position of maximum action are connected in parallel, and in series (by external connection of the brushes) with that set which is actually in the position of maximum action.

In this style of winding, any coil, such as *A*, is in the position of least action when the projection on which it is wound

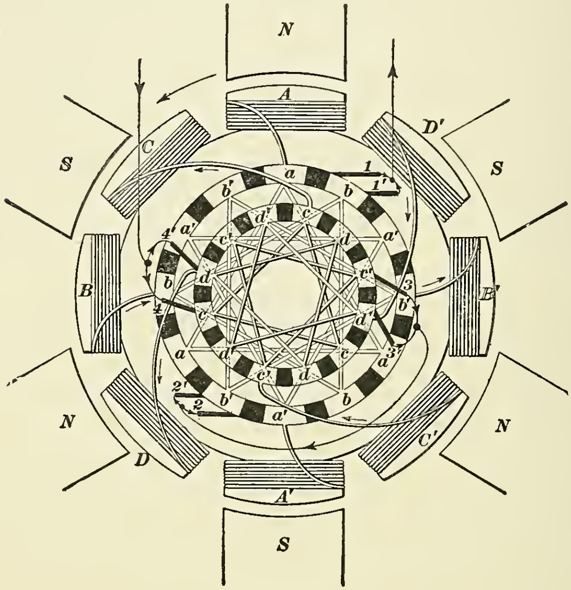


FIG. 1157.

is directly under a pole-piece, as at *N*; for when in this position all the lines of force from the pole-piece *N* pass directly through the center of the coil, which, therefore, cuts none of the lines of force. As soon as the coil moves from this position, one side begins to cut the lines of force of the pole-piece *N* as it passes from in front of it, and as it moves still farther, the *other* side of the coil begins to cut the lines of force of the pole-piece *S*, *towards* which it is moving, so that when half way between the two poles *N* and *S*, both sides of the coil are cutting lines of force equally and at the maximum rate, and this is, therefore, the position of *maximum* action. With eight coils and six pole-pieces, only two (opposite) coils can each be directly under a pole-piece at the same instant.

3092. Referring to Fig. 1157, the two pairs of coils A and A' and B and B' make up one winding, and are connected to one commutator, as shown. The two opposite coils A and A', and also B and B', are connected in series by connections across the back of the armature core (not shown in the diagram).

The other winding is made up of the two pairs of coils C and C' and D and D', the coils of each pair being connected in series, and to the other commutator, as before.

It will be seen that each commutator is made up of twelve segments separated by a considerable width of insulating material (indicated by the solid black parts). These twelve segments are connected together by cross-connecting wires in four sets (one for each coil of the windings), of three segments each (one for each pair of poles).

Two sets of brushes are used on each commutator, each set consisting of two brushes, permanently connected together, which rest on the commutator a distance apart equal to the span of one segment, as shown at *1*, *1'* or *2*, *2'*.

It will be seen that the positions of the commutator segments and brushes are so arranged that when the brushes of one commutator bear on the ends of single segments, the brushes of the other commutator bear on the middle points of the adjacent segments, and *vice versa*.

3093. When the armature is in the position represented in Fig. 1157, coils A and A' are in the position of least action, and are disconnected from the external circuit. The other set of coils B and B' of this winding is, however, in the position of maximum action, and is connected to the circuit through brushes *1* and *1'* and *2* and *2'*, which rest on segments b and b', respectively. Of the second winding, each set of coils C and C' and D and D' is equally distant from the position of maximum action, and these two sets are, therefore, connected in parallel with each other through brushes *4* and *4′*, which rest on segments *c* and *d*, and brushes *3* and *3′*, which rest on segments *c′* and *d′*, and are connected in series with the set of coils *B* and *B′* by the external connection between the two sets of brushes *2* and *2′* and *3* and *3′*.

To follow out the changes in the connections of the coils, consider that the armature is moving in the direction indicated by the arrow.

As coils *B* and *B′* move away from their position of maximum action, brushes *1′* and *2′* are disconnected from segments *b* and *b′*, and as the armature moves, finally come into contact with segments *a* and *a′*, thus throwing the two sets of coils *A* and *A′* and *B* and *B′* in parallel. At the same time, brushes *4* and *3* being disconnected by the insulating segments from segments *c* and *d*, only coils *D* and *D′* of the second winding are connected to the circuit through brush *4′* and in series with the coils of the other winding (now connected in parallel) through brush *3′* and its connection with brushes *2* and *2′*, coils *C* and *C′* being entirely disconnected.

It will be seen that these successive combinations of coils are precisely the same as take place in the Brush machine, except that each combination takes place six times in each revolution instead of twice, due to the multipolar field.

CLOSED-COIL BIPOLAR ARMATURES.

3094. The greater part of the applications of the electric current demands that the current shall not only be *direct*, i. e., unchanging in direction, but *continuous*, i. e., maintaining a constant voltage. (See Art. **3056.**)

Of the two types of direct-current armatures considered, the unipolar is limited in its application by reason of the impossibility of coiling its conductors; in the open-coil type this difficulty does not exist, but in order to avoid serious pulsations in the current output, it is necessary to use a large number of armature windings, each with its commutator and brushes, which soon leads to undesirable complication in the construction.

3095. It is easily seen that with bipolar or multipolar induction, it is necessary to employ some form of commutator in order to obtain a direct current in the external circuit, since the E. M. F. induced in the coils of the winding is alternating; further, to obtain a practically *continuous* current, it is evident that a considerable number of coils must be used, and the number of active coils that are connected between the brushes must be as nearly as possible the same, and connected in the same manner, in all parts of a revolution, and not alternately connected in parallel and in series with other coils, or disconnected entirely from the circuit, as in the open-coil windings. In order to maintain this equality in the number and connections of the coils, they must be connected in series with one another, otherwise their unequal E. M. F.'s would give rise to wasteful local currents. This being the case, and in order that the connection between any particular coil and the external circuit be reversed as the E. M. F. of the coil is reversed, i. e., as the coil passes through the neutral space, without disconnecting the coil altogether from the circuit, it is necessary that both of the commutator segments to which it is attached should come in contact with the same brush at the time the coil is in the neutral space; that is, the coil must be short-circuited by the brush.

3096. In the windings illustrated in Fig. 1149, two separate coils are connected between each pair of commutator segments, the coils being connected in series. It is evident, however, that each separate coil may be supplied with a two-segment commutator, as represented in Fig. 1158.

Here, as in Fig. 1149, the four coils are represented at A, A', B, and B', and each coil is connected to a commutator of two segments, coil A to segments a and a, etc.

This results in four commutators (which are, for convenience, represented as being concentric, as before), upon each of which rests a pair of brushes, *1*, *2*, *3*, and *4*, representing the — brushes and *1'*, *2'*, *3'*, and *4'* the + brushes. Since in the ring winding coils *A* and *B* are, respectively, diametrically opposite coils *A'* and *B'*, and in the drum winding the same coils practically coincide in position, it is evident that the E. M. F.'s of each of these *pairs* (*A* and *B* and *A'* and *B'*) will be the same, although the E. M. F.'s of the individual coils may differ in different parts of the revolution. Consequently, the winding may now be connected up (by suitable external connections between the brushes) so that coils *A* and *B* and *A'* and *B'* will be in series, while the two *pairs* of coils are in parallel. This condition is represented in Fig. 1158, coils *A* and *B* being connected in series by the connection between brushes *1'* and *2*, and coils *A'* and *B'* being similarly connected in series by the connection between brushes *3* and *4'*. The two pairs of coils are connected in parallel by connecting together brushes *1* and *4* and brushes *2'* and *3'*, these pairs of brushes then serving as the terminals + and — of the external circuit.

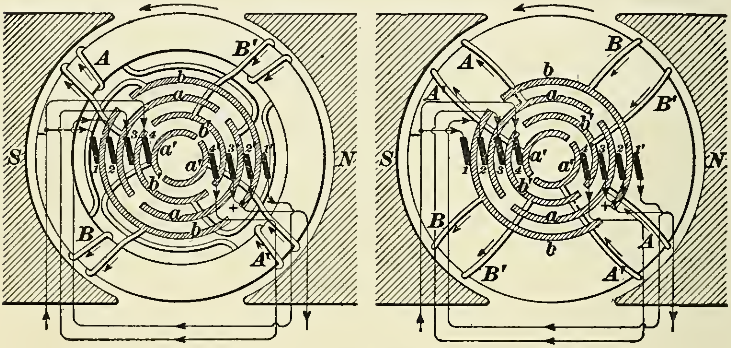


FIG. 1158.

3097. From the above it follows that there are two paths for the current in passing through the armature; in the position shown in the figure, one of these is from brush *1*, through coil *B* to brush *1'*, thence through the external connection to brush *2*, thence through coil A and out through brush *2'*. The other is from brush *4*, through coil A' to brush *4'*, thence through the external connection to brush *3*, thence through coil B' and out through brush *3'*.

As the armature revolves (in the direction indicated by the arrows), coils B and B' approach the neutral position, and when their E. M. F.'s are reduced to zero, they are momentarily short-circuited as brushes *1* and *1'* and *3* and *3'* bridge over the gaps between the ends of segments b and b and b' and b'. Immediately after they are again connected in circuit, but in the opposite direction, so that the E. M. F. generated in them as they pass under the S pole still adds to the E. M. F. acting on the external circuit.

3098. It will be seen that this winding fulfils some of the conditions laid down in Art. **3095,** namely, that the connections between the coils are not disturbed throughout a revolution, and that the connections between a coil and the external circuit are reversed as there described.

There is not a sufficiently large number of armature coils to give anything but a pulsating current, and any increase in the number of armature coils, and, consequently, commutators, would lead to a very complicated and undesirable construction, if arranged in the same manner as those already used.

However, as the external connections between the various brushes (in Fig. 1158) serve to connect certain commutator segments together, these segments might be permanently connected without affecting the action of the armature *when in the position shown* in Fig. 1158, and only two brushes would then be required, as *1* and *2'*. Under these conditions the commutator would have but four pairs of segments, the segments of each pair again forming practically one segment. Thus, these four segments could be made in the form of a single four-segment commutator, as represented in Fig. 1159.

3099. In this new arrangement each segment of the commutator is made up of two of the segments shown in the original scheme. These parts have been lettered to correspond with the segments shown in Fig. 1158.

Further, as will be seen by comparing the two figures, these parts of the original segments are so located that the brushes (*1* and *2*, Fig. 1159) bridge over the gap between them at exactly the same part of the revolution as the *terminal* brushes (the brushes connected directly to the external circuit, *1* and *4* and *2′* and *3′*, Fig. 1158) in the original scheme shown in Fig. 1158, and the connections made by the intermediate brushes (*2* and *3* and *1′* and *4′*, Fig. 1158) are replaced by the permanent connections between the two parts of a segment.

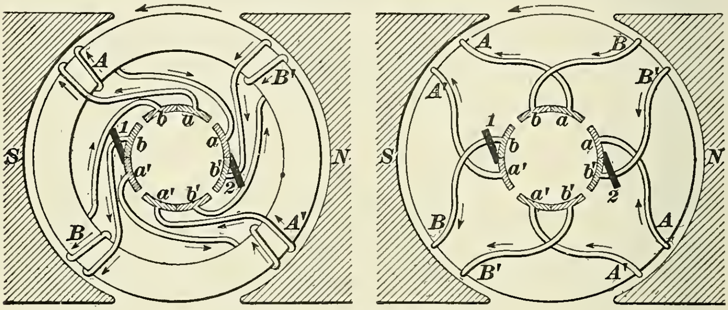


FIG. 1159.

The action of this new arrangement is then identical with that shown in Fig. 1158, since the coils are connected directly to the brushes, short-circuited when in the neutral space, etc., at exactly the same part of the revolution and in the same manner.

3100. In order to more clearly show the similarity of the two methods, the brushes have been shown in the same position relative to the pole-pieces in each figure, the only requirement in this respect being that the coils shall be short-circuited by the brushes when in the neutral space. The connections between a coil and its commutator segments can be so carried out that the brushes may have any position with respect to the pole-pieces.

Thus, in the ring winding shown in Fig. 1159, the commutator and the brushes may be moved bodily around to the left until each of the gaps between the commutator segments stands opposite the coil which is connected across the gap, which would materially lessen the length of the connections between the coils and the commutator, and the brushes would then rest at the ends of the vertical diameter instead of the horizontal, as at present. In the case of the drum winding, the connections between the ends of a coil and the commutator segments are now of equal length, so that any change in the position of the commutator would only result in shortening one connection and lengthening the other.

3101. This new arrangement is still open to the objection of having too few coils, but it will be evident from an inspection of Fig. 1159 that to increase the number of coils it is only necessary to divide each segment in the middle and connect a new coil across the gap so formed. The effect of this is shown in Fig. 1160, which represents the same windings as Fig. 1159, with four new coils *C*, *C'*, *D*, and *D'* inserted in each. In the case of the ring winding the commutator and brushes have been turned to the left to bring the segments to which a coil is connected directly opposite the coil. In the drum winding the position of the commutator and brushes remains unchanged.

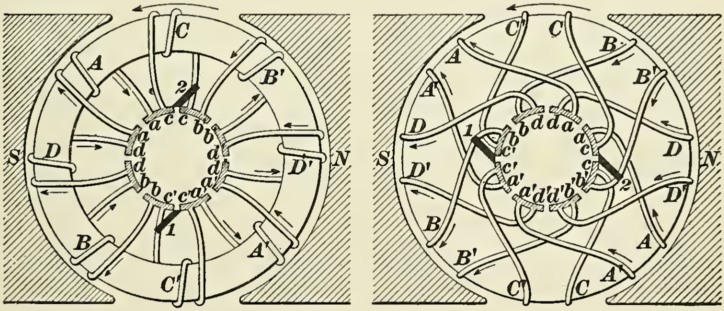


FIG. 1160.

3102. From an examination of Fig. 1160, it will be seen that there are two **leads** (pronounced *leeds*) connected to each commutator segment; that is, the segment a c has the two leads c C and a A.

It is evident, however, that instead of connecting a lead from each coil directly to the commutator, the coils may be connected directly together, and a single wire run from this connection down to the corresponding commutator segment. In this case the current would traverse the commutator segment only during the time that the segment is in contact with a brush.

The result of this arrangement is that the winding itself forms a continuous spiral around the armature core, with the end joined to the beginning; that is, the winding is *reentrant*, or *closed upon itself*. From this feature, this form of winding, whether applied to a ring or a drum core, is called a **closed-coil winding.** Having the closed coil wound upon the core, the connections or leads to the commutator segments may be tapped in at convenient points; the number of turns of the winding which exist between two adjacent points where the leads to the commutator segments are attached constitute an armature coil.

By winding the turns close together on the core so as to utilize the whole of its surface, and bringing out a sufficient number of leads to the commutator, a winding consisting of a large number of coils may be obtained.

3103. From the nature of the winding, these coils will be *equally* divided into two parallel circuits around the armature. Beginning at one brush, the current generated passes from one commutator bar through one lead, and splits (or, if there be two leads to each bar, splits at the commutator bar), passing on around and around the armature till it finds an outlet through the lead (or leads) to the commutator bar at the opposite side, and arrives at the other brush. If there is an even number of bars, the brushes will cover opposite bars at the same instant, which means that there is an even total number of coils, and that each branch of the circuit contains the same number of coils. If there is an odd number of commutator bars, and, consequently, an odd number of coils, the instant when one brush makes contact with one bar the opposite brush makes contact with two bars, thus cutting one coil out of action for the time being, which leaves as before an equal number of coils in each circuit of the armature. As the brushes pass on from bar to bar, in either case, it is seen that the brush must touch two bars, and thus short-circuit a coil, or cut it out of action. For this reason the number of coils employed in generating the current will not be constant, and, consequently, the voltage will vary. This variation is very slight, however, when the number of coils is high, and is of no material importance.

The E. M. F. of each half of the armature winding is made up of the sum of all the E. M. F.'s generated in the separate conductors that are connected in series between the brushes, and as the various commutator segments are connected to the winding at successive intervals, the difference of potential between the brushes rises in a series of steps from the negative to the positive brush, the difference of potential between adjacent segments being equal to the E. M. F. generated in the coil which is connected between them. The greatest difference of potential between adjacent segments is, therefore, in the forms of winding so far considered, only equal to the maximum E. M. F. that is generated in a single coil.

CALCULATION OF E. M. F.

3104. Although at any instant the E. M. F.'s of all the separate conductors may be quite different, owing to variations in the density of the field, the sum of all the separate E. M. F.'s is practically constant. It is only necessary, then, to calculate the *average* E. M. F. in each conductor, and to multiply this average E. M. F. by the number of conductors connected in series to obtain the E. M. F. of the armature.

Let $N =$ the total number of lines of force that emanate from one pole-piece and are cut by the conductors; then, each conductor cuts 2 N lines of force per revolution, since it cuts each line twice. Let $S =$ the number of revolutions per minute of the armature; then, the *average rate* at which each conductor cuts the lines of force is $\frac{2\,N\,S}{60}$ lines of force per second. The *average* E. M. F., e, generated in each conductor is, then, $e = \frac{2\,N\,S}{60 \times 10^8}$. Let $c =$ the total number of conductors on the surface of the armature; then, in each an E. M. F. $= e$ is generated, but in only $\frac{c}{2}$ conductors are these E. M. F.'s added, since the conductors are arranged in two parallel circuits. The total E. M. F. generated in the armature E is, then,

$$E = \frac{c}{2} \times \frac{2\,N\,S}{60 \times 10^8}.$$

It will be seen from the above that the fact that each conductor cuts the lines of force twice is balanced by the fact that only half the conductors are in series, so, by canceling the 2's, the formula becomes

$$E = \frac{c\,N\,S}{60 \times 10^8}. \qquad \textbf{(480.)}$$

From this formula, having given any three of the four quantities, E, c, N, and S, the fourth may be readily found.

COMMUTATION OF CURRENT.

3105. From Fig. 1160 it will be seen that although a coil when approaching the middle of the neutral space may have little or no E. M. F. generated in it, it must carry the entire current which is flowing in the part of the armature in which it is included, until the *following* segment of the pair to which it is connected comes into contact with a brush, which allows the current from that part of the armature to flow through the brush without passing through the coil under consideration. When the *leading* segment of the pair to which the coil is connected passes out from under the brush, the coil is again inserted in the armature circuit, but in the other side, so that the current flowing through it is in the opposite direction. From this it follows that the current in the coil must be reversed in direction during the time that a brush is resting on *both* of the segments to which the coil is connected. Since the coil, whether of the ring or the drum winding, consists of one or more turns of wire wrapped around an iron core, its inductance is an appreciable quantity; hence, when the coil is short-circuited by the brush, the current does not immediately drop to zero, but continues to circulate through the local circuit formed by the coil, the two commutator segments, and the brush, it being maintained by the self-induced E. M. F. of the coil.

3106. Further, when the leading segment of the coil passes out from under the brush and introduces the coil into the armature circuit again, the inductance of the coil will tend to prevent the current in it from suddenly attaining the same value as that flowing in the part of the armature circuit into which the coil is introduced. The result of this action is that the apparent resistance of the coil is very largely increased at that time, so that only a part of the current passes through the coil to the brush when the leading segment of the coil first passes out from under it, but, instead, continues to flow directly between the leading segment and the brush through the narrow air-space that separates them, thus causing a spark. As this distance gets greater, more and more of the current flows around through the coil which has been short-circuited, and the spark becomes less and less intense, and finally disappears. All these operations take only a very short time, usually about as long as is required for the armature to rotate through the angle embraced by one or two commutator segments, so that the spark lasts only a small fraction of a second; but as it is repeated at every brush for every coil that is short-circuited, the aggregate result is that the sparking eats away both brushes and commutator segments, thus causing a considerable deterioration of the machine, besides wasting a certain amount of energy.

3107. If an E. M. F. is introduced into the local circuit (formed by the coil, its commutator segments, and the brush), which is opposite in direction to the E. M. F. of self-induction, not only may the current in the coil be brought more quickly to zero, but it may even be reversed and caused to flow in the opposite direction, while the coil is still short-circuited by the brush. Evidently, then, if this reverse current is brought up to the same value as that flowing in the part of the armature into which the coil is inserted at the moment the brush leaves the leading segment, there is no change in the amount or direction of the current flowing in the coil, consequently no sparking as commutation is effected.

If the value of the reverse current in the short-circuited coil at the moment the brush leaves the leading segment is less than the current it will have to carry, the spark will occur as before, but to a much less degree, since the amount of change in the current of the coil is much less, consequently its E. M. F. of self-induction and its apparent resistance are much reduced; if the current in the coil is greater than that which it is to carry, the excess of current can not immediately disappear as the brush leaves the leading segment, owing, as before, to the inductance of the coil, but continues to flow, the local circuit now including the small air-space between the leading segment and the top of the brush. In other words, sparks appear at the brushes, as before.

3108. Fig. 1161 illustrates both these conditions, representing a section of a ring armature with the coils *a*, *b*, *c*, *d*, and *e* and the corresponding commutator segments. In each figure (*A* and *B*) each half of the armature winding is supposed to have flowing in it 10 amperes, as indicated by the numbers near the arrows that show the direction of the current. In each figure, coil *c* is represented as just passing out of the condition of short circuit, i. e., the brush *C* is just leaving the leading segment of coil *c*. In *A*, coil *c* is supposed to have an E. M. F. acting in it which is sufficient to reverse the current and bring it up to a value of 5 amperes at the instant its short circuit is broken, so that at that instant, owing to the inductance of the coil, only 5 amperes of the main current can flow through the coil, the balance passing through the tiny air-space *from* the leading segment *to* the tip of the brush *C*, as represented. In *B*, the E. M. F. acting in coil *c* is supposed to be great enough to bring the current in the coil up to 15 amperes, so that when the brush leaves the leading segment the 15 amperes continue to flow for an instant, 10 being supplied by the half of the armature into which the coil is connected, and the other 5 passing across the tiny air-space *from* the tip of the brush *C* *to* the leading segment of the coil, as represented.

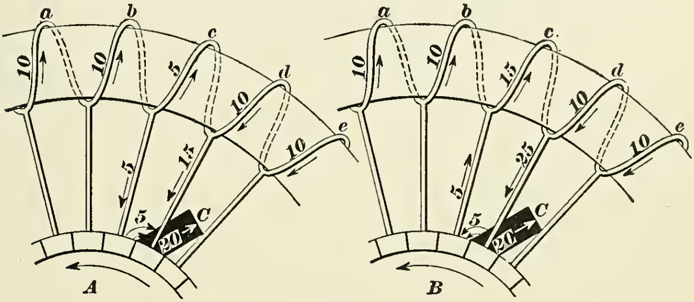


FIG. 1161.

Of course, this condition of affairs lasts for only a moment, the current in coil *c* quickly adjusting itself to the armature current (10 amperes in this case).

3109. The E. M. F. necessary to reverse the current in the coil may be supplied in a variety of ways. It will be seen that the *direction* of this E. M. F. must be the same as that generated in the coils of the part of the armature circuit into which the coil is to be introduced. Consequently, by moving the brush *ahead*, i. e., in the direction of rotation of the armature, until the coil, when short-circuited, is in the magnetic field, the necessary E. M. F. will be generated in the coil. When the armature is furnishing a very small current, the E. M. F. required to reverse the current in the short-circuited coil will be comparatively small, and as the current output of the armature increases, this E. M. F. will have to be similarly increased. There is no abrupt change from the neutral space to the magnetic field; that is, at the edge of the field the density of the lines of force gradually shades off to zero in the neutral space. Consequently, when only a small current is flowing in the armature, it is not necessary to push the brushes ahead very far to move the short-circuited coil into a field of sufficient strength to supply the necessary E. M. F. during the period of short circuit; as the current in the armature increases, however, it is necessary to move the brushes ahead still farther, in order that the field in which the coil moves while short-circuited may be of sufficient density to supply the necessary E. M. F. for reversing the current.

This movement of the brushes is usually obtained by mounting the holders for the several brushes on a common support that can be made to turn around the axis of the armature and be clamped in any desired position. By rotating this support, the brushes are simultaneously moved to the desired position, as evinced by the cessation of the sparking whenever a change in the current calls for such an adjustment.

3110. Another method of introducing an E. M. F. into the short-circuited coil results from the fact that the local circuit through which the current in the short-circuited coil flows is in part through the brush. In addition, the current from both parts of the armature winding also flows through the same two commutator segments into the brush. It is evident, then, that as the leading segment passes out from under the brush, the area of brush surface that is in contact with this segment grows rapidly less, thus increasing the resistance in the path of the current that is flowing from the part of the armature circuit into which the short-circuited coil is about to be inserted. If the brush is made of a material of high conductivity, this increase in the resistance will be slight, and will produce little or no effect until the leading segment is actually leaving the brush; but if of comparatively low conductivity, the increase in the resistance will be more pronounced, and will cause a drop, or difference of potential, between the commutator segment and the brush. The other segment, however, is all the time moving more and more under the brush, thus reducing the resistance at that point, so that the difference of potential between the *leading* segment and the brush tends to send the current around through the short-circuited coil and into the brush through the *other* segment.

3111. In other words, this difference of potential acts as an E. M. F. to reverse the current in the short-circuited coil; consequently, it will prevent sparking just as setting up an E. M. F. in the coil itself will, if of the right amount. It will be seen that as the current in the armature increases, thus requiring a greater E. M. F. to reverse the current in the short-circuited coil, the difference of potential between the leading segment and the brush also increases at the same rate, so that this method of preventing sparking is, to some extent, self-regulating. However, this method by itself can not well be used, as it is impracticable to so adjust the nature and extent of the contact surface of the brush as to obtain the right E. M. F. for reversing the current; and even if this adjustment were once made, it could not be permanent, owing to changes in the extent and nature of the contact surface of the brush and its variations in pressure incident to the continual operation of the machine, the amount of the contact resistance depending upon all of these factors.

3112. In practice, commutation is effected by a combination of these two methods; that is, the brushes are shifted until the E. M. F. induced in the short-circuited coil, aided by the difference of potential between brush and segment, is sufficient to ensure sparkless commutation. With metallic brushes, the contact resistance is usually so low as to render the difference of potential between the brush and segment of very little value in commuting the current. In other words, the E. M. F. induced in the coil itself must do the reversing; hence, the brushes must be newly shifted for each small change in the current output to proportionately change the E. M. F. acting in the short-circuited coil. With brushes of higher resistance, however, the difference of potential developed at the contact surface furnishes such a large proportion of the total E. M. F. required that the brushes may remain in one position during considerable changes in the current output. High-resistance brushes, therefore, require less shifting to obtain sparkless commutation than do those of low resistance.

3113. It will be seen that to commute a given current in a given length of time, the E. M. F. required will be proportional to the inductance of the armature coil. Consequently, it is desirable that the armature coils have as little inductance as possible, since there will then be less E. M. F. required to commute the current; or, in other words, there will be less shifting of the brushes necessary for sparkless commutation.

From the very nature of the factor, it is evident that as the armature coils are wound on an iron magnetic circuit, they must be of few turns in order that their inductance may be low; consequently, the winding should be divided into as many coils as convenient, thus making the number of commutator segments comparatively large.

3114. There are other considerations which influence the number of commutator segments to be used in any particular case; for example, if the maximum difference of potential between adjacent commutator segments is 20 volts or greater, any sparking at the tip of the brush is liable to continue between the commutator segments as the armature turns; as each segment passes out from under the brush a similar arc may be maintained, until they all extend from brush to brush, short-circuiting the whole armature. To prevent this, the *average* difference of potential between the segments should not be greater than about 15 volts, if the machine is to give a current of more than about 20 amperes. With less current output, a greater average difference of potential may be used if necessary.

ARMATURE REACTION.

3115. It has been discussed and shown previously that when a current is flowing in a conductor located in a magnetic field a reaction exists between the current and the field, so that a force must be applied to the conductor in order to move it through the field. This motion being opposed by the lines of force of the field, the force applied to the conductor ultimately acts on the lines of force of the field, tending to crowd them ahead in the direction that the conductor is moved. Consequently, when a current is flowing in the conductors of a dynamo armature (either ring or drum wound), the lines of force are crowded around in the direction of rotation, thus causing the field to be less dense under the *leading* pole tips, i. e., the pole tips *towards* which coils that are in neutral spaces are moving, and to be more dense under the *following* pole tips, than would be the case were the field symmetrically distributed. This is indicated in Fig. 1162, in which an armature core situated in a bipolar field is represented, with the conductors equally spaced around its periphery; these conductors are supposed to be connected up as a closed-coil winding, either ring or drum, and a current is supposed to be flowing in the winding. By applying the thumb-and-fingers rule to this figure, it will be seen that the direction of the current in the conductors under the *N* pole is up, through the paper, while under the

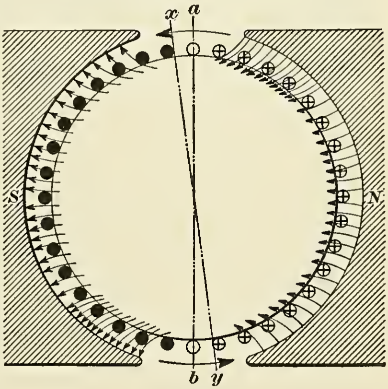


FIG. 1162.

S pole the current is in the opposite direction. This is indicated by marking the conductors with a + and a solid black dot, respectively.

The relative distribution of the lines of force is indicated by the lines from the pole-pieces to the core; their distribution within the armature core is immaterial at present. It will be noticed that the effect of the distortion of the field is to alter the relative density of the lines of force under the pole tips, and to shift the true neutral line (*x y*) from its theoretical position half way between the pole tips (line *a b*) in the direction of rotation.

3116. It does not matter how the conductors are joined together, so long as the current in each conductor is as represented. Consequently, supposing the armature to be stationary, the same shifting of the field would result if the conductors were so connected as to form a spiral coil wrapped around the core, with its axis along the line *a b*, as represented in Fig. 1163, the brushes being on the theoretical neutral line *a b*. The magnetizing force of this coil acts along the line *a b*, that being the axis of the coil. It will be seen that this introduces no magnetomotive force that is *opposed* to the lines of force passing through the armature, since on each side of a line connecting the centers of the pole faces there are the same number of conductors carrying the current in each direction. The only effect of this armature magnetomotive force is, then, to distort the field.

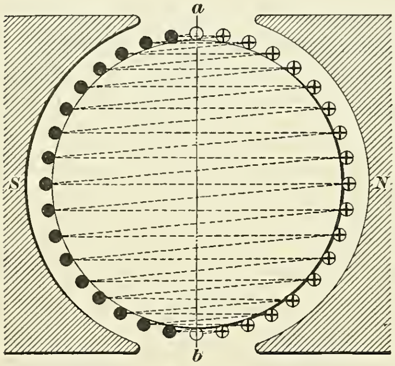


Fig. 1163.

3117. As soon, however, as the brushes are shifted ahead to effect sparkless commutation, this condition does not hold; the shifting of the brushes introduces into that part of the armature that lies on each side of the center line of the pole faces an excess of conductors carrying a current in one direction, as is shown in Fig. 1164, which represents an armature coil in a bipolar field, with 36 conductors, as before. It is supposed in this case that the brushes are shifted until they bring the short-circuited coils on the line $x\ y$, which is ahead of the theoretical neutral line $a\ b$ by the angle r. From an inspection of this figure, it will be seen that on each side of the center line of the pole-pieces the number of conductors carrying the current in one direction exceeds that of the conductors carrying the current in the opposite by the number of conductors included between the lines $x\ y$ and $s\ t$, which make equal angles with the line $a\ b$. Further, it will be seen that the current flowing in these conductors sets up a magnetomotive force which is *directly opposed* to the magnetic field in which the armature revolves. Hence, these conductors may be considered as forming a spiral coil around the armature, whose axis coincides with the center line of the pole-pieces, as represented in the illustration, Fig. 1164.

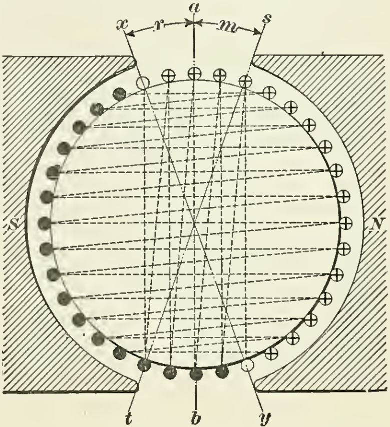


FIG. 1164.

3118. It will be seen, then, that when the brushes of an armature are shifted ahead from the position where the short-circuited coils are in the theoretical neutral line, in order to effect sparkless commutation, the magnetizing effect of the armature current may be divided into two components; one of these acts in a direction at right angles to the field in which the armature revolves, and so distorts it, while the other acts in a direction opposite to the field, hence, reduces its strength. The angle r, Fig. 1164, is evidently the angle through which the brushes are shifted; hence, it is called the *angle of lead* of the brushes. It has been shown that the current in all the conductors included in *twice* the angle of lead makes up the magnetomotive force that directly opposes the field, and this is called the **counter magnetomotive force** of the winding. It is measured (in ampere-turns) by the product of the number of conductors included in twice the angle of lead and the current in each. The current flowing in the rest of the conductors makes up the **cross magnetomotive force** of the winding, its value in ampere-turns, as before, being the product of the number of conductors and the current in each.

3119. It has been shown that the brushes must be shifted ahead of the neutral line in order to bring the short-circuited coil into a field of sufficient density to set up the proper E. M. F. in it; it will be seen that as the armature current increases, the density of the field under the leading pole tip is decreased more and more, so that the brushes must be shifted farther to bring the short-circuited coil into a field of the proper density. This introduces a greater and greater counter magnetomotive force, which reduces the strength of the field still more, and makes the effect of the counter magnetomotive force greater; and it will be readily seen that the armature current might rise to such a value that any amount of shifting of the brushes would not be sufficient to bring the short-circuited coil into a field of sufficient density for sparkless commutation.

Thus, the armature reaction introduces a factor which tends to limit the amount of current which the armature can supply, by making an excessive shifting of the brushes necessary to effect sparkless commutation, this limit of load being known as the *sparking limit.*

3120. It has already been pointed out that force is required to move a conductor through a magnetic field when a current is allowed to flow through the conductor.

Applying this principle to the armature winding of a dynamo, it will be seen that the current in each conductor gives rise to a force acting approximately tangent to the surface of the armature; the amount of the force on each conductor depends upon the strength of the current in each conductor and the strength (density) of the field in which it lies, and the sum of all these forces (in pounds) multiplied by the velocity of the conductors (in feet per minute) is the power (in foot-pounds per minute) necessary to apply to the conductors to move them through the field against the force set up by the current.

It will be seen that the calculation of the force acting on each conductor at any instant would be difficult, requiring a knowledge of the density of the field in which each conductor is moving. But this is not necessary, for, as has already been pointed out (Art. **3030**), the power required to move the conductor is equal (when reduced to the same units) to the product of the E. M. F. generated in the conductors and the current flowing, which are quantities readily measured.

3121. The total power required to drive the armature, or the **input,** is equal to the power required to drive the conductors, which may be found as pointed out above, plus whatever power is necessary to overcome the friction of the journals and the hysteresis and eddy-current losses (see Arts. **3050** to **3052**) that take place in the armature core. These quantities may be found or calculated separately by methods which will be taken up later. It has already been shown that the **output** of a dynamo is the product of the difference of potential between its terminals and the current flowing in the external circuit; the **efficiency** of the dynamo is, of course, the ratio between the output and the input.

CLOSED-COIL ARMATURE WINDINGS.

3122. Thus far only the simplest forms of ring and drum windings for bipolar field-magnets have been considered. These are susceptible of many modifications, however, especially when used with multipolar fields, some of which are essential for certain applications.

In the following discussion of the most generally used windings, for the sake of simplicity, only a few conductors will be represented in each winding, showing the *principle* of the winding and arrangement of the connections. The conditions which govern the design of a winding for a commercial machine and the actual construction of the winding will be taken up later.

RING WINDINGS.

3123. The simplest form of ring winding is that already described, in which the conductor forms a continuous closed spiral, with leads brought out at a series of equidistant points to the commutator segments, and with bipolar fields, and this form of winding is not susceptible of much modification.

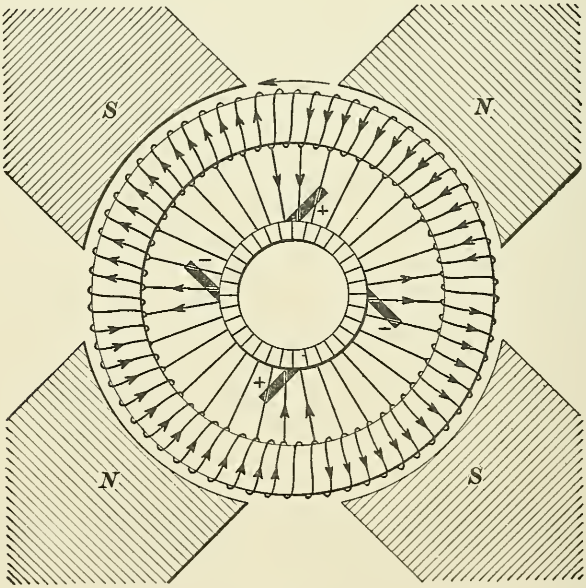


Fig. 1165.

If a simple ring-wound armature is placed in a multipolar field, each adjacent pair of poles will act on the winding in the same manner as a bipolar field, so that *that section* of the armature will be divided into two parallel circuits. The whole winding will, therefore, be divided into as many circuits as there are poles, consequently requiring as many brushes. This is represented in Fig. 1165, which shows a four-pole field, with a ring-wound armature of 32 coils, each of two turns. With the larger number of coils the device used as the commutator heretofore, namely, metallic segments placed side by side and separated by air-spaces, can not be used to advantage; instead, a large number of segments of approximately rectangular section are placed side by side, separated by thin strips of insulating material. This is indicated in Fig. 1165.

3124. It will be seen that in this arrangement the armature is divided up into four circuits, and four brushes are required, which must be placed so as to short-circuit a coil when in the neutral space, as represented in the figure.

There is no difference of potential between the *opposite* brushes, + and +, or − and −, so that each of these pairs may be connected together in parallel, to supply a single external circuit. Similarly, with 6, 8, 10, or more poles, a corresponding number of brushes must be used, of which all those of like sign are connected together in parallel. In this form of four-pole armature (Fig. 1165), opposite commutator bars are always at the same potential; consequently, there is no difference of potential between them, and they may, therefore, be permanently connected together. This is accomplished by means of cross-connecting wires, and does away with the necessity of more than two brushes. This can be done only when the winding is made up of an even number of coils, for with an odd number there will always be one segment "left over."

3125. In general, with *any* number of poles this form of winding has the segments that are always at the same potential situated $\frac{360°}{p}$ apart, p being the number of *pairs* of poles in the field; and these segments may be connected together by cross-connecting wires; only two brushes are used, provided the number of segments is a multiple of p.

3126. The E. M. F. generated in a simple ring armature rotated in a multipolar field may be found from formula **480,** given in Art. **3104.** The total number of cuttings of lines of force by each conductor in one revolution is $2p\,N$, p being the number of *pairs* of poles and N the number of lines of force that emanate from *one* pole face; but since only $\frac{c}{2p}$ conductors are connected in series, the term $2p$ cancels out, and $E = \frac{c\,N\,S}{60 \times 10^8}$, as before.

3127. Thus the *total* number of lines of force in the armature of a multipolar machine is equal to $p\,N$, p and N having the values given above. Each line of force is cut twice by each conductor in each revolution, however, from which results the value $2\,p\,N$, given above.

The same E. M. F. will be generated in each of the four circuits of the winding, provided that the number of lines of force through each gap space under the poles is the same, which is usually the case, although, as will be pointed out later, it is quite possible for it to vary.

TWO-CIRCUIT WINDINGS.

3128. If the number of lines of force through each gap space is not the same, then the E. M. F. generated in each circuit will not be the same; consequently, the higher E. M. F. of one circuit will tend to make it furnish more than an equal share of the current output when connected to the external circuit.

To obviate the possibility of such an event occurring, several systems of multipolar ring windings are in use, all of which are based on the general principle of connecting each coil of the armature in series with one which is in another field, of either the same or opposite polarity. This divides the armature winding into two parallel circuits, a part of each circuit being in two different fields, so that even if the fields are individually not of the same strength, the E. M. F. of each armature circuit is the same.

A winding which is divided into only two circuits in parallel, whatever the number of pairs of poles in the field, is known as a **two-circuit** winding, to distinguish it from the form in which the winding has as many circuits in parallel as there are poles, which is called a **multiple-circuit** winding.

3129. One form of two-circuit winding, in which coils situated in fields of *like* polarity are connected in series, is illustrated in Fig. 1166. A four-pole field is shown with an armature having 17 coils numbered from 1 to 17. Each

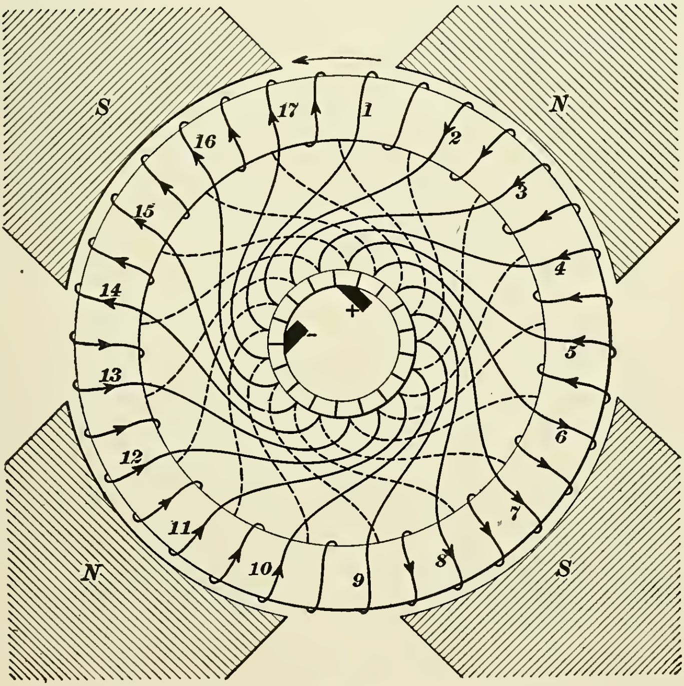


FIG. 1166.

coil is connected to two commutator segments, and each *segment* being connected to two *coils*, there are, therefore, as many segments as coils, i. e., 17.

It will be seen that the *end* of any one coil is connected to the *beginning* of a coil which is a certain number of coils away from it; for example, the end of coil *1* is connected to the beginning of coil *9*, which is $9 - 1 = 8$ coils to the right of coil *1*. This spacing is called the **pitch** of the winding; that is, in the above case, the pitch is 8, and the end of each coil throughout the winding is connected to the beginning of the 8th coil to the right, as shown in the figure.

3130. Whether the pitch in this form of winding be odd or even, in order that all the coils may be included in the winding before it closes upon itself, the number of coils must be one more or one less than the product of the pitch and the number of *pairs* of poles. Then, if $p =$ the number of pairs of poles, $y =$ the pitch, and $s =$ the total number of coils, in general,

$$s = py \pm 1. \qquad \textbf{(481.)}$$

Thus, in the case illustrated in Fig. 1166, where $p = 2$ and $y = 8$,

$$s = (2 \times 8) \pm 1 = 15 \text{ or } 17.$$

In this case 17 was the number used, as shown. It will be seen from this formula that when p is an *even* number, s must be an odd number; while, if p is *odd*, s may be odd or even, depending on whether y is even or odd.

There being but two circuits through the armature, two brushes only need be used, as represented.

Note.—To prevent confusion, the brushes have been represented as *inside* the commutator in this and other figures.

In the position represented in the figure, coils *1* and *9* (in series) are short-circuited by the −brush; as the armature continues to rotate, coils *14* and *6* (in series) would next be short-circuited by the +brush, and so on.

3131. In general, the brushes for this style of winding may be located as follows: If one is to the *left* of a pole-piece, the other must be to the *right* of a pole-piece of *like* polarity. In a four-pole machine, this allows of an angle of only 90° between the brushes; but in a six-pole machine, it allows of an angle of either 60° or 180°, and in an eight-pole machine an angle of either 45° or 135°. With a greater number of poles, a greater number of different angles between brushes may be used.

3132. Another form of two-circuit ring winding is represented in Fig. 1167. Here the number of coils is the same as before, but twice the number of commutator segments are employed. In this case each end of each coil is carried straight down to a separate commutator segment, and, in addition, a cross-connection is made between each

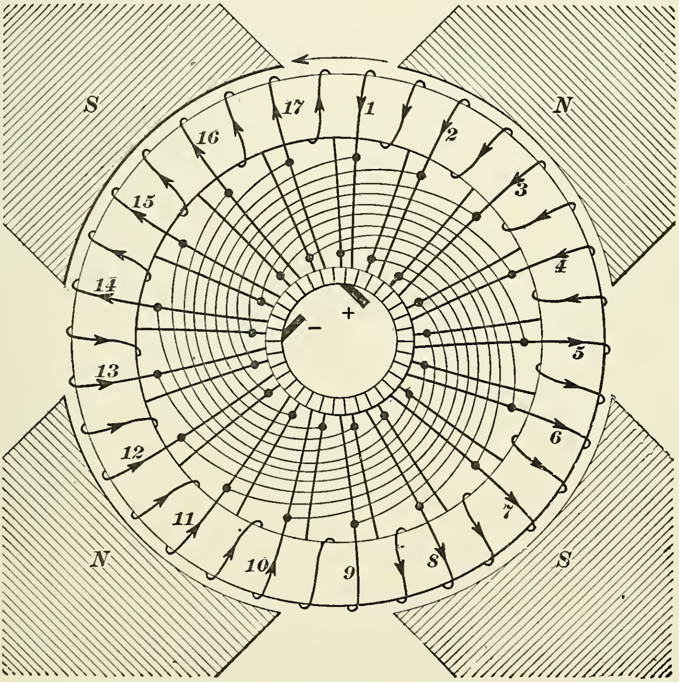


FIG. 1167.

commutator segment to the one directly opposite it. In practice, it is customary to place the cross-connections inside the end of the commutator, instead of between the leads to the commutator, which is the method represented in the diagram.

Two brushes are used, located (with the four-pole field) 90° apart on the commutator. In the position of the armature represented in the diagram, coil *9* is short-circuited by the +brush; a moment later the −brush will short-circuit coil *5*, then the +brush will short-circuit coil *1*, then the −brush will short-circuit *14*, and so on, from which it will be seen that the coils in the successive neutral spaces are short-circuited one at a time.

3133. The formula for the total number of coils in the winding given in Art. **3130** (formula **481**) also applies to this winding; but if the direction of the winding in the coils is always assumed to advance in the same direction as the numbering of the coils, so that the *end* of coil *1* adjoins the *beginning* of coil *2*, and so on (which is the logical way of considering it), then the number of coils which can be used is only that given by the formula $s = p y - 1$.

Thus, in Fig. 1167, $p = 2$ and $y = 9$, and $s = (2 \times 9) - 1 = 17$ coils. If +1 had been used instead of −1, the number of coils would have been 19, and a closed-coil winding would have resulted, but the distribution of potentials between commutator segments would have been very irregular, since several coils would be included between adjacent bars in some instances.

3134. Since the object of the cross-connections is to connect together in series coils which lie in fields of like polarity, it is evident that opposite segments are connected together only in the case of a four-pole field, for which this form of winding is generally used.

In general, for any number of poles, the segments connected together are $\frac{360°}{p}$ apart, p being the number of pairs of poles, as before. Two segments for each coil may be used, but if this scheme of winding is laid out for a field with 6 poles, it will be seen that the distribution of potentials around the commutator is irregular. By introducing a third commutator segment for each coil at a point $\frac{360°}{p} =$ 120° removed from each of the other two, the distribution of potentials will become uniform. In general, then, the number of segments in the commutator for this form of winding will be equal to p times the number of coils.

3135. There are several forms of two-circuit ring windings besides the two given, but as they introduce new complications in the way of cross-connections, they are of limited practical application. Of the two given, the latter is very generally used, since it is very simple and the cross-connections are very regular. By making the cross-connections a part of the commutator construction, as is generally the practice, the winding itself is as simple as a plain ring winding.

The fact that there are twice as many commutator segments as coils is also advantageous in reducing the difference of potential between segments. (See Art. **3114.**)

3136. In a two-circuit ring winding, a greater E. M. F. will be generated than in a multiple-circuit winding with the same number of conductors, since in the two-circuit winding the number of conductors connected in series is always $\frac{c}{2}$, while in the multiple-circuit winding it is always $\frac{c}{2p}$. The E. M. F. of the two-circuit winding is then p times as great as that of the multiple-circuit winding with the same number of conductors, and by introducing this term in formula **480,** given in Art. **3104,** it becomes

$$E = \frac{cpNS}{60 \times 10^8}, \qquad \textbf{(482.)}$$

which is the formula for determining the E. M. F. of any *two-circuit* winding.

BIPOLAR DRUM WINDINGS.

3137. From the nature of the drum winding, each coil must have at least two active conductors, in order to bring both ends of the coil to the front of the armature core; further, these two conductors must lie in fields of opposite polarity, and the E. M. F.'s generated in the two conductors must be as nearly as possible in phase, in order that they may add together without opposition. From an examination of the drum winding shown in Fig. 1160, it will be seen that the winding is constructed as follows: The surface of the armature core being divided into a number of *winding spaces* equal to *twice* the number of coils the winding is to have, then, starting at, for example, segment *d a*, coil *A A* is formed by carrying the conductor along one of the winding spaces to the back of the core, across the back to the winding space alongside the one diametrically opposite the one in which the coil was begun, then along this winding space to the front and up to commutator segment *a c*, the one next on the right of segment *d a*. From this point the next coil (*C C*) is started, the conductor being carried along the core from front to back, not in the winding space *next* to that occupied by the conductor first considered, but in the *second* winding space to the right of that one; the one skipped over will be filled by another coil. The coil is completed in the same manner as the first, the end being carried to the next segment to the right of *a c*. By proceeding with the remainder of the coils in the same manner, it will be seen that when half the coils (*A A*, *C C*, *B' B'*, and *D' D'*) are wound on the core, there is an even spacing of conductors all around, but only half the commutator segments are utilized, and only alternate winding spaces occupied. To proceed with the winding, coil *A' A'* is wound, starting at segment *a' d'* and carrying the conductor along the core from front to back in the winding space between spaces occupied by the parts of coils *A A* and *C C* that return from back to front, then across the back and along the core from back to front in the winding space left between the first parts of coils *A A* and *D D* that were wound, and then to segment *a′ c′*. The remainder of the coils, *C′ C′*, *B B*, and *D D*, are wound on in a similar manner, and the end of coil *D D* connects with segment *d a* from which the winding started, thus forming a closed-coil winding.

3138. If it were desirable to make each coil of more than two turns, the extra turns would be wound around the core in the same winding spaces occupied by the first two before carrying the lead to the commutator segment and proceeding with the next coil. In practice, this is generally done, the size of the coils being so calculated that the whole of the armature surface is covered. In the diagrams used to represent the various drum windings, to prevent confusion, only a few coils, with two conductors per coil, will be represented.

3139. In drum windings there are two different factors which correspond to the *pitch* as used in two-circuit ring windings, namely, the number of winding spaces skipped over in connecting together the oppositely situated conductors of the *same* coil, across the back (and also across the front, if each coil has more than two conductors), which is called the **back pitch,** and the number of winding spaces skipped over in connecting together *succeeding* coils, across the front of the core, which is called the **front pitch.** In the diagram given in Fig. 1160, the back pitch is 7 and the front pitch 5. In the case of the two-circuit ring windings the pitch was always taken in the same direction, i. e., if a coil was connected to a coil situated *y* coils to the *right*, this latter was in turn connected to a coil *y* coils to the right again. In the drum winding given in Fig. 1160, however, the front pitch is in the opposite direction to the back pitch, and this is indicated by giving the front pitch a — sign. Thus, the back pitch being 7, the front pitch is —5.

3140. The method of representing drum windings used in Fig. 1160 is not convenient, since it is difficult to represent the connections across the back of the core without a confusion of lines. Fig. 1168 shows the method of diagrammatically representing drum windings that will be used in this discussion. This winding is the same as that represented in Fig. 1160, it being imagined that the armature and winding is expanded from the back until it becomes a flat disk. The heavy radial lines represent the conductors on the face of the core, the lighter lines represent the connections between them; the ring represents the cylindrical surface of the core, and the shaded parts represent the portions of the core that are covered by the pole-pieces; that is, they represent the magnetic fields. The commutator and brushes are represented in a similar manner as for the ring windings.

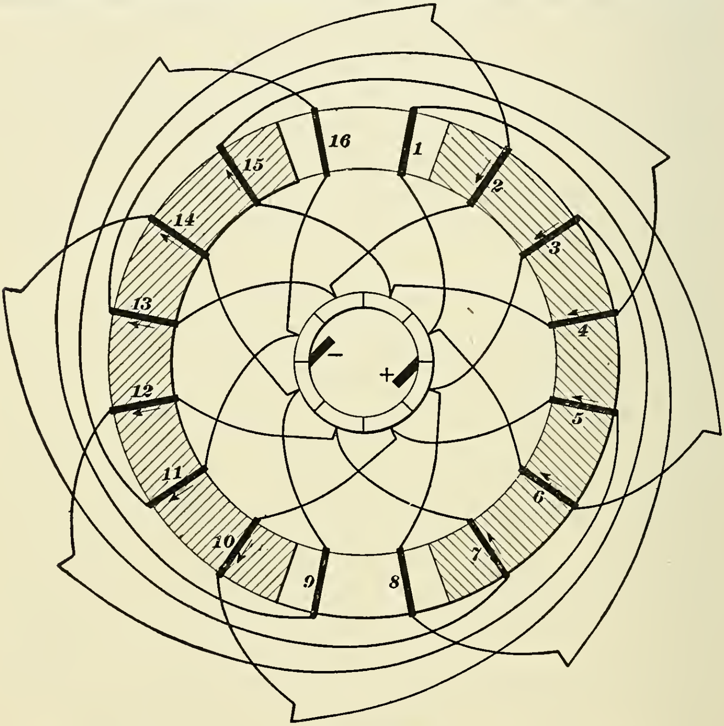


FIG. 1168.

3141. It will be seen from this diagram (in which the 16 conductors are evenly distributed on the surface of the armature) that the conductors of each of the short-circuited coils *1–8* and *9–16* do not lie in the same part of each neutral space, because they are not diametrically opposite on the core.

With an even number of coils, and with the conductors placed in one layer on the surface of the core, opposite conductors can not be connected together and give a symmetrical winding. With an odd number of coils, however, opposite conductors may be connected together, as illustrated in Fig. 1169, which shows a winding with 9 coils, i. e., 18 conductors. In this winding the back pitch is 9 and the front −7. In the position shown, the coil formed of conductors *1* and *10*, which lie directly in the center of the neutral spaces, is short-circuited by the +brush. There being an

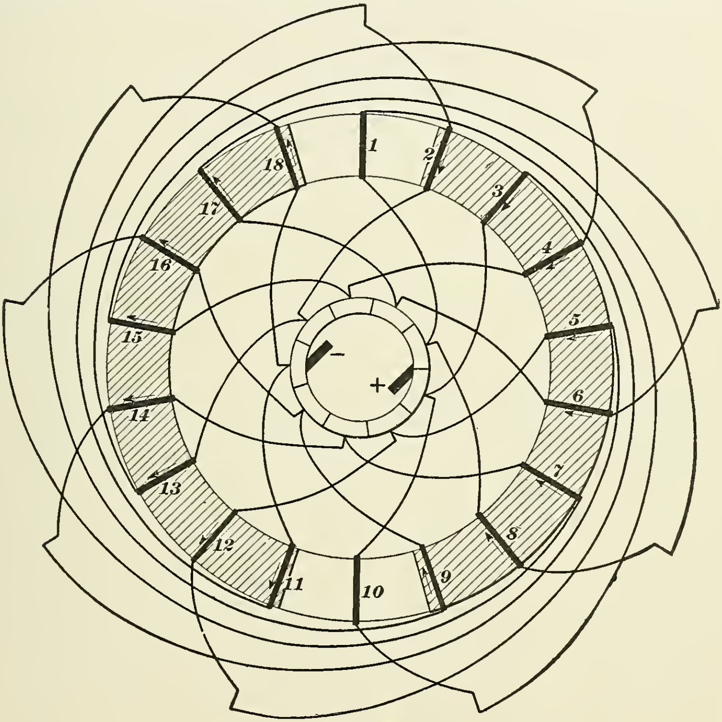


FIG. 1169.

odd number of commutator segments, only one coil is short-circuited at a time.

3142. If, in the winding illustrated in Fig. 1168, alternate conductors, e. g., those with odd numbers, are moved around to the left until they coincide in position with the even numbered conductors, then the two conductors in each coil would be directly opposite each other, as in the case of the winding with the odd number of coils. With this form of winding, half of each coil is in the outside and half in the inside layer of windings, which introduces no difficulty in winding if each coil consists of but one turn; but if each coil consists of two or more turns, then the fact that the conductors of the coil that are in the inner layer must all be wound on before the outer layer can be wound causes serious difficulties in the winding.. It is usually better to wind the coils side by side, as represented.

When the coils are wound in slots cut in the periphery of the armature core instead of being wound continuously over the surface, it becomes an easy matter to construct coils of many turns of wire. These coils are wound on a form and taped, after which they may be slipped into place and connected up.

3143. The possible variations in the method of winding bipolar drum armatures are many. In general, the number of conductors must always be even, although the number of coils may be either odd or even. The back pitch determines the relative position on the core of the members of a coil, from which it follows that in order to have both members in the neutral spaces at the same time the back pitch should be very nearly equal (in bipolar fields) to $\frac{w}{2}$, w being the number of winding spaces.

The back pitch obviously can not be exactly equal to $\frac{w}{2}$, unless s (the number of coils) is odd. With an even number of coils the nearest approach to this value is evidently $\frac{w}{2} \pm 1$; if $+1$ is used, the end connections are longer and make more crossings, and the winding has no particular advantage over that resulting from the use of -1 in the above formula. It is better, then, to make the back pitch $= \frac{w}{2} - 1$.

3144. The front pitch determines the position of a coil relative to the coils with which it is immediately connected, and should, therefore, differ from the back pitch by 2. (See Art. **3137.**) If the front pitch is less than the back pitch, each of the coils, taken in the order in which they are connected, lies to one side of the coil preceding it in the *same direction* as the back pitch; for example, in Figs. 1168 and 1169 the direction of the back pitch is to the *right*, and the front pitch being in each case less than the back pitch the successive coils each lie to the *right* of that preceding it, e. g., coil *3–12* lies to the *right* of coil *1–10* (Fig. 1169). This is called the **advance** of the winding.

3145. If the front pitch is greater than the back pitch, the advance is opposite in direction to the back pitch. There is no particular advantage in this, however, and it has the disadvantage that the connections across the ends of the core are longer for the same winding than in the case where the back pitch and the advance are both in the same direction, thus requiring a greater length of wire for the winding and increasing the number of crossings of the end connections. It is better, then, to make the front pitch less than the back pitch, in which case its value would be $= \frac{w}{2} - 3$, when the back pitch $= \frac{w}{2} - 1$, as noted above.

3146. It is possible to use values for the back and the front pitch which are less than those given by the above formulas, as indicated in Fig. 1170, which gives a winding in which $s = 10$ and $w = 20$; the back pitch $= +7$ and the front pitch $= -5$.

In the position represented in this figure the coil formed of conductors *1* and *8* is short-circuited by the +brush, and that formed of conductors *18* and *11* is short-circuited by the —brush. It will be seen that these coils do not lie alongside one another, as has been the case in all the previous windings where *s* is even, but instead are separated by conductors in which the armature current is flowing. This results in causing the short-circuited coils to lie on the edge of, or even in, the magnetic field, unless the width of the field is made smaller than has been represented.

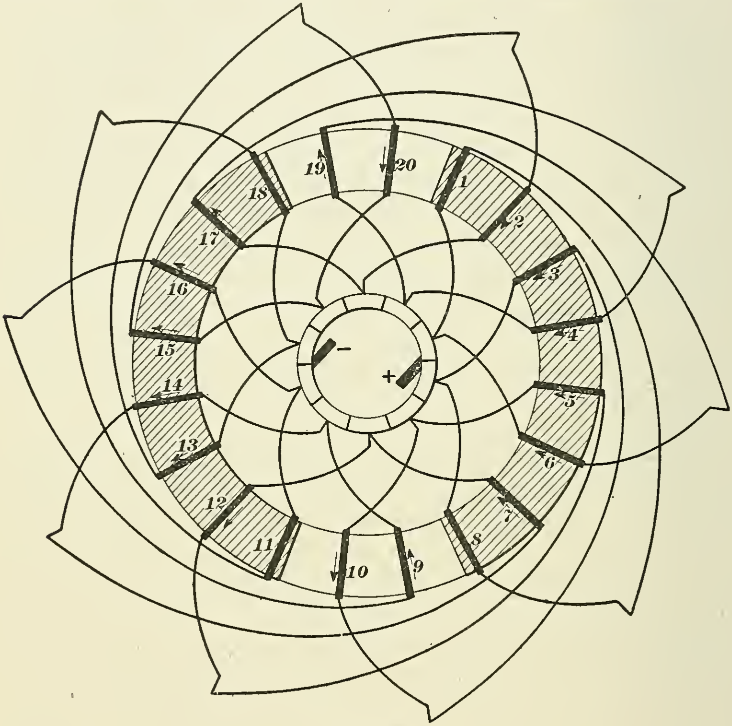


FIG. 1170.

3147. This form of winding, where the back pitch is *less* than $\frac{w}{2} - 1$, is called a **chord winding,** and the disadvantage of having the short-circuited coils slightly out of the neutral space (which with a greater number of coils would be much less than with the few coils represented in the diagram) is to some extent balanced by the shorter length of wire required for the end connections and the fewer crossings made by them, providing the space between the two winding spaces occupied by a coil is not less than the width of the field.

In addition, it will be seen that where the brushes are shifted the current in some of the armature conductors included in twice the angle of lead of the brushes is opposite in direction to that in the others, which reduces the counter magnetomotive force of the armature winding.

3148. Another modification of the drum winding consists in giving to both the front and the back pitches the *same* direction. Fig. 1171 represents a winding in which $s = 10$, as in Fig. 1170, $w = 20$, and the front and the back pitches are

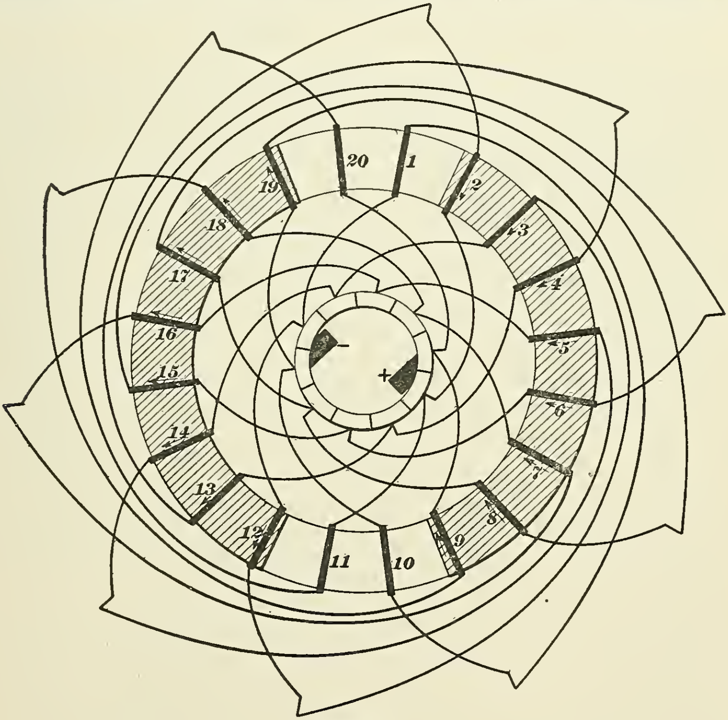


Fig. 1171.

each equal to +9. It will be seen that one effect of giving both pitches the same direction is to make it possible to have both pitches equal, which is obviously not the case when they are given opposite directions. As in the windings previously considered, the winding space occupied by the *first* half of a coil must be the *second* winding space away from that occupied by the *first* half of the preceding coil. (See Art. **3137.**) Since both pitches are in the same direction, this condition makes it necessary that the total number of winding spaces be equal (in bipolar fields) to the sum of the front and back pitches, ± 2. If $y =$ the *average* pitch, i. e., half the sum of the front and back pitches, then the number of winding spaces which must be used is given by the formula

$$w = 2y \pm 2. \qquad \textbf{(483.)}$$

In Fig. 1171, $y = 9$, and w might then have been 16 or 20. If -2 is used, the advance of the winding is in the same direction as the pitch; but if $+2$ is used the advance is in the opposite direction.

This is more advantageous, since the end connections are a little shorter for the same number of conductors; hence, a less length of wire is required for the winding.

3149. In order that all of an even number of winding spaces may be passed over in connecting up this form of winding, both front and back pitch must be odd, so that if they are equal the average pitch will also be odd. If the front and the back pitches differ by 2, the average pitch may be even.

For example, with an average pitch of 8, the number of winding spaces might be $w = (2 \times 8) \pm 2 = 18$ or 14; since each pitch must be odd, the back pitch might be taken as +9 and the front pitch as +7, or *vice versa*.

In this style of winding, both pitches having the same direction, it will be seen that when the *average pitch* is *even* the number of *coils* is *odd*, but when the *average pitch* is *odd* the number of *coils* is *even*. Further, when the average pitch is odd, the end connections on both ends are of the same length, which is often an advantage in manufacture.

END CONNECTIONS OF COILS.

3150. Though it has not thus far been represented, the end connections of the drum winding must be made to avoid the armature shaft in crossing the ends of the core. This is accomplished in a variety of ways. In the ordinary forms of drum winding with coils made up of several turns each, the end connections of each coil are simply carried across the end of the core and bent out to one side to avoid the shaft; as each coil is wound, its end connections are laid over the end connections of the coils previously wound, the whole being so disposed as to make as nearly as possible a symmetrical-looking winding when done. In this form of winding, the several coils may be of quite different lengths, those wound on last being longer than those first wound.

Further, the end connections lap over and cross each other in all directions, and special precautions must be taken to insulate carefully between coils, and in case of accident to one of the coils first wound, the rest of the coils must be removed before the injured coil can be repaired.

3151. It is apparent, then, that on account of these difficulties some other method of winding is desirable which shall not be open to some or all of the above mentioned objections. In case each coil consists of but one turn (two conductors), the end connections may be arranged as represented in Fig. 1172, in which *C* is the drum armature core; *a* and *b* are the two conductors of an armature coil, and *c* and *d* the two conductors of the coil next succeeding coil *a b* in the winding. Considering that the view represents the *back* end of the core and that *b* is the conductor from which the winding starts, then the winding is proceeded with as follows: Conductor *b* is bent down at right angles at the end of the core (see plan, Fig. 1172) and carried in a spiral curve along the end face of the core to a point *n*, which is on the diameter about at right angles to the plane of the coil, and at a sufficient radial distance from the axis of the core to clear the shaft. At this point the conductor is bent outward at right angles, carried along away from the core parallel to the shaft for a short distance, as represented at *n* (in the plan), then bent at right angles again and carried in a spiral curve parallel to the end face of the drum and bent over to form conductor *a*. In forming the

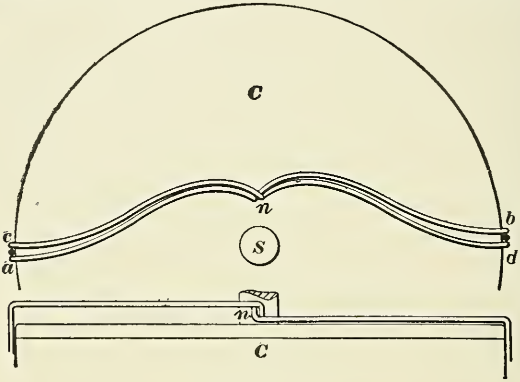


FIG. 1172.

next coil *c d*, which is located two winding spaces away from coil *a b* (see Art. **3137**), a similar process is gone through with, the two spiral connectors being carried along parallel with those of the first coil to be wound, as represented.

3152. It will be seen that by completing the whole winding in a similar manner, the end connections are situated in two parallel planes, those in the inner plane being the connections from the *first* conductors of the coils to the center, and those in the outer plane being the connections from the center to the *last* conductors of the coils. This forms a very symmetrical winding, and the end connections cross in such a manner that it is a very simple matter to insulate them thoroughly. It is evident that the number of short lengths of conductors at *n* (Fig. 1172) is equal to the number of coils; hence, the distance of *n* from the axis must be great enough to allow this number of conductors to lie side by side, with insulation between. If the conductors are of uniform section throughout, and are spaced closely on the surface of the core, this might require that the point *n* be too far from the axis. To avoid this difficulty, the end connections may be made in the form of separate connectors, of thin sheet copper, wide enough to give the necessary cross-section, and bent to the proper shape; these are placed in position, with the width of the copper strip parallel to the shaft, and fastened to the conductors on the face of the armature core. In such a "built up" winding, the active conductors are often made of heavy copper bars of rectangular section, to which the end connectors are riveted or soldered. Fig. 1173 shows one form of end connector made from a rectangular piece of sheet copper, which is slotted for almost its entire length. The two tongues of metal thus formed are bent over, one to the right and one to the left, forming the end connector represented in the figure. The proper curves for the spiral parts of the end connector may best be determined by laying them out on the drawing-board, and by trials determining which curve will give the most uniform clearance between adjacent connectors. When using these end connectors, the shorter conductors of the winding may as well be *under* as *alongside* the longer conductors, if their number is properly chosen, thus forming a two-layer winding.

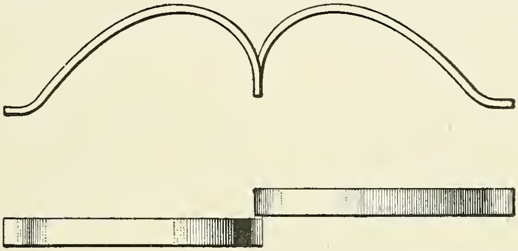


FIG. 1173.

3153. In case it is desired to use more than one turn in each coil, a winding which is usually attributed to Eickemeyer may be used. In this winding each coil is wound on a wooden form to the proper shape, and the proper number are then placed in position on the armature core. The shape of the coil as completed is approximately rectangular, the ends of the rectangle being bent in a manner similar to the end connections illustrated in Fig. 1172, so that when the coils are placed in position on the core, the end connections of each coil as a whole cross in the same manner as in the winding described in Art. **3151,** where each coil consists of a single turn. Of the two sides of the coil which form the active conductors, one is shorter than the other by reason of difference in the planes in which the two parts of the end connections lie. (See Fig. 1172.) This shorter side, in the completed winding, may lie beneath or alongside the longer side of the neighboring coil, thus forming a two-layer or a single-layer winding, as the designer may decide.

3154. By referring to the various diagrams for drum windings which have been given, it will be seen that in all the windings the adjacent conductors which lie in the neutral spaces have between them nearly or quite the full difference of potential that exists between the brushes. This is not quite so marked in the chord winding (Fig. 1170), which is one advantage of this form of winding.

In the two-layer winding it will be seen that as the two adjacent conductors are placed one over the other, the full difference of potential exists between the members of the two layers at such time as they are in the neutral spaces. This feature requires that the adjacent conductors in single-layer windings and the two layers in two-layer windings be carefully insulated one from the other.

MULTIPOLAR DRUM WINDINGS.

3155. The use of the drum winding for large multipolar armatures has become very general, as it possesses many advantages. In these larger machines the number of conductors in the winding may usually be so chosen that only one turn is required for each coil, and each coil is made up of two active conductors made from copper bars and two separately formed end connectors.

For greater mechanical security, these bars are let into grooves cut or punched in the periphery of the core, which grooves are necessarily (as will be pointed out later) narrow, close together, and comparatively deep. Such a winding as this is known as a **bar winding,** and if the grooves or slots in the armature core have overhanging tops, so that the conductors are thoroughly enclosed, the armature is said to be **iron-clad.**

3156. One of the principal features of a drum winding is that opposite sides of a coil must be in magnetic fields of opposite polarity. It follows that a drum winding that is designed for a two-pole field will not give any E. M. F. if rotated in a four-pole field, since opposite sides of a coil would then be in fields of like polarity.

In order to obtain a drum winding for a multipolar machine, it is necessary then that conductors which are similarly situated with respect to fields of opposite polarity should be connected together to form the armature coils. To accomplish this, the pitch of the winding must be something near the value given by $\frac{w}{2p}$, w being the number of winding spaces and p the number of pairs of poles, as before.

As in the bipolar windings, the front pitch should equal the back pitch ± 2 when the two pitches are in opposite directions, and as the total number of winding spaces must be even, both pitches must be odd in order that all the winding spaces may be passed through.

LOOP WINDING.

3157. When the front and the back pitches are of opposite sign, *half* the conductors under any pair of adjacent poles are connected together in series, and, therefore, form one circuit of the armature. This results in there being as many armature circuits as poles, as in the simple multipolar ring, with the same necessity for either as many brushes as poles or a cross-connected commutator and a single pair of brushes.

This type of multipolar drum winding is called a **loop winding,** since, in following the course of the winding, a series of loops is formed, caused by the opposite sign of the two pitches.

3158. Fig. 1174 is a diagram of a four-pole loop winding, in which 32 conductors are represented. The back pitch is taken as +9 and the front pitch as −7.

As in the previous diagrams, the conductors which make up the coils that are short-circuited by the brushes have no arrows, the arrows near the other conductors indicating the direction of the E. M. F. induced in them.

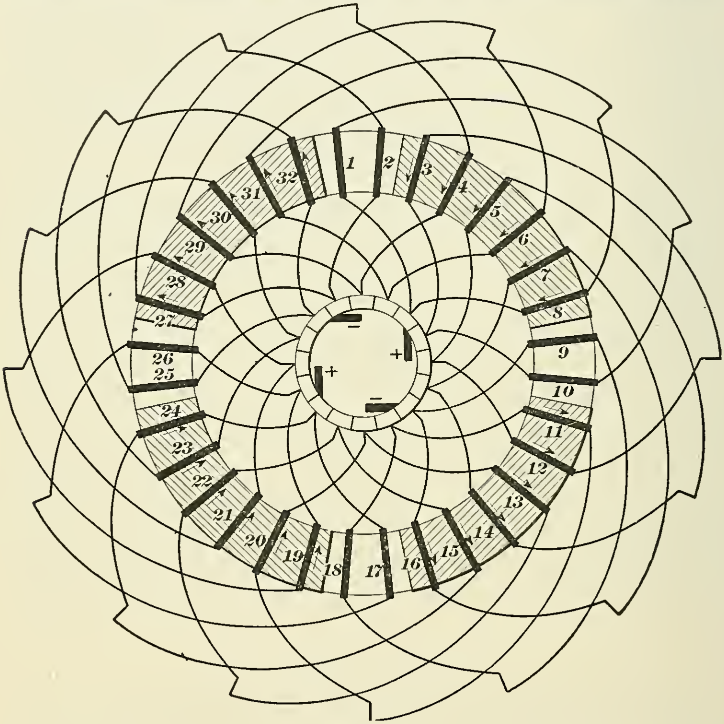


FIG. 1174.

This form of winding needs no further elaboration. It can be used for 6, 8, or a greater number of poles, and its E. M. F. may be calculated in the same manner as for the multipolar ring winding with as many armature circuits as poles. (Art. **3126.**)

When this form of winding is used in slotted armatures (see Art. **3155**), the only requirement that has not already been given is that the total number of conductors must be some even number, and the total number of conductors must be a multiple of the number of slots.

WAVE WINDING.

3159. In bipolar drum windings, giving both pitches the same direction has little effect on the resulting winding; in multipolar drum windings, the effect is marked. If both pitches have the same direction, the winding, passing from front to back under one pole-piece and returning to the front again under the next pole-piece, would continue by passing from front to back under the *next* pole-piece, progressing in the same direction as before, and thus forming a series of *waves*, instead of loops, as when the pitches have opposite directions. (Art. **3157.**) This style of winding is then called a **wave winding.**

As in the bipolar winding, the pitch may be the same, both front and back, in which case it must be odd, or the front and the back pitch may differ by 2, in which case they must both be odd, making the average pitch even.

3160. It has been pointed out that the wave winding advances in a series of waves or steps, and it is evident that, after making a number of steps equal to the number of poles, the winding must come to the *second* winding space from that containing the conductor with which the winding started. From this it follows that the total number of winding spaces possible with this form of winding is equal to the product of the number of poles and the average pitch, ± 2, or, as expressed in the symbols previously used,

$$w = 2py \pm 2. \qquad \textbf{(484.)}$$

It will be noted that this is the same formula as that used for the bipolar drum winding, in which both pitches were given the same direction (formula **483**), with the addition of the term p. (See Art. **3148** and Fig. 1171.)

3161. Fig. 1175 is a diagram of a four-pole wave winding, in which $y = 9$. Therefore, $w = 2py \pm 2 = 34$ or 38. The former number (34) is used in this diagram.

It will be seen from this diagram that the wave winding results in a two-circuit winding, requiring only two brushes, just as the two-circuit multipolar rings. This holds true whatever the number of poles of the field.

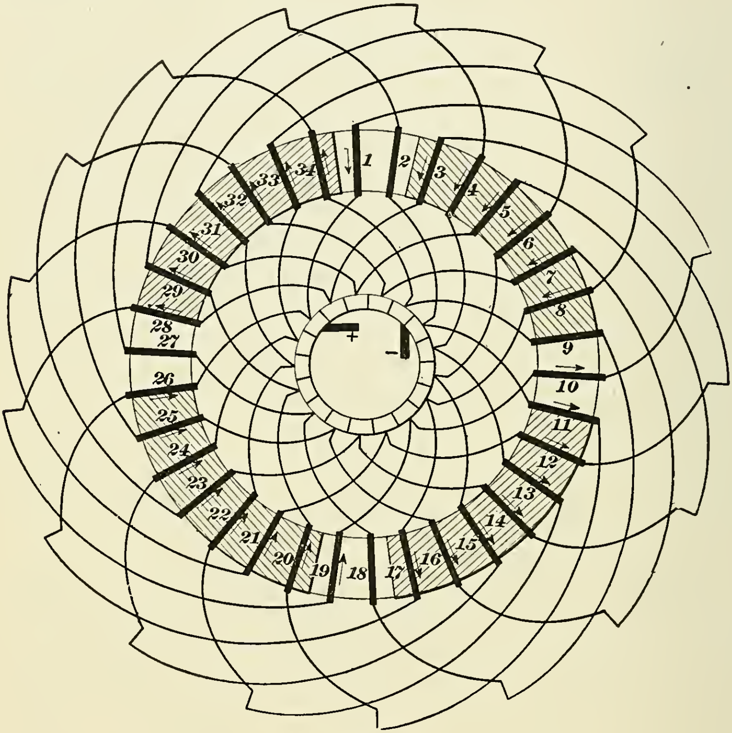


FIG. 1175.

The advance of this winding is in the same direction as the pitch. If +2 had been used in the formula, 38 conductors would have been required, and the advance would have been opposite in direction to the pitch.

If the average pitch had been taken as 8, using +9 for the back and +7 for the front pitch (or *vice versa*), the same number of conductors might have been used; i. e., $w = 2py \pm 2 = (2 \times 2 \times 8) \pm 2 = 30$ or 34.

For bar-wound armatures, it is better to use $+2$ in the formula, and the same pitch on both ends, if the number of conductors required will allow, since that will give the most economical system of end connections.

From Fig. 1175 it will be seen that each brush alternately short-circuits two coils that are in series, and the point where these two coils are connected is the commutator segment that is as nearly as possible opposite the brush that is short-circuiting the coils.

3162. Sometimes it is desired to use a two-circuit armature, but the ordinary form would give too great a difference of potential between segments. Since in this form of winding there are p coils included between every adjacent pair of commutator segments, an additional commutator segment may be inserted, in such a case, between each pair of segments of the winding as already given, each of these interpolated segments being connected with the segment of the original commutator that is directly opposite it. This is illustrated in Fig. 1176, which shows a four-pole wave winding with 30 conductors, in which the pitch (both front and back) is $+7$. A number of commutator segments equal to the number of conductors (30) is used; alternate segments are connected to the winding, and each of the rest is connected to the segment directly opposite, which is one of those connected directly to the winding. The result of this interpolated segment construction is that, unless the brushes are wider than one segment, only one coil, consisting of two conductors, is short-circuited at a time, and the difference of potential between adjacent segments is only that generated in one coil, instead of that generated in p (2) coils, as would be the case if the interpolated segments were not used.

3163. When in the position shown in the figure, the coil formed of conductors *1* and *8* is short-circuited by the — brush. If the armature is rotated in the direction indicated by the arrow, the next coil to be short-circuited is that formed of conductors *9* and *16*, by the +brush; the next is the coil formed by conductors *24* and *17*, by the −brush;

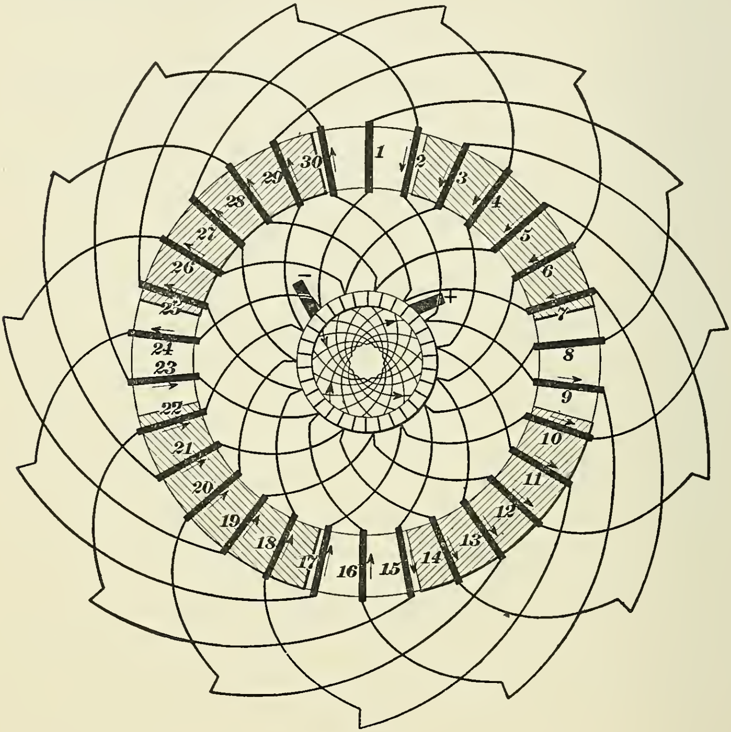


Fig. 1176.

and so on, as the armature rotates. (Compare this with Art. **3132.**)

Only two of the cross-connectors carry the current at any one time, as indicated by the arrow, Fig. 1176.

3164. With more than two pairs of poles, an additional set of interpolated segments must be used for each pair of poles increase over two, and these must be located $\frac{360^\circ}{p}$ apart on the commutator, and connected together. This makes such a complicated system of connections that the interpolated segment construction is seldom used for fields with more than four poles, although, when the number of pairs of poles (p) is *even*, one set of interpolated segments connected to the segments directly opposite may be used, thus halving the difference of potential between segments and the number of conductors short-circuited at a time. Hence, with an eight-pole field, one set of interpolated segments would reduce the difference of potential between adjacent segments to the E. M. F. generated in two coils.

3165. From the formula for the number of winding spaces in the wave winding, $w = 2py \pm 2$, it will be seen that w is always twice an odd number when p is even, as in 4, 8, or 12 pole machines; while w may be twice either an odd or an even number when p is odd, as in 2, 6, or 10 pole machines. From this it follows that with bar-wound armatures arranged for two-circuit single winding, w must be such a number that the number of conductors per slot and $2p$, the number of poles, can not have a common factor greater than 2. For example, four conductors per slot can not be used in an 8-pole machine, as 4 and 8 have a common factor greater than 2. Four conductors per slot can, however, be used with six poles. It would seldom be the case that a greater number of conductors per slot than four would be desired, owing to mechanical difficulties in constructing the winding.

Unless an interpolated segment commutator is used, the number of commutator segments is equal to $\frac{w}{2}$, hence is odd when p is even, and may be either even or odd when p is odd.

3166. In multipolar drum armatures, end connections similar to those described in Art. **3152** are almost invariably used, especially as almost all the larger sizes of drum-wound armatures employ bars for the active conductors and flat strip end connectors, the armature coils then consisting of but two active conductors each.

In case it is desirable to use more than two active conductors per coil, the type of winding described in Art. **3153,** in which the coils are wound to shape on a separate form and afterwards placed in position on the core, may be very advantageously used, especially with slotted armatures.

MULTIPLE WINDINGS.

3167. Sometimes in large machines for large current output the size of the conductors required and the volume of current that must be commuted at the brushes are both inconveniently large with the ordinary forms of winding, as already described. To avoid these difficulties, two or more separate windings on the same armature may be employed, each of which will then furnish its share of the required current. A separate commutator may be employed for each winding, in which case the corresponding brushes of each commutator must be connected in parallel; but as this leads to undesirable complications, it is much better to combine the various commutators into one, by inserting the successive segments of one commutator between the similar segments of the other. The various windings are then connected in parallel by using a wide brush, which must evidently be of sufficient span to be always in contact with at least one segment that is connected to each winding, so that if there are m separate windings, each brush must have a span not less than that of m segments. Under these conditions, the coils of the successive windings will be short-circuited one at a time, and the volume of current commutated will be only $\frac{1}{m}$ of that which would be short-circuited if a similar form of single winding were employed for the same current output.

Such a winding as has been described is known as a **multiple winding,** to distinguish it from those forms in which the conductors are so connected as to form a *single* closed-coil winding.

Any specific winding is usually spoken of as a *double,* *triple*, etc., winding, according to the number of separate windings employed.

3168. If a given number of conductors which, when connected up into any particular form of single closed-coil winding, will give an E. M. F. of V volts, are so connected as to give m separate windings of the same form, all connected in parallel, there will be but $\frac{1}{m}$ as many of the conductors connected in series as in the single winding, hence the E. M. F. will be only $\frac{V}{m}$ volts. To apply the formulas given for finding the E. M. F. developed in a winding consisting of a certain number of conductors (formulas **480** and **482**), it is only necessary then to introduce the term m (the number of separate windings) into the denominator of the formula, so that for multiple-wound *multiple-circuit* windings the formula becomes

$$E = \frac{c N S}{60 \times 10^8 m}, \qquad \textbf{(485.)}$$

and for multiple-wound *two-circuit* windings it becomes

$$E = \frac{c p N S}{60 \times 10^8 m}. \qquad \textbf{(486.)}$$

3169. The principle of multiple winding may be applied to any form of closed-coil winding, if desired, and, further, by properly selecting the number of coils and their order of succession, the end of one winding may be joined to the beginning of the next, and so on, thus forming a single reentrant system of the whole series of conductors. This may also be modified, as will be pointed out, to make the windings form a number of separate reentrant systems which will be some whole factor of m. That is to say, the conductors of a multiple-wound armature having m windings may be combined as m separate reentrant systems, 1 reentrant system, or a number of separate reentrant systems equal to some whole factor of m. In practice, it is seldom that m exceeds 3 or 4, although it may be any whole number within reasonable limits.

The application of the principle of multiple windings to the various types of armature windings will now be taken up.

MULTIPLE-WOUND MULTIPLE-CIRCUIT RING WINDINGS.

3170. The multiple-circuit winding is the simplest form of ring winding, and, as has already been pointed out, it may be used in fields having any number of pairs of poles without changing the connections.

Since the adjacent coils of a single-wound ring are connected together, and for multiple windings the separate coils of each winding are supposed to lie between successive coils of the others, it follows that, in connecting up the coils of a ring winding to form m separate windings, each coil is connected to the mth coil on each side; that is, $m - 1$ coils are *skipped over* in connecting successive coils of the wind-

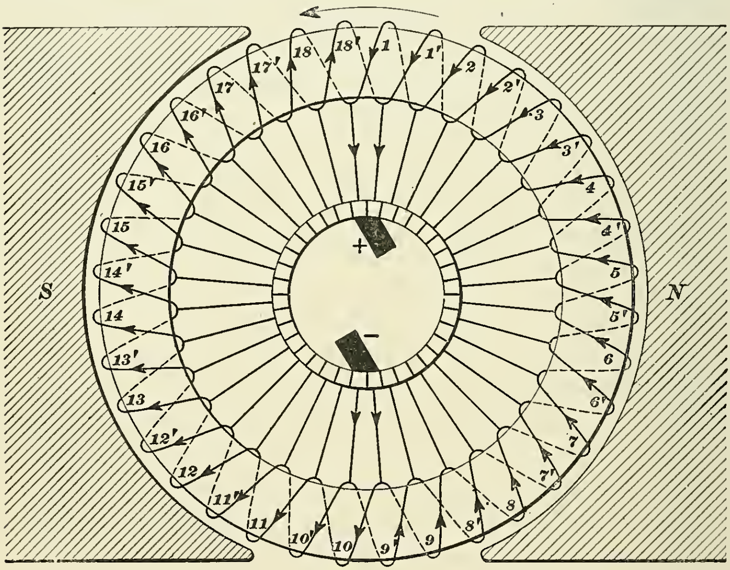


FIG. 1177.

ing. This is shown in Fig. 1177, which represents a two-pole multiple-wound multiple-circuit ring armature of 36 coils, in which $m = 2$. Consequently, in connecting successive coils, $2 - 1 = 1$ coil is skipped once, and alternate coils are connected in each winding. Coils numbered *1*, *2*, *3*, etc., represent the one winding, and coils *1′*, *2′*, *3′*, etc., represent the other.

3171. It will be seen that, in connecting alternate coils of the even number (36) which is used in this case, the end of the 18th coil is connected to the first coil, thus forming one reentrant system, so that a fresh start must be made to form the second winding, which, therefore, forms a second reentrant system.

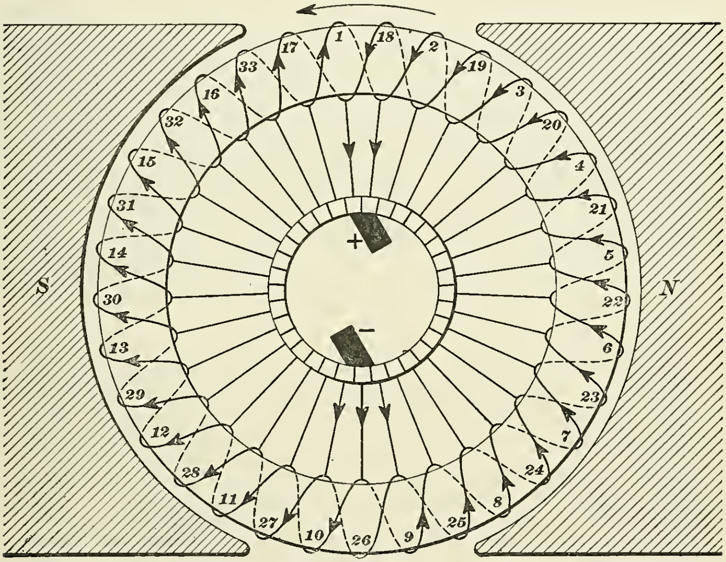


Fig. 1178.

This results from the fact that the total number of coils is divisible by the number of windings, without a remainder.

If the number of coils is so chosen that there is a remainder, then, after passing through alternate coils once around the armature, the end of the last coil connected will not connect with the beginning of the coil from which the winding was started, but with one on one side or the other of it, thus starting the second winding, which ends at the beginning of the first coil of the first winding; the two windings thus form a single reentrant system.

This is illustrated in Fig. 1178, which represents a two-pole multiple-circuit multiple-wound armature having 33 coils, in which $m = 2$, as before. The coils are numbered from 1 to 33, inclusive, in the order in which they are connected. It will be seen that, after passing through alternate coils once around the armature, thus passing through 17 coils, the next coil in succession is coil *18*, immediately to the right of coil *1*, which is then the beginning of the second winding, which ends with coil *1*.

3172. To make a single reentrant winding, when $m = 2$, the number of coils must be odd. This being the case, the number of commutator segments is odd, and but one coil is short-circuited at a time, unless the brush has a span greater than that of two segments. In the case illustrated in Fig. 1178, coil *26* is short-circuited by the $-$brush; a moment later, the $+$brush will short-circuit coil *18*, then the $-$brush will short-circuit coil *10*, then the $+$brush coil *2*, and so on.

3173. In general, for this class of windings (which, as already stated, may be applied to fields having any reasonable number of pairs of poles), if the number of coils, s, is a multiple of the number of windings, m, the conductors will connect together into m separate reentrant systems; while, if the number of coils is mutually prime with m, the conductors will join together into a single reentrant system. For example, a multiple-circuit multiple-wound armature where $m = 3$ is to have in the neighborhood of 50 coils. If 48 or 51 coils is the number used, three separate reentrant systems will result, each containing $\frac{48}{3} = 16$ or $\frac{51}{3} = 17$ coils. If 49 or 50 coils are used, a single reentrant system will result. When $m = 4$, or any even number, the number of reentrant systems that will result with any given number of coils, s, will be equal to the *greatest common factor* of m and s. Thus, when $m = 4$ with 48 coils, the greatest common factor being 4, that number of separate reentrant systems will result; with 49 coils, the greatest common factor is 1, and one reentrant system will result. With 50 coils, however, the greatest common factor is 2, so that two separate reentrant systems will result, each made up of 2 of the 4 windings. (Compare this with Art. **3171.**)

MULTIPLE-WOUND TWO-CIRCUIT RING WINDINGS.

3174. The application of the principle of multiple windings to this form of armature winding is not materially different from the cases just considered.

In the single winding, described in Arts. **3129** to **3133,** the number of coils in the winding is found from formula **481,** $s = p\,y \pm 1$, the last term (± 1) being introduced in order that the winding should form a *single* two-circuit winding. To apply this formula to multiple-wound two-circuit windings, it is only necessary to substitute m, the number of separate windings desired, for 1, which gives the following formula :

$$s = p\,y \pm m. \qquad \textbf{(487.)}$$

If y (the pitch) is a multiple of m, then s will also be a multiple of m, and, as in the multiple-circuit windings, m separate reentrant systems will result ; while if y and m are mutually prime, then s will not be a multiple of m, and a single reentrant system will result. In fact, the number of separate reentrant systems which will result with any given number of coils will be equal to the greatest common factor of m and y.

3175. For example, a four-pole two-circuit ring winding, with a pitch of 11 and 3 windings ($y = 11$, $m = 3$) could have $s = p\,y \pm 3 = 22 \pm 3 = 25$ or 19 coils, and 11 and 3 being mutually prime, a single reentrant system would result with either number. Fig. 1179 represents the above case, 25 being the number of coils used. It will be seen that this winding is of the same type as the single winding illustrated in Fig. 1166. In this case the coils are numbered from 1 to 25; in addition, the numbers *1′*, *2′*, *3′*, etc., show the order in which the successive coils are connected. This being a triple winding, the brushes are made of the same span as three segments; the + brush short-circuits coils *8* and *19*, and the − brush short-circuits coils *2* and *13*, the rest of the coils having arrow-heads showing the direction of the current in them.

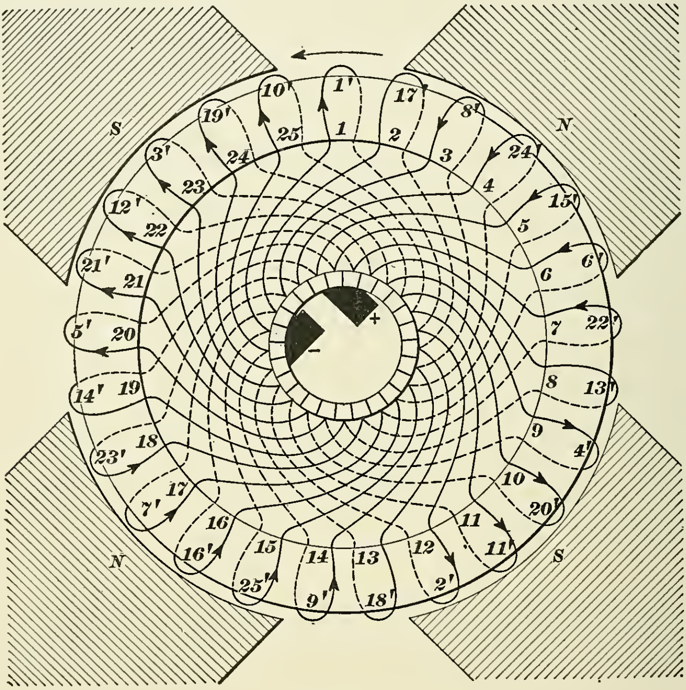


Fig. 1179.

Each of the three windings of this example being a two-circuit winding, there are six circuits through the armature. On tracing these out, starting from the − brush, it will be found that the various coils are divided among the circuits as follows:

$$-\left\{\begin{array}{l}\left\{\begin{array}{l}1-12-23-9-20\\15-\ \ 4-18-7\end{array}\right\}\\ \left\{\begin{array}{l}25-11-22\\14-\ \ 3-17-6\end{array}\right\}\\ \left\{\begin{array}{l}24-10-21\\16-\ \ 5\end{array}\right\}\end{array}\right\}+$$

This indicates an extreme irregularity in the number of coils in each circuit, but this is only due to the small number of coils necessarily used in the diagram. In any winding as actually used the irregularity would be almost inappreciable.

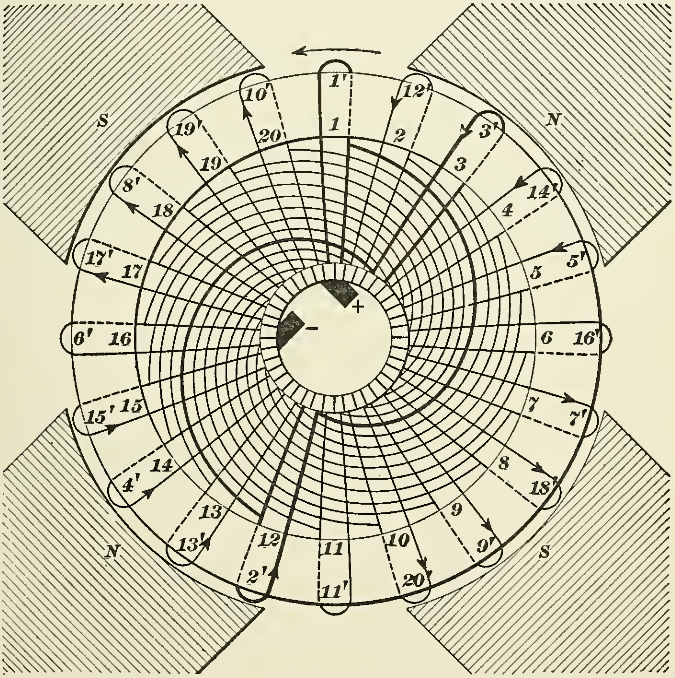


FIG. 1180.

3176. Fig. 1180 is a diagram of a four-pole two-circuit double-wound armature of the same type as that illustrated in Fig. 1167. For this type of two-circuit ring winding, formula **487** is used; but to obtain an even distribution of potentials around the commutator only the −sign should be employed; i. e., $s = p\,y - m$. In this case $y = 11$; hence, $s = 22 - 2 = 20$. As y and m are in this case also mutually prime, the winding forms a single reentrant system. As in Fig. 1179, the coils are numbered *1*, *2*, *3*, etc., and the numbers *1′*, *2′*, *3′*, etc., show the succession in which the coils are connected. Coils *1*, *12*, and *3*, and their connections, are drawn in heavier lines than the rest, to better show the plan of connection.

Although this is a double winding, the brushes must be made of a span equal to at least that of 3 segments, as shown, in order that they may be in connection with both windings all the time. This width is necessary because each coil is connected to two adjacent segments.

In the position shown, the +brush short-circuits coils *1* and *11*, and the −brush short-circuits coils *16* and *6*; the direction of the current in the remaining coils is indicated by the arrow-heads, as before. The four circuits of this armature are made up as follows, starting from the −brush:

$$-\left\{\begin{array}{l}\left\{\begin{array}{l}17-8-19-10\\15-4-13-\ \ 2\end{array}\right\}\\ \left\{\begin{array}{l}7-18-\ \ 9-20\\5-14-\ \ 3-12\end{array}\right\}\end{array}\right\}+$$

This winding is much more regular than that shown in the previous figure, but this is not an essential feature of this form of winding, being due to the even number of coils and windings.

3177. In case it is desired to make the cross-connections a part of the commutator construction, which is usually more desirable, the angular span of the cross-connections should be the same throughout, in order that the cross-connections may be symmetrical.

With the winding as shown, this is not the case, for the leading segment of coil *1* is connected to the *19th* segment to the right, while the following segment of the same coil is connected to the *21st* segment, also to the right.

If, instead of connecting the two ends of each coil to adjacent segments, they are connected to two segments which are separated by a third, the inequality in the spans of the cross-connections disappears, and they become symmetrical. This, however, causes the leads from the armature coils to the commutator segments to cross, requiring extra precautions in insulating.

In case the winding were triple, quadruple, etc., the two ends of each coil would be connected to two segments separated by 2, 3, etc., others; that is, in general, the two commutator segments to which each coil of the m windings is connected would be separated by $m - 1$ other segments, if it be desired to make the cross-connections a part of the commutator construction.

MULTIPLE-WOUND MULTIPLE-CIRCUIT DRUM WINDINGS.

3178. The conditions governing the multiple-circuit multiple-wound ring windings also apply to this class; in addition, the influence of the difference between the ring and the drum form of coil must be taken into account. As each coil of the drum winding is made up of *two* active parts, each occupying a winding space, the number of winding spaces, w, must be *even*.

The back pitch, which determines the number of winding spaces included between the two active parts of a coil (see Art. **3139**), needs only to be made of such value that the two parts of the coil shall not be in any one field at the same time, which implies that the angular span of the coil should not be much greater or less than $\frac{360°}{2p}$.

The front pitch, which determines the number of winding spaces included between similar parts of two successive coils, is determined by the number of separate windings used. In the multiple-circuit *single-wound* drum winding, the front pitch = back pitch ± 2; that is, a winding space, belonging to another part of the winding, intervenes between the adjacent parts of successive coils.

In the multiple windings, in addition to the winding space for another coil of the *same* winding, there must also be included between the adjacent parts of successive coils two winding spaces for each of the *other* windings. Consequently, the difference between the front and back pitches must be $2m$. In practice, the front pitch is made less than the back pitch for reasons already given. (Art. **3145.**) Both pitches must be odd, and the front pitch must be opposite in direction to the back pitch.

3179. As in the multiple-circuit multiple-wound ring windings, the number of separate reentrant systems formed by the windings will equal the greatest common factor of the number of coils and the number of windings; the number of coils being equal to one-half the number of winding spaces, the number of reentrant systems is equal to the greatest common factor of $\frac{w}{2}$ and m. Any even number of winding spaces may be used, whatever the number of poles.

In order to prevent opposing E. M. F.'s in a coil, the number of winding spaces should be about equal to the product of the number of poles and the average of the front and back pitches. (Compare Art. **3156.**) It is usually rather better to make the number of winding spaces a little greater than this product, as in this case the end connections are a little shorter.

3180. Fig. 1181 shows a diagram of a four-pole multiple-circuit double-wound drum armature having 20 coils ($w = 40$). The back pitch is taken as $+13$; hence, the front pitch $= -(13 - 2m) = -(13 - 4) = -9$. $\frac{w}{2}$ (20) being a multiple of m (2), this gives two separate reentrant systems. A single conductor is represented in each winding space, numbered *1*, *2*, *3*, etc.; the order in which the conductors making up the first of the two windings are connected is indicated by the numbers *1'*, *2'*, *3'*, etc., and the order of connection of the conductors of the second winding is indicated by the numbers *1″*, *2″*, *3″*, etc. Each brush short-circuits a single coil, and the short-circuited conductors

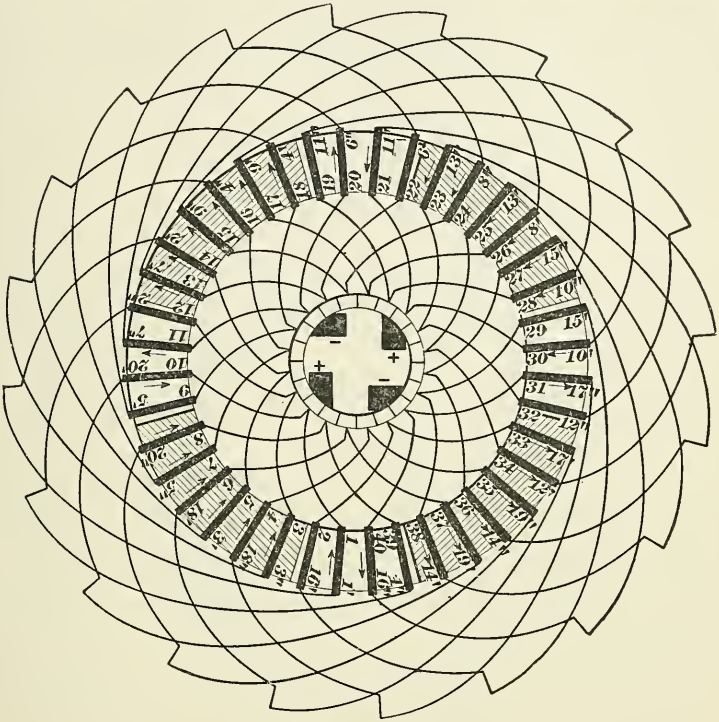


Fig. 1181.

2, *9*, *12*, *19*, *22*, *29*, *32*, and *39* are indicated by the absence of the arrow-heads, which on the rest of the conductors indicate the direction of the current in them.

MULTIPLE-WOUND TWO-CIRCUIT DRUM WINDINGS.

3181. The principles and formulas given for two-circuit single-wound drum windings require only slight modifications to adapt them to this class of windings.

The front and the back pitches being in the same direction may be alike or may differ by 2. In either case, each pitch must be odd; so, if both pitches are alike, the average pitch must be odd, but if they differ by 2, the average pitch may be even.

In the single-wound two-circuit drum winding it was pointed out that, in passing through the winding, the second winding space to one side or the other of that at which the start was made would be arrived at after passing under each pole in succession, and from this the formula given for the number of winding spaces was derived.

In the multiple-wound two-circuit drum windings, in addition to this one winding space belonging to the *same* winding, two others for each of the *other* windings of the armature must also intervene between the winding space started with and that passed through after making one series of steps around the armature. From this it follows that the total number of winding spaces allowable will be given by the formula

$$w = 2\,p\,y \pm 2\,m, \qquad \textbf{(488.)}$$

y being the *average* pitch, and p and m being the number of pairs of poles and the number of windings, respectively, as before. As in all two-circuit windings, only two brushes are necessary, although two for each pair of poles may be used if desired.

The number of separate reentrant systems formed will be equal to the greatest common factor of m (the number of windings) and y (the average pitch).

3182. In Fig. 1182 is shown a diagram of a four-pole, double-wound, two-circuit drum armature, having the same number of coils (20) as the multiple-circuit armature illustrated in Fig. 1181. In this case the pitch, both front and back, is taken as 9, and the number of winding spaces found from formula **488**, as follows:

$$w = 2\,p\,y \pm 2\,m = 36 \pm 4 = 40 \text{ or } 32.$$

In this case 40 winding spaces was the number used. As before, one conductor in each winding space is represented, they being numbered *1*, *2*, *3*, etc. Since the greatest common factor of y (9) and m (2) is 1, this winding results in a single reentrant system, the order in which the conductors are connected being indicated by the numbers *1′*, *2′*, *3′*, etc. Two brushes are shown, the +brush short-circuiting the coils formed from conductors *13*, *22*, *31*, and *40*, and the −brush short-circuiting the coils formed from conductors *3*, *12*, *21*, and *30*, these being indicated by the absence of the arrows which, with the other conductors, indicate the direction of the current in them. The path of the current through the four circuits of this armature, starting from the − brush, is as follows:

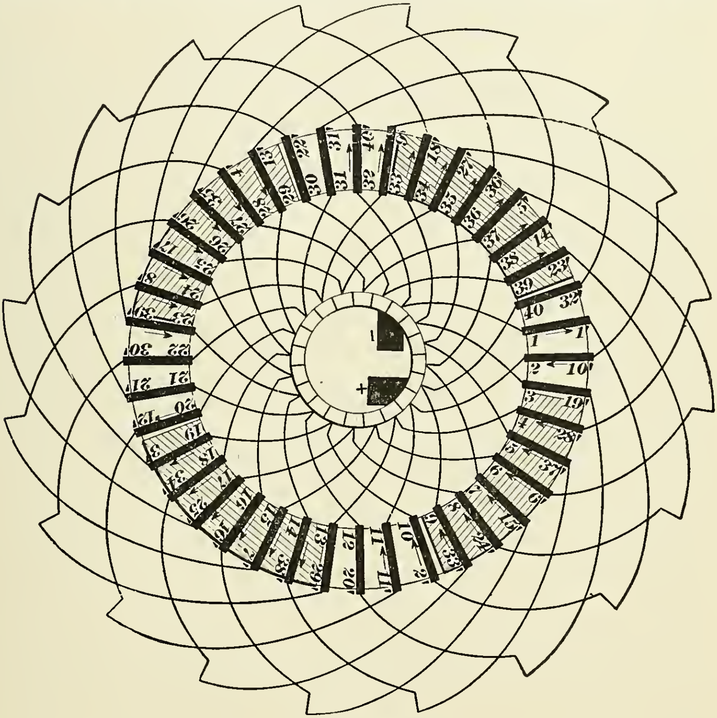


FIG. 1182.

$$-\left\{\begin{array}{l}\left\{\begin{array}{l}1-10-19-28-37-\ 6-15-24-33-2\\32-23-14-\ 5-36-27-18-\ 9\end{array}\right\}\\ \left\{\begin{array}{l}39-\ 8-17-26-35-\ 4\\34-25-16-\ 7-38-29-20-11\end{array}\right\}\end{array}\right\}+$$

It will be seen that some irregularity is indicated, owing to the coils short-circuited by the —brush being taken wholly from the second winding. With the necessarily large number of conductors used in practice, the difference between the number of conductors in the different branches of the winding forms such a small percentage of the whole number employed as to make its effect negligible.

3183. One of the principal advantages of multiple winding as applied to drum armatures appears when bar windings (a single conductor per winding space, with separate end connections) are used.

In this form of winding, the bars are usually set in slots cut in the periphery of the armature core, and it is very desirable that the number of slots adopted for any particular size of armature be such that they may be used for windings giving different voltages, without change. Thus, for example, of the two windings illustrated in Figs. 1181 and 1182, the two-circuit winding (Fig. 1182) will evidently give twice the E. M. F. that the multiple-circuit winding (Fig. 1181) will with the same number of revolutions and in a magnetic field of the same strength, the only change made in the winding being in the span and arrangement of the end connectors. The same result may be attained by changing the multiple-circuit winding from a multiple-wound to a single-wound armature, which would be accomplished in this case (Fig. 1181) by reducing the back pitch to 11, or increasing the front pitch to —11. The two-circuit winding can not be so changed, however, in a four-pole machine, as an odd number of *coils* is required for the single winding (Art. **3165**); but when the number of pairs of poles is odd, as in a six-pole machine, an even number of coils may be employed for the single winding, and this may be changed to a double winding by changing the end connections, if desired, and the features of the two-circuit winding retained.

3184. For example; suppose that, having decided on a certain number of revolutions and a certain number of lines of force in the field, it is found that 358 conductors are required for a six-pole, single-wound, two-circuit, bar-wound drum armature, to give 500 volts.

From formula **484,** $w = 2\,p\,y \pm 2$, the required pitch may be found; since $w = 358$ and $p = 3$, $358 = 6 \times y \pm 2$, from which $y = \frac{356}{6} = 59.3 +$ or $\frac{360}{6} = 60$, which latter value would necessarily be used, fractional pitches being an absurdity. As the front and back pitches must each be odd, to have the average pitch 60, the front and back pitches may be 59 and 61, respectively.

In case it was desired to use the same armature for a 250-volt machine, the same number of conductors might be used, by so changing the pitch as to make a double winding. The proper pitch to use would be found from formula **488,** $w = 2\,p\,y \pm 2\,m$; $w = 358$ as before, $p = 3$, and $m = 2$, and $358 = 6 \times y \pm 4$; hence, $y = \frac{354}{6} = 59$, or $\frac{362}{6} = 60.33 +$. 59 would be taken as both front and back pitch. It would thus be only necessary to slightly change the end connectors for the *back* pitch, to use the same armature for either a 250-volt or a 500-volt machine.

THE MAGNETIC CIRCUIT.

3185. As far as the generation of the E. M. F. of the dynamo is concerned, it is only essential that the lines of force of the magnetic field be present at the points where they are cut by the conductors, and have the proper direction and distribution. However, since each line of force is continuous, forming a closed circuit, provision must be made for a complete path for the lines of force to and from the points where they are cut by the conductors, and through the magnetizing coil or coils wherein they are generated. Of course, they might be left to find their own circuit through the surrounding air, but in order to realize the large number of lines of force required with the expenditure of a reasonable amount of magnetizing force, it is necessary that the path of the lines of force be of as great a permeability as possible; i. e., through an iron or steel *magnetic circuit.*

In addition to the armature and its winding, a bipolar or multipolar dynamo must then have an iron or steel *frame*, or *field-magnet*, which completes the magnetic circuit outside the armature. This frame is made up of one or more pairs of *pole-pieces*, from (or into) which the lines of force pass to (or from) the armature through the spaces between the faces of the pole-pieces and the surface of the armature core, which are called the **air-gaps**; it must also have a part upon which the *magnetizing coils* are wound, which part is called the **field core.** The part of the frame that joins together the field cores, if more than one is used, or that joins the pole-pieces and the field cores, is called the **magnetic yoke.**

CONSTRUCTION OF FRAME.

3186. It will be seen that the object of the frame, as a whole, is to so guide the lines of force that are generated by the current in the magnetizing coils that they will enter and leave the armature at the proper points, forming the magnetic field in the air-gaps of the required distribution and density.

It is not essential to the operation of the machine that the frame be of any given form or size, so long as the lines of force are properly delivered to the armature; economy in materials or labor, mechanical strength, and other considerations determine the form and size of frame to be adopted.

3187. Since the magnetic circuit may be considered analogous to the electric circuit, it will be seen that in order to obtain a large number of lines of force with a moderate magnetizing force, the reluctance of the circuit must be low; that is, the iron should be of considerable cross-section and the circuit of moderate length. It should be remembered that, since the permeability of the best of iron is only, perhaps, 1,500 times that of air, a considerable number of lines of force that pass through the magnetizing coil complete their circuit around through the air without passing through the air-gaps. To reduce this magnetic leakage as far as possible, surfaces between which there is a great difference of *magnetic potential* should be kept as far apart as the design of the magnet will allow, and made of as small area as possible. In any case, some leakage is bound to occur, and this must be provided for by making those parts of the frame through which the leakage lines pass of sufficient area for both the useful and the leakage lines. The conditions which govern the leakage will be more fully discussed later; in general, the area of the iron in the frame must be sufficient for from 15 to 50% more lines of force than are used in the armature.

DENSITY OF LINES OF FORCE.

3188. Referring to Fig. 952, it will be seen that the saturation curves there shown all rise in a nearly straight line for some distance from *O*, then curve away from the axis of the ordinates and follow another approximately straight line, which makes a much greater angle with the axis of the ordinates than does the first-mentioned line. This effect is much more marked in the case of wrought iron and cast steel than with cast iron, but in any case it will be seen from this feature of the saturation curves that the most economical density at which to work the iron of the magnetic circuit is that in the vicinity of the bend or "knee" of the curve. A much lower density could not be economically used, because a considerable increase in the number of lines of force could be obtained with comparatively little increase in the magnetizing force required; and on this account accidental small changes in the magnetizing force would produce a considerable change in the number of lines of force, so that the magnetic circuit of the machine would be in an *unstable* condition. A much higher density would not be economical, because the increase in the number of lines of force could be obtained only by a very considerable increase in the magnetizing force.

3189. Applying these statements to the curves given in Fig. 952, it will be seen that, in general, cast steel and wrought-iron forgings should be worked at densities of between 80,000 and 100,000 lines of force per square inch, while sheet iron may be worked higher, between 90,000 and 110,000 lines of force per square inch. With cast iron, the curves being flatter, the allowable range is somewhat greater, the usual range in practice being from 25,000 to 50,000 lines of force per square inch, the latter value being used only in the case of the best grades of soft, gray cast iron.

The best densities to use are, therefore, not those that give the maximum permeability of the iron used, as at that point the iron would be in the unstable condition referred to previously.

3190. From the above and from the curves referred to, it appears that for the same expenditure of magnetizing force a cast-iron magnetic circuit must have about twice the sectional area of one of cast steel or wrought iron, in order to realize the same number of lines of force, so that the cast-iron magnetic circuit would be about twice as heavy as one of steel or wrought iron; its less cost per pound, however, may often counterbalance this extra weight, and, in fact, the choice of materials for the frame, as well as almost all the other features of a dynamo, depends upon the local conditions governing each particular case.

3191. The density used in the air-gaps varies, but the best practice fixes it at somewhere in the neighborhood of 30,000 lines of force per square inch; this depends, however, on many other features of the design, as will be pointed out later.

In any case, the amount of the magnetizing force that is required to force the magnetic flux through the air-gaps is a large proportion of the total amount, since the permeability of the air-gaps is 1, which much more than compensates for their comparatively short length.

FORM OF MAGNETIC CIRCUIT.

3192. The form of the magnetic circuit is subject to many variations; there are, however, two general classes into which they may all be divided. In the first, a single source of magnetizing force for each pair of poles (which may reside in one or more magnetizing *coils*) sends the lines of force around through a magnetic circuit, of which the air-gaps and armature directly form a part. Such an arrangement is said to have **salient poles.** In the second type, at least two magnetizing forces are necessary for each pair of poles; these magnetizing forces act in opposite directions upon a complete magnetic circuit, and the opposing lines of force cause consequent poles to appear at points on the magnetic circuit, which points are properly provided with pole-pieces, between which the armature is located. Such an arrangement is said to have **consequent poles.**

3193. One of the simplest forms of salient-pole bipolar field-magnets is represented in Fig. 1183. In this form the magnetizing force is supplied by the single coil shown in section at *W* and *W*. This surrounds the field core *C*, to which are attached the magnet yokes *M* and *M*, which terminate in the pole-pieces *N* and *S*. Between these pole-pieces the armature *A* revolves. The mean paths of the lines of force through the magnetic circuit (neglecting leakage lines) are indicated by the dotted lines having the arrow-heads, which indicate the direction of the lines of force, assuming the polarities of the pole-pieces to be as indicated by the letters *N* and *S*. In this figure the field core is represented as being vertical, and this type of magnet is so used in certain machines of English make. It may, however, be either vertical or horizontal, and be above, below, or on either side of the armature, as desired. The Jenney motors, the Wood bipolar machines, the Holtzer-Cabot small motors, and others

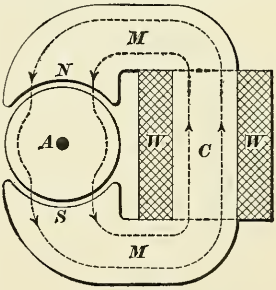


FIG. 1183.

made in this country use this type of magnets with the coil horizontal and below the armature. Further, the armature shaft may either have the direction indicated or be at right angles to that direction, if desired, without changing the character of the field-magnet. The mechanical construction in this last case would evidently be bad, and, in general, this is the principal feature which determines the disposition of the magnet frame with regard to the armature.

3194. A form of consequent-pole field-magnet which is derived from that just described is shown in Fig. 1184. This form of field-magnet is known as the "Manchester type," and is used by the Mather Electric Co., the Westinghouse Co., and others in this country.

This is practically the same form of magnet as that shown in Fig. 1183, with the addition of a second similar magnet situated on the opposite side of the armature *A*, as indicated by the letters N', M', C', M', and S'.

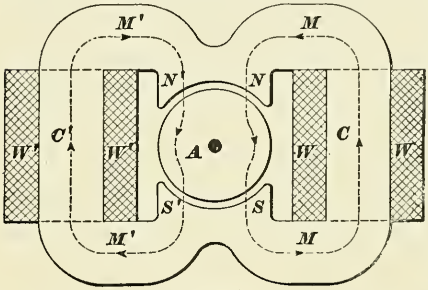


FIG. 1184.

Assuming that the same total number of lines of force passes through the armature in each case, it follows that with the consequent-pole magnet (Fig. 1184) each half of the magnetic circuit contains half the total number of lines, and needs, therefore, to be of but half the sectional area of the frame of the salient-pole magnet, which carries all the lines of force, as is indicated by the relative proportions of the two magnets. (See Figs. 1183 and 1184.) Consequently, the weight of the frame in either case is about the same.

3195. In the consequent-pole magnet, the magnetic circuit in each half is approximately the same length but of half the area as that of the salient-pole magnet; its reluctance is about twice as great, but since it carries half the number of lines of force, it follows that the magnetizing force required for *each half* of the consequent-pole magnetic circuit is the same as that required for the whole of the salient-pole magnet. However, the magnetizing coils on the consequent-pole magnet are of smaller diameter than those used in the salient-pole magnet, so that the weight of copper used for the magnetizing coils of the former type of magnet is not double that required for the latter type. The actual ratios of weights of copper and iron may be readily calculated for any particular case, but there are other conditions that influence the choice of the form of magnet to be used, which must be taken into account.

3196. Fig. 1185 shows the adaptation of these two forms of field-magnets to a multipolar machine. In the figure, the part to the left of the vertical diameter represents the salient-pole magnet, and that to the right represents the consequent-pole magnet, each being laid out as for an eight-pole magnet.

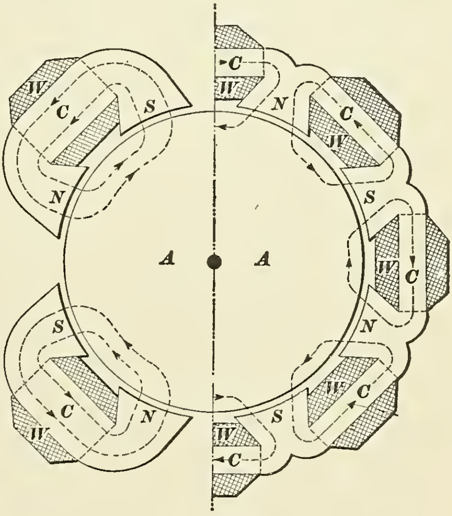


FIG. 1185.

The salient-pole magnet consists of a number of separate magnets, each with its magnetizing coil. It is, therefore, necessary to supply some separate support for these magnets. In the consequent-pole magnet, however, the whole frame is continuous, each pole-piece being supported by a field core on each side, the frame, therefore, being of sufficient mechanical strength for its own support.

In the latter form, the mean length of the magnetic circuit for each pair of poles is less than with the salient-pole magnets, which results in a slight saving in magnetizing force, other things being equal.

Of the above types of magnets for multipolar machines, the salient-pole type is used in the "Perrett" machines, built by the Electron Manufacturing Co., and the consequent-pole type is used by the Standard Electric Co., in this country, and in several types of machines made abroad.

3197. The two simple forms of field-magnets which have been described may be considerably modified by changing the position or increasing the number of the field coils. For example, the magnetizing coil of the salient-pole magnet (Fig. 1183) may be wound over the entire frame from pole-piece to pole-piece, as in the "ring-type" machine of the Mather Electric Co. Similarly, the magnetizing coil on each half of the consequent-pole magnet (Fig. 1184) may be wound over the entire frame from pole-piece to pole-piece, as in the "C & C" machines. In both these examples, the field cores are made approximately circular in outline.

Further, by dividing the magnetizing force between two coils, and locating these coils in the part indicated as the magnet yoke in Fig. 1183 (*M* and *M*), a type of field-magnet results which is commonly known as the **horseshoe** type, as illustrated in Fig. 1186. It will be seen that in these two forms the magnet yoke (*M*) of each corresponds to the field core of the other. This type of field-magnet is very extensively used for bipolar machines, the Thomson-Houston, Crocker-Wheeler, Connecticut, Keystone, and other makes of machines using it in the position shown, i. e., with the magnet frame beneath the armature.

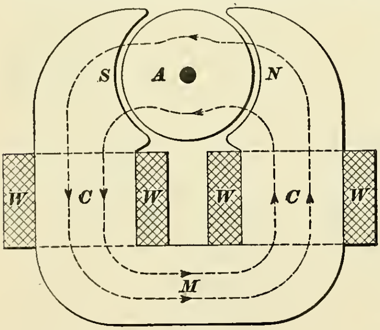


Fig. 1186.

The General Electric Co. in their Edison machines, the Commercial Electric Co., the Eddy Electric Manufacturing Co., and others, use the same form of magnet in the reverse position, i. e., with the magnet frame above the armature.

The Excelsior arc machine employs the same type of magnet, but with the armature shaft parallel to the field cores, passing, therefore, directly through the magnet yoke. The pole-pieces are necessarily modified in shape to suit the changed position of the armature, and are extended to embrace three sides of the armature, which is ring wound.

3198. The consequent-pole magnet that results from combining two horseshoe magnets of the types illustrated in Fig. 1186 is shown in Fig. 1187. Here the various letters have the same reference as in the previous figures. As in that previously described, the consequent-pole arrangement requires only half the cross-section of metal in each half of the magnetic circuit, but the total amount used is about the same. This is also a commonly used type of bipolar field-magnet. Among others, it is used in the Wood arc machine of the larger sizes, in the position represented in the figure, i. e., with the field cores (*C*, *C*, *C*, *C*) vertical. The Weston and the Schuyler arc machines use the same form of field-magnet, but with the field cores horizontal, and it has also been used in this same position for various special machines built by the General Electric Co. and others.

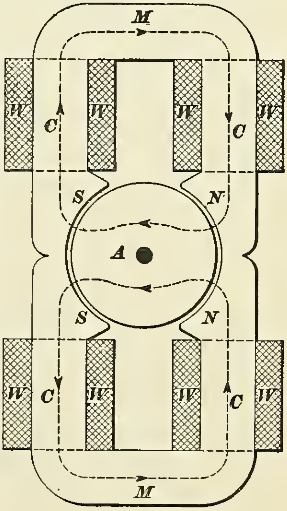


FIG. 1187.

The smaller sizes of the Wood arc machine use this form of magnet with the field cores horizontal, and with the shape of the pole-pieces modified so as to allow of the armature shaft being parallel to the field cores, it passing through and having its bearings in the yokes (*M* and *M*). The Brush arc machine uses a similar construction, but the armature is made in the form of a ring-wound disk, and the pole faces face the end faces of the armature, as represented in the diagram, Fig. 1188. The magnet in this case might be considered to be two separate bipolar, salient-pole, horseshoe magnets.

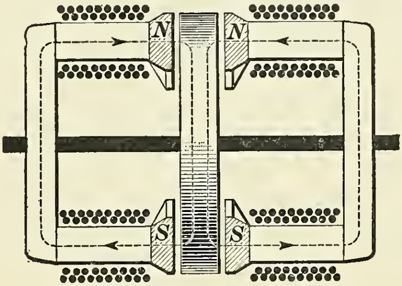


Fig. 1188.

3199. By carrying the magnetizing coils still further along the frame, until they are as close as possible to the ends of the pole-pieces, still another type of field-magnet results, as represented in Fig 1189.

As shown, this is a very heavy and clumsy magnet, requiring a large amount of material on account of the length of the magnet yoke, *M M*. If, however, half the material in this yoke be located on the other side of the armature, so that the magnetic circuit through the frame from field core to field core consists of two branches, a much neater and lighter magnetic circuit, that is quite extensively used, results, as represented in Fig. 1190.

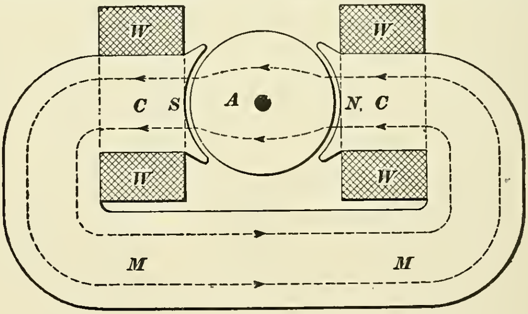


Fig. 1189.

This form of circuit still has salient poles, since the poles are produced by the direct action of the magnetizing forces, and not by the opposition of two magnetizing forces.

It has the advantage that the magnetizing coils and armature are enclosed by the frame, thus affording them mechanical protection.

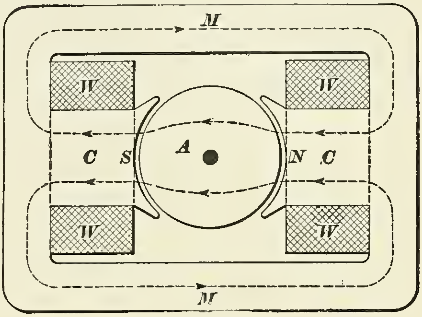


Fig. 1190.

This type of magnet is used (in the position shown) by the makers of the "Detroit" dynamos, by the Western Electric Co., and by others in this country and abroad.

The Thomson-Houston arc-lighting dynamos also employ this type of field-magnet, the form being modified by making the magnet yokes of a series of round, wrought-iron bars, which connect together circular flanges on the ends of the field cores, thus making the general outline cylindrical.

Eickemeyer has used it for very compact machines in which the magnetizing coils actually enclose the armature, the field cores being very short.

The same form of magnet, but with the magnetizing coils above and below the armature, was used in the old Hochhausen dynamos, also by the Thomson-Houston Company for their old "S. R. G." railway motors, and by others.

3200. With this arrangement of the magnetizing coils, a consequent-pole bipolar magnet is not possible; but by reversing one of the coils so that the two magnetomotive forces are opposite, two consequent poles will be formed on the magnet yokes *M* and *M*, Fig. 1190, at a point opposite the neutral spaces of the bipolar form; and by locating

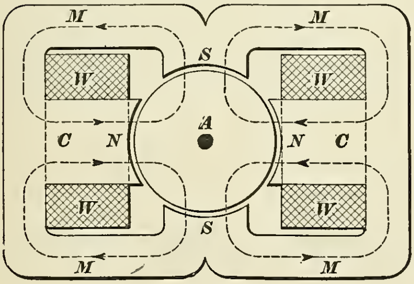


Fig. 1191.

suitable pole-pieces at these points, a four-pole magnet results, as represented in Fig. 1191. It will be seen that this magnet has one pair of salient poles *N* and *N*, and one pair of consequent poles *S* and *S*. This gives a very compact form of four-pole magnet, and is used in several types of railway motors, in the "Eddy" slow-speed stationary motors, and by other makers. The "Wenstrom" dynamos also employ a somewhat modified form of this type of field-magnet, the magnet yoke being barrel-shaped and completely enclosing the magnetizing coils and pole-pieces, spaces being left in the sides for the removal of the armature.

3201. By winding magnetizing coils around the consequent poles of the type of magnet illustrated in Fig. 1191, they become salient poles, giving still another type of field-magnet, illustrated in Fig. 1192. The same letters of reference are used in this figure as in the previous ones. This is a very useful form of field-magnet, and is that most generally used in this country for multipolar machines of any number of poles, almost every maker using it for multipolar generators and alternators.

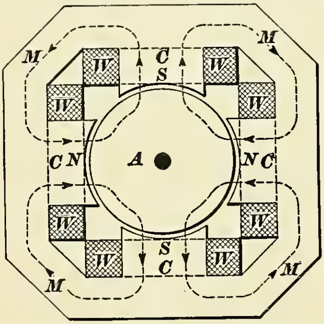


FIG. 1192.

The various magnet yokes form a complete ring, which is often, especially when six or more poles are used, made circular in outline. A modification of this form of magnet is used by the Siemens & Halske Company, in which the field cores project radially outward from a common hub, instead of inward, the armature revolving outside the poles of the magnet.

3202. The number of possible forms of field-magnets is very great, although they may all be classed as either salient or consequent pole magnets, or combinations of the two. Many of the forms of magnets which have been and are used seem to have been designed merely with a view to getting something different from any other maker, and considerations of economy of material or of mechanical fitness, which should prevail in the selection of a design, have been largely neglected. These forms described are the basis of the designs of field-magnets in modern construction.

METHODS OF EXCITING THE FIELD.

3203. The requisite number of ampere-turns for exciting the field of a dynamo-electric machine may be obtained in a variety of ways. In the first place, the current which flows through the magnetizing coils may come either from some separate external source, the machine being then said to be **separately excited,** or it may be furnished by the armature of the machine itself, it being then said to be **self-excited.** In some cases a combination of separate and self-excitation may be used. A diagram illustrating separate excitation is given in Fig. 1193. The current required is in this case supplied by the primary or secondary battery *B*, although another dynamo may be used, if desired. In order to adjust the current in the magnetizing coils to the proper value, or to vary it if necessary, an adjustable resistance *r* is included in the field circuit.

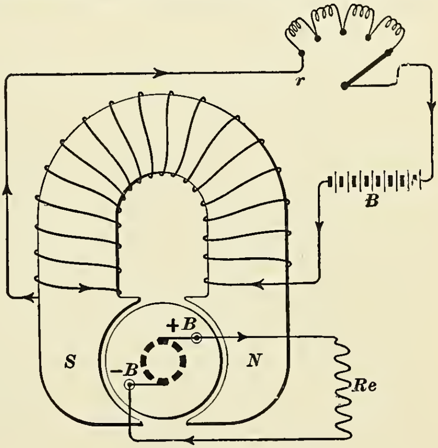


FIG. 1193.

The armature has no connection whatever with the field circuit, but supplies the external circuit, *Re*, directly.

3204. It is evident that with self-excitation a small or a large current may be used in the magnetizing coils, according to the nature of the source of the current, a large or a small number of turns being used in the magnetizing coils to give the necessary magnetizing force.

Alternators are usually separately excited, since the current given out by the machine, being alternating, can not be used directly for the purpose. Separate excitation has also the advantage that variations of the output of the armature of the machine, caused by changes in the speed or of the current, do not directly affect the field excitation.

SERIES WINDING.

3205. There are three general methods by which self-excitation is accomplished. In the first, the whole of the current flowing through the armature also flows through the magnetizing coils; such a machine is said to be **series wound,** from the fact that the armature and magnetizing coils are connected in series. This arrangement is represented in the diagram shown in Fig. 1194.

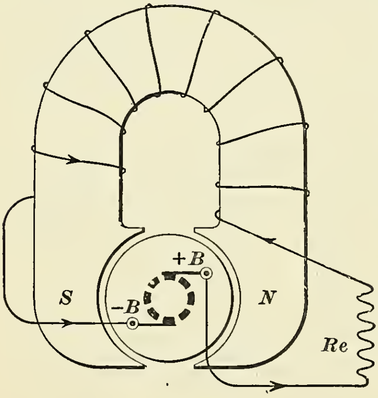


FIG. 1194.

With this arrangement, the magnetizing force acting on the magnetic circuit, consequently the number of lines of force in the magnet, varies with the current which the machine furnishes to the external circuit; therefore, when the armature is running at a constant speed, the E. M. F. which is generated in it varies as the current varies, though not necessarily in the same proportion. This is not usually desirable, since most applications of direct current require that either the E.M.F. or the current be maintained approximately constant.

3206. To realize either of the above conditions in a series-wound dynamo, it is necessary to adopt some method of regulation, whereby either the effect of variations in the current on the magnetizing force of the field may be neutralized or the *effective* E. M. F. of the armature may be altered to suit the conditions. The former result may be obtained by placing an adjustable resistance in parallel with the magnetizing coil, as represented in Fig. 1195. In this diagram, *S F* represents the magnetizing coil, or series field, and *R* is the adjustable resistance, connected in parallel with the magnetizing coil, as described. It will be seen that the current divides between the two branches of this part of the circuit, and by varying the resistance *R* the proportion of the whole current that flows through the magnetizing coil *S F* may be varied as required.

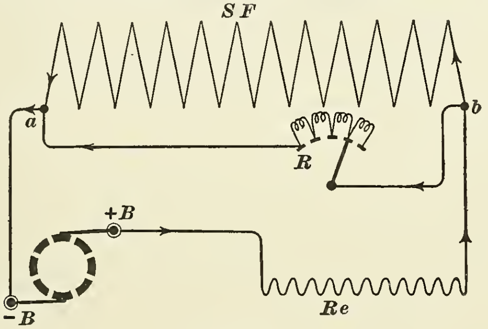


FIG. 1195.

The method of varying the effective E. M. F. that is used in the Thomson-Houston open-coil armature has already been described (Art. **3086**). Another method of accomplishing the same result with closed-coil armatures is to shift the brushes away from the neutral point, which entails special construction and precautions against destructive sparking, etc.

3207. Series winding is very little employed in dynamos, except for machines designed to give a constant current, such as is used for operating lamps or other devices that are connected in series. For motors, however, series winding is very useful, since when starting up under heavy load, or whenever taking a current in excess of the normal amount, the field strength is increased, which increases the amount of the reaction between the armature winding and the field, that is, increases the turning force of the armature.

SHUNT WINDING.

3208. The second method of self-excitation consists of forming a separate circuit of the magnetizing coils, which are connected directly between the brushes, or in shunt to the external circuit, this style of winding being, therefore, known as **shunt winding.** This is illustrated in Fig. 1196. It will be seen that the magnetizing-coil circuit is in a measure independent of the external circuit (*Re*), it being exposed at all times to the full difference of potential that exists between the brushes ($+B$ and $-B$); from this it follows that changes in the current flowing in the external circuit do not affect the magnetizing force acting on the field, except as they may change the difference of potential between the brushes. Changes in the current of the external circuit do affect this quantity in several ways, namely, by varying the drop due to the resistance of the armature winding, by varying the counter magnetomotive force of the armature winding, and by varying the length of the path of the lines of force by the variations in the amount by which they are distorted by the cross magneto-

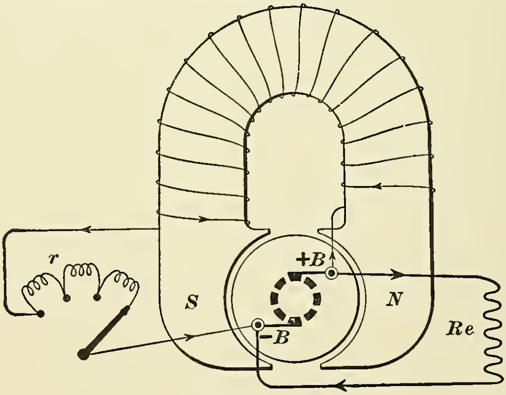


FIG. 1196.

motive force. (See Arts. **3115** and **3118.**) This last is comparatively unimportant, but the other two require careful consideration in the design of dynamo machinery, as will be pointed out.

3209. In a shunt-wound motor the conditions are different, the magnetizing-coil circuit being supplied directly from the mains; the magnetomotive force then depends simply upon the difference of potential between the supply mains, which is usually kept constant, so that in general a shunt-wound motor may be considered to have a constant magnetizing force acting on its field-magnet.

COMPOUND WINDING.

3210. From the above statements it will be seen that in order to maintain a constant difference of potential between the brushes of a dynamo (assuming a constant speed), the magnetomotive force of the magnetizing coils must be increased as the current increases, both to increase the number of lines of force so as to increase the E. M. F. generated, and to make up for the counter magnetomotive force of the armature winding. One way to accomplish this result is to place an adjustable resistance (*r*, Fig. 1196) in the magnetizing-coil circuit, which may be gradually cut out as the current output increases, thus reducing the resistance of the magnetizing-coil circuit, and increasing thereby the current flowing through it. This, however, requires personal attention, automatic devices for varying the resistance not being satisfactory, and in case the current from the dynamo fluctuates rapidly, it is difficult to operate the resistance with sufficient rapidity. Since the amount by which the mag-

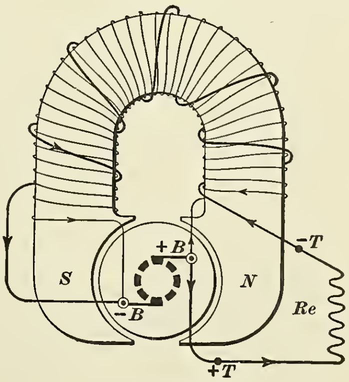


FIG. 1197.

netomotive force of the magnetizing coils must be varied is closely proportional to the current flowing, which follows from the nature of the causes which require the variation, it is possible to obtain the required variation by providing additional magnetizing coils through which the main current passes. This is known as **compound winding,** and is illustrated in Fig. 1197.

3211. It is evident that this is a combination of series and shunt winding, the shunt winding furnishing a constant magnetizing force and the series winding an additional magnetizing force which is proportional to the current output of the machine. This latter winding is so proportioned that it furnishes the proper increase in the magnetomotive force, as the current increases, to make up for the dropping off of the difference of potential between the brushes that would otherwise occur. For certain classes of work, a little more than this amount is provided, so that the difference of potential between the brushes rises slightly as the current output increases. In such a case the machine is said to be **over-compounded.**

3212. Compound winding is seldom used for motors, as either a series or a shunt winding serves for almost all conditions of operation. Nevertheless, for application to such machinery as printing-presses, a compound winding is extremely useful, as the series turns produce a powerful field at starting and at slow speed, and they may gradually be cut out or connected in various combinations to produce different working speeds without the necessity of inserting an external resistance in the armature circuit, except for starting up, when a resistance may be temporarily used.

BUILDING UP THE FIELD.

3213. Any iron, after being magnetized, retains a certain amount of residual magnetism, so that there will be a small E. M. F. generated in the armature winding when the armature is rotated and the field circuit left open; this is utilized to start the current in the magnetizing coils. In the case of a shunt-wound dynamo, when the machine is started and the magnetizing-coil circuit closed, the small E. M. F. generated in the armature by the residual magnetism sends a small current through the magnetizing coils, producing a small magnetizing force. If this magnetizing force tends to send lines of force through the magnetic circuit in the *same* direction as the residual magnetism, the number of lines of force will be increased; this will increase the E. M. F., which increases the current in the magnetizing coils, and still further increases the number of lines of force and the E. M. F., which process continues until further increase in the magnetizing force results in so little increase in the number of lines of force that the E. M. F. generated becomes steady, the windings being so designed that this shall be the E. M. F. at which it is desired to run the machine.

It will be seen that if the external circuit is open, all of the current that the E. M. F. (due to the residual magnetism) produces flows through the magnetizing coils; if, however, the external circuit is closed, only a part of the current flows through the magnetizing coils, so that the field will "build up" more slowly than with the external circuit open, and, in fact, will not build up at all if the external resistance is low as compared with the armature resistance. From this it follows that a shunt-wound machine should be started up with its external circuit open.

A series-wound machine, on the contrary, must have its external circuit closed in order that any current may flow through the magnetizing coils, and the lower the resistance of the external circuit the more quickly will the machine build up.

From the above it will be seen that a compound-wound dynamo may be started with its external circuit either open or closed, since it has both series and shunt wound coils. Usually, however, such machines are started and brought to their full E. M. F. with the external circuit open.

3214. At starting, while the current is increasing in the magnetizing-coil circuit, the inductance of the magnetizing coils increases its apparent resistance, and a part of the energy supplied to the coils is stored up in the magnetic field which is being established. As soon as the current in the magnetizing coils reaches its maximum value, however, and so long as it remains constant at this value, there is no reactance present, and the entire amount of energy delivered to the coils is expended in heating the wire; that is, it requires (directly) no energy to *maintain* a magnetic field at a constant value, the field depending on the *ampere-turns* that are acting on the magnetic circuit. It is obvious, however, that in order to force the current through the wire of which the magnetizing coil is composed, energy must be expended, but this energy appears *entirely* as heat, and, consequently, is wasted as far as any practical application of it is concerned. The number of watts expended in sending the current through the magnetizing coils should, therefore, be made as small as the design of the machine will permit, both to prevent any excessive waste of energy and to prevent possible damage by the heat liberated. In practice, the loss of energy from this cause varies from about 2 per cent. of the total output of the machine in larger sizes, to 5 or more per cent. in the smaller.

3215. In shunt-wound machines the magnetizing coils are exposed to the full difference of potential that exists between the brushes of the machine, and, consequently, should use only a small amount of current in order that the loss in watts should be the required small percentage of the output. From this it follows that the wire used for the magnetizing coils should be of small size and of considerable length, making a large number of turns around the magnets, both to give the necessary resistance to keep the current at its proper value and to allow of this small current furnishing the requisite number of ampere-turns. In series-wound machines, however, as the total current flowing gives the magnetizing force, the magnetizing coils need to have comparatively few turns, which should be of correspondingly large wire, in order that the watts loss (which is equal to $C^2 R$) should be kept within the desired limits.

It will be seen that in series-wound dynamos the difference of potential between the *terminals* of the machine is less than that which appears between the brushes by the amount of the drop in the magnetizing coils.

The above remarks concerning the magnetizing coils of shunt and series wound dynamos also apply to those of compound-wound machines, since they are made up of a shunt and a series winding.

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